

Off-equilibrium dynamics of a four-dimensional spin glass with asymmetric couplings

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Abstract. We study the off-equilibrium dynamics of the Edwards–Anderson spin glass in four dimensions under the influence of a non-Hamiltonian perturbation (i.e. with asymmetric couplings). We find that for small asymmetry the model behaves like the Hamiltonian model, while for large asymmetry the behaviour of the model can be well described by an interrupted ageing scenario. The autocorrelation function $C(t_w + \tau, t_w)$ scales as τ/t_w^β , with β a function of the asymmetry. For very long waiting times the previous regime crosses over to a time translational invariant regime with stretched exponential relaxation. The model does not show signs of reaching a time translational invariant regime for weak asymmetry, but in the ageing regime the exponent β is always different from 1, showing a non-trivial ageing scenario also valid in the Hamiltonian model.

1. Introduction

The study of spin models with asymmetric couplings began in the 1980s in connection with neural network models. It is known that the synapses (the connections between neurons) are not symmetrical. In order to analyse an asymmetric network one is forced to go beyond equilibrium statistical mechanics, since the system cannot be described by a Hamiltonian: a purely dynamical analysis is, in general, a very complex task. The first analyses in this direction were made in [1]. Within the framework of a dynamical mean-field theory, first developed in the case of spin glasses [2], it was possible to study analytically a very simplified asymmetric model with a spherical constraint. The authors reached the very interesting conclusion that the spin-glass phase is suppressed by an arbitrary small amount of asymmetry. This property was considered relevant in the neural network community, as in this context the spin-glass phase corresponds to spurious states that deteriorate the memory properties of the network. Alternatively, without a spin-glass phase, the dynamical properties of asymmetric networks seemed to be trivial and the interest in such systems diminished. Recently we have improved our understanding of the dynamics of glassy systems, in particular of the so-called *ageing phenomena* (see for example [3]). It is now clear that the mean-field dynamics of spin glasses has to be modified in order to account for the fact that spin glasses are out of equilibrium. This implies, for example, that the dynamics is not time translational invariant and as a consequence the classic fluctuation–dissipation theorem is no longer valid.

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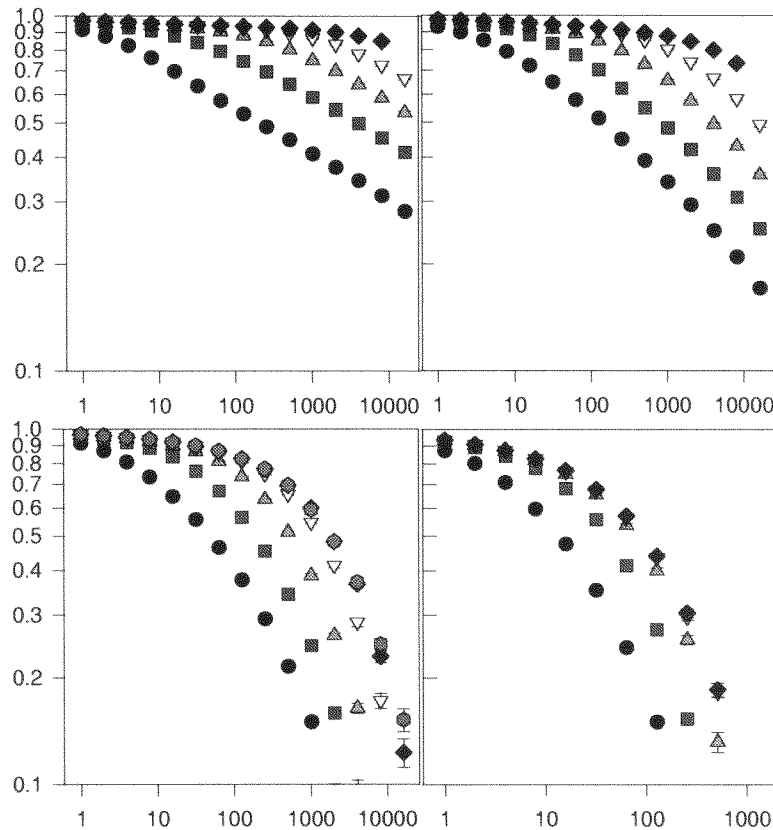


Figure 1. Autocorrelation functions for $T = 0.5$, asymmetries $\epsilon = 0.1, 0.2, 0.3$ and 0.4 and different waiting times (see text).

In view of asymmetric systems, one of the main questions is to what extent the asymmetry affects the ageing scenario. The conclusions of [1] appeared to indicate that a system with asymmetry will not age (there is not a spin-glass like behaviour).

The work of the following years has mainly been concerned with extreme situations such as the fully asymmetric model and the $T = 0$ case (see, for example, [4] and references therein). Recently the dynamical and static properties of the Sherrington–Kirkpatrick (SK) model with asymmetric couplings have been studied in [5]. It has been found that the off-equilibrium dynamics of the model is definitely dissimilar to that predicted by the spherical approximation. An ageing scenario indistinguishable from the one of the Hamiltonian mean-field model is clearly present for a wide range of small asymmetries.

In another recent paper [6], from an analysis of the off-equilibrium dynamics of an asymmetric p -spin spherical model, the authors find that, although for random initial conditions ageing is destroyed, by carefully tuning the initial conditions one is able to find regions where the system ages forever, never reaching equilibrium.

These findings leave open the discussion about the nature of the dynamics of asymmetric networks. Since almost all the known analytic properties of asymmetric networks are based on mean-field spherical models, one has to be careful in extrapolating the conclusions to other, more realistic, models.

In this paper we discuss the issue of whether the overall behaviour seen in mean-field models survives in systems with finite-range interactions. We have carried out computer

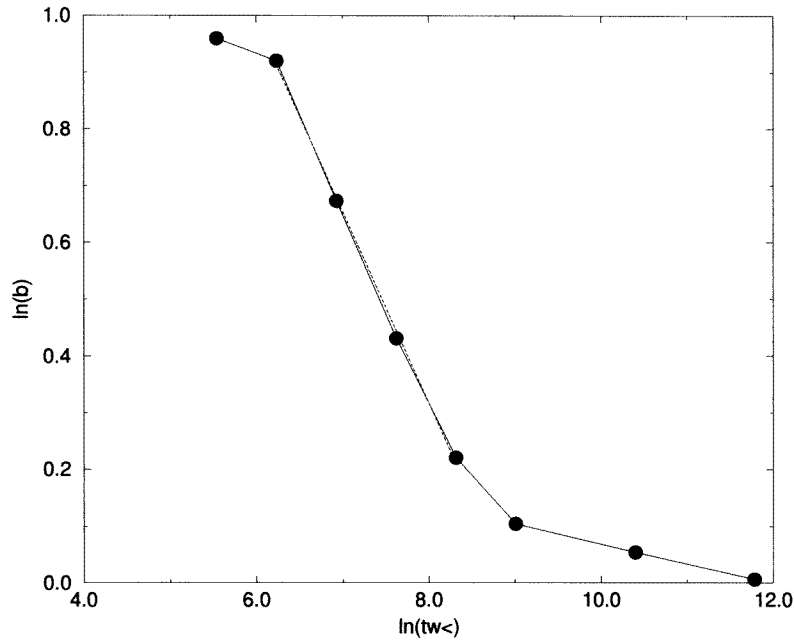


Figure 2. Data collapse of the autocorrelation for $\epsilon = 0.3$ (see text).

simulations of the four-dimensional Edwards–Anderson (EA) model with asymmetric interactions. By careful analysis of the off-equilibrium dynamics we are able to present a clear picture of what happens in finite dimensions. The choice of $D = 4$ is because the spin-glass transition in the symmetric EA model is clearly established while the more realistic case of $D = 3$ is still a matter of (little) debate [7].

Our simulations of the $D = 4$ model clearly show that the autocorrelation functions behave according to a typical ageing pattern[†], for several values of the asymmetry. For strong asymmetries ageing is interrupted and time translational invariance (TTI) is restored after a transient timescale. The scaling in the ageing region is not a simple one. We note that when assuming the form $C(t_w + \tau, t_w) = \tilde{C}(\tau/t_w^\beta)$, with the exponent β depending on the degree of asymmetry (and probably on the temperature), one obtains very good fits. For long enough waiting times in the case of strong asymmetry the dynamics becomes stationary (TTI) and the decay can be well fitted by a stretched exponential. For weak asymmetry the systems cannot reach the stationary regimes for the timescales simulated but the ageing scenario observed is clearly not a simple one (i.e. with scaling τ/t_w): in this case we find a behaviour that cannot be distinguished from that of the Hamiltonian model [5].

2. The model

Our model system is defined by the EA spin-glass Hamiltonian:

$$H = - \sum_{\langle i,j \rangle}^N s_i J_{ij} s_j \quad (1)$$

[†] It is now clear that ageing for the SK model is not simple ageing, but a more complex behaviour [8]. This is what we mean herein by *typical ageing*.

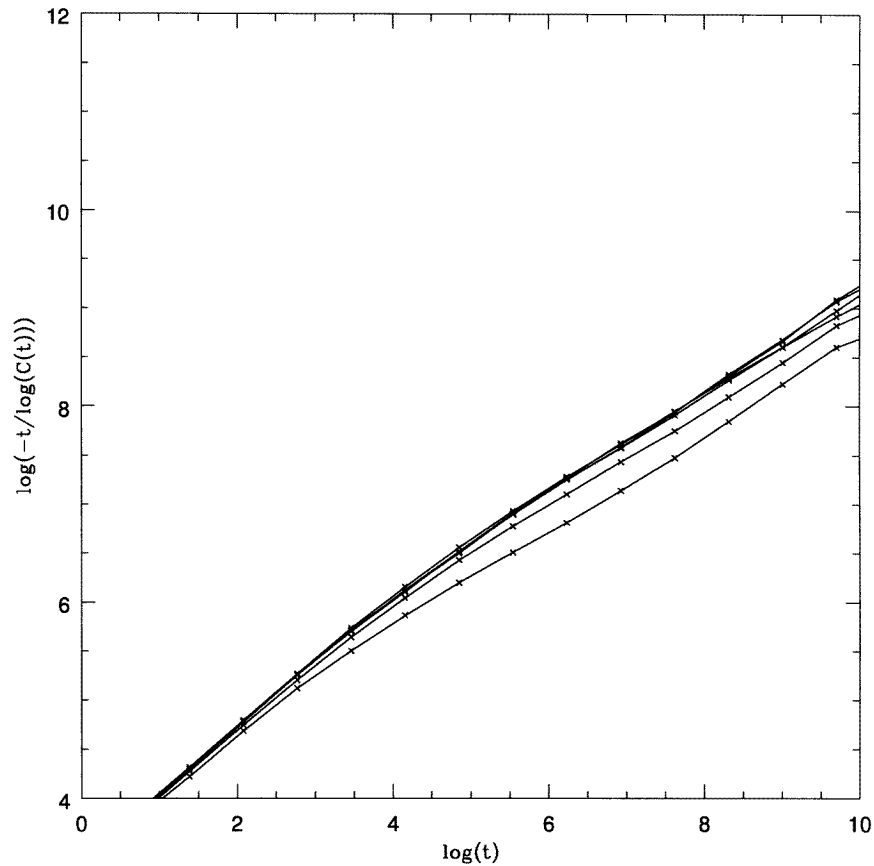


Figure 3. Stretched exponential behaviour of the autocorrelations for $\epsilon = 0.3$ and waiting times $t_w = 2^9, 2^{11} \dots 2^{19}$ from bottom to top.

where $\{s_i = \pm 1, i = 1 \dots N\}$ are N Ising spins and $\langle i, j \rangle$ denotes a sum over nearest neighbours. The couplings J_{ij} are chosen as in [5]. They are a weighted sum of a symmetric and a completely asymmetric part:

$$J_{ij} = \frac{1}{\sqrt{1 - 2\epsilon + 2\epsilon^2}} [(1 - \epsilon)J_{ij}^{(S)} + \epsilon J_{ij}^{(NS)}]. \quad (2)$$

The symmetric part of the interaction is given by the symmetric couplings $J_{ij}^{(S)} = J_{ji}^{(S)} = \pm 1$ with probability 0.5. The coupling $J_{ij}^{(NS)}$ gives the non-symmetric part and it is drawn independently of $J_{ji}^{(NS)}$. Finally, ϵ measures the strength of the non-symmetric part.

For the simulations we have chosen a standard heat-bath dynamics and, starting from a random initial condition, we let the system evolve at temperature T during a waiting time t_w . Then we measure the autocorrelation function at times $t = t_w + \tau$:

$$C(t_w + \tau, t_w) = \frac{1}{N} \sum_{i=1}^N \overline{\langle s_i(t_w) s_i(t_w + \tau) \rangle}. \quad (3)$$

As usual, the overline means a disorder average while the brackets mean a thermal average. We measure the autocorrelation for several values of the waiting time, of the form times 2^n .

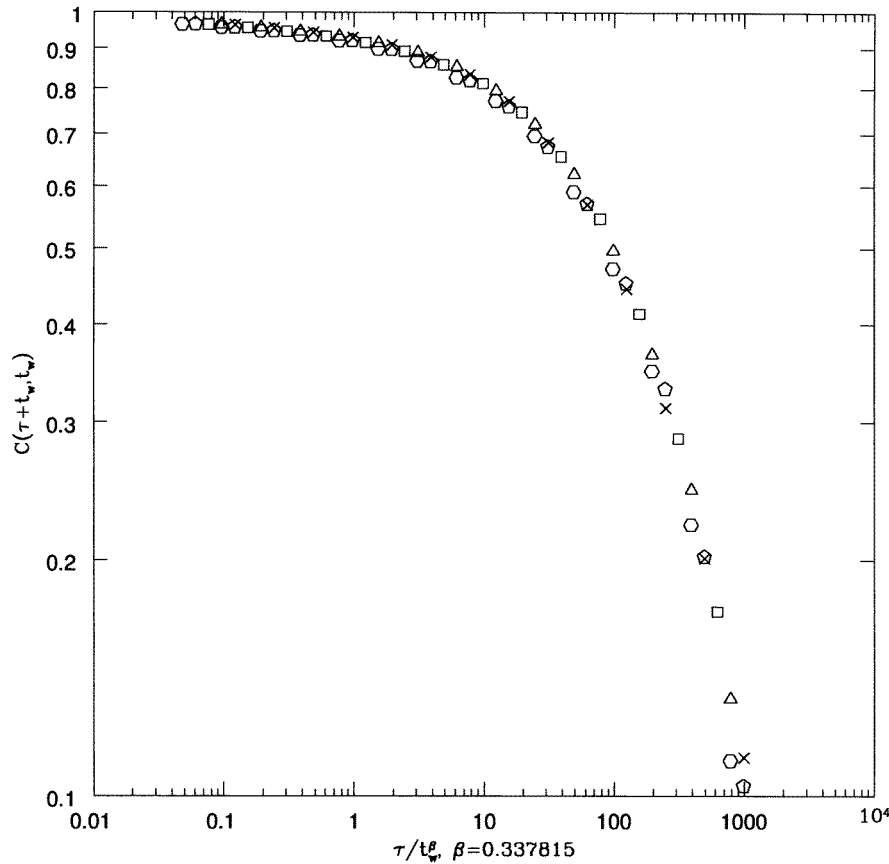


Figure 4. Scaling plot of the autocorrelation function in the ageing regime for $\epsilon = 0.3$.

Hereafter we will describe the results obtained for a lattice of linear dimension $L = 7$. We have also done simulations for $L = 5$ and $L = 6$ and confirmed that finite-size effects do not play an important role in our conclusions. Recalling that the critical temperature of the symmetric model with $\pm J$ interactions is approximately $T_c \approx 2$, the simulations were carried out at a fixed temperature $T = 0.5$.

Before presenting our results let us briefly describe, for reference purposes, the behaviour found in the model with $\epsilon = 0$, i.e. the Hamiltonian symmetric case, whose dynamic properties (for a Gaussian distribution of the couplings) were studied in [9]. The authors found that the four-dimensional spin glass in the low-temperature phase presents a typical ageing behaviour that can be well characterized by the following scaling of the autocorrelation:

$$C(t_w + \tau, t_w) = \tau^{-x(T)} \tilde{C}(\tau/t_w) \tag{4}$$

with a scaling function

$$\tilde{C}(z) = \begin{cases} \text{constant} & \text{for } z \rightarrow 0 \\ z^{x(T)-\lambda(T)} & \text{for } z \rightarrow \infty. \end{cases} \tag{5}$$

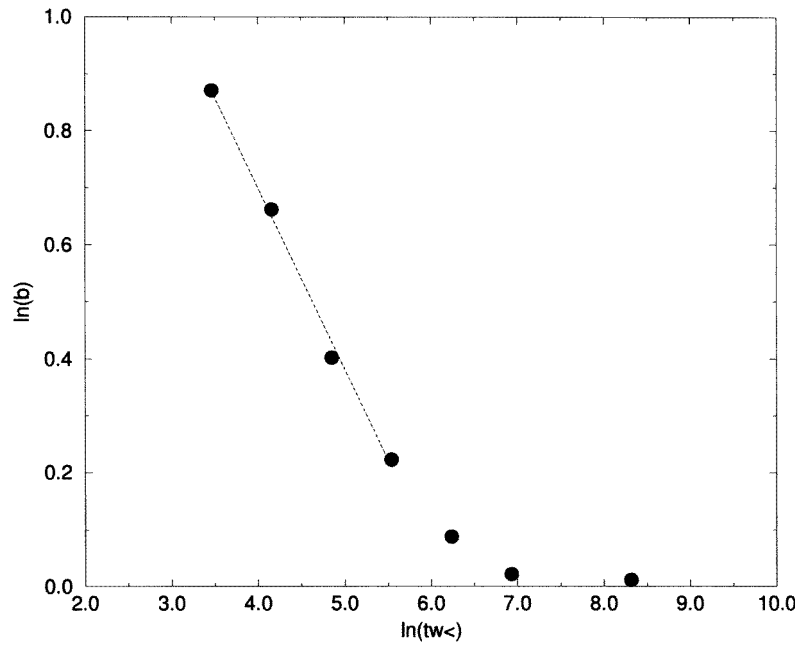


Figure 5. Data collapse of the autocorrelation for $\epsilon = 0.4$ (see text).

In the ‘quasi-equilibrium’ regime where $z \rightarrow 0$ one can write that $\lim_{t \rightarrow \infty} \lim_{t_w \rightarrow \infty} C(t, t_w) = q_{EA}$, where q_{EA} is the EA order parameter. In the other regime there is a faster decay of the autocorrelation toward zero, i.e. $C \approx t^{-\lambda}$ with $\lambda(T) \gg x(T) \forall T$.

Hereafter we will discuss how this picture is modified when an asymmetric perturbation is introduced into the system.

3. Results

In figure 1 we show a log–log plot of the autocorrelation function $C(t_w + \tau, t_w)$ versus τ , for several values of the asymmetry $\epsilon = 0.1, 0.2, 0.3$ and 0.4 . For each case we plot the curves for $t_w = 2^n$ with $n = 2, 5, 8, 11, 14, \dots$. As the asymmetry grows we can readily see a departure from the two regimes scenario which is valid in the symmetric model. For the cases with larger asymmetry the scaling of the symmetric model is no longer valid and, at least for $\epsilon > 0.2$, an interrupted ageing scenario is evident: for large t_w correlation functions no longer change. The situation is more subtle for smaller values of ϵ and, for the timescales reached in this simulations, this effect is not detectable: here we find that on our timescales the *typical ageing scenario* persists. The overall picture of the high asymmetry case is reminiscent of the one found for the two-dimensional spin glass [10], where there is no spin-glass phase but at low temperatures one still observes interrupted ageing scenario.

In order to make quantitative predictions we ran longer simulations for the model with $\epsilon = 0.3$ increasing in time to $t_w = 2^{19}$. We then tried a data collapse in the following form: take the data for a particular t_w , that we have chosen for practical purposes, to be

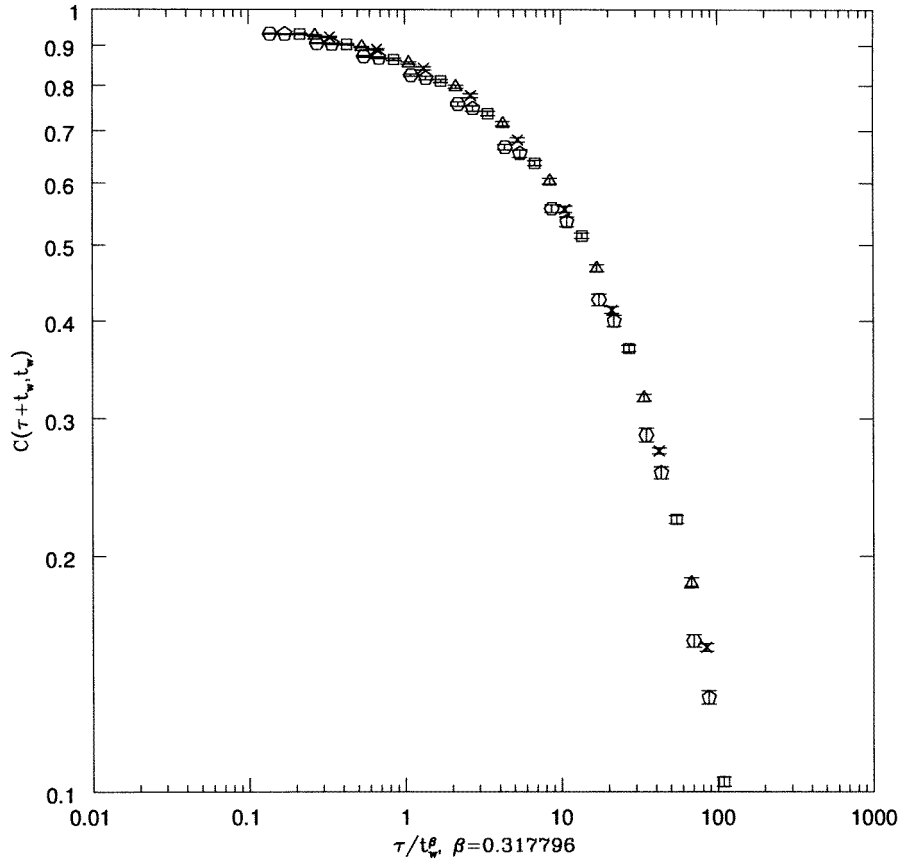


Figure 6. Scaling of the autocorrelation in the ageing regime for $\epsilon = 0.4$.

the largest one (2^{19}). Then for each other value of t_w we tried a transformation of the form $C(x) \rightarrow aC(bx)$, adjusting the parameters a and b to make the curves for the two waiting times collapse. We plot $\log b$ versus $\log t_w$ in figure 2. The result is very interesting. First we notice an intermediate region, for $2^9 \leq t_w \leq 2^{12}$, where the function is linear $b \propto t_w^\beta$. This suggests that in this region the autocorrelation scales are:

$$C(t_w + \tau, t_w) \sim \tilde{C}\left(\frac{\tau}{t_w^\beta}\right). \quad (6)$$

Performing a linear fit of these four points we obtain for the exponent $\beta = 0.34$. For waiting times greater than $t_w = 2^{13}$ the system enters another regime with a nearly constant value of $b \approx 1$. This means that for $t_w \approx 2^{15}$ the dynamics changes qualitatively and enters a time translational invariant regime, thus the ageing is interrupted. So there is a typical relaxation time t_w^{MAX} which signals the onset of a stationary regime. In order to test if this stationary regime presents a simple exponential relaxation we did a log-log plot of the function $-\tau/\log(C(t_w + \tau, t_w))$ versus τ for $t_w = 2^9, 2^{11} \dots 2^{19}$ as shown in figure 3. As expected the curves saturate in a limit curve for $t_w \geq 2^{15}$ but the relaxation turns out not to be a simple exponential. If this were the case the limiting curve would be constant, with zero

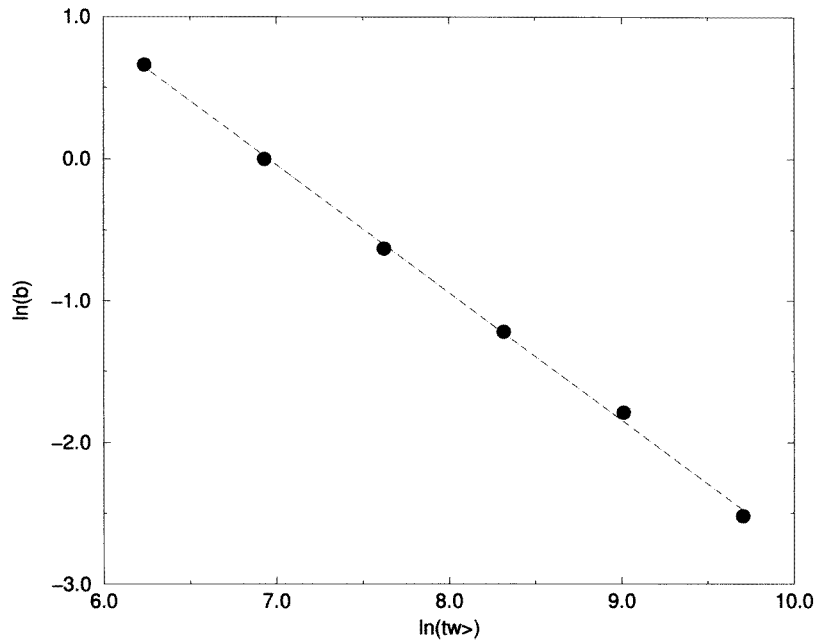


Figure 7. Data collapse of the autocorrelation for $\epsilon = 0.1$ (see text).

slope. The straight lines with finite slope observed are evidence of a stretched exponential relaxation,

$$C(\tau) = e^{\tau^\alpha/t^*} \quad (7)$$

where t^* is the characteristic time of the relaxation.

In figure 4 we show a scaling plot of the autocorrelation function in the ageing region. This figure has to be compared with figure 5 of [9] for the symmetric model. It is now clear that there are not two sharply defined time regimes and the scaling with τ/t_w^β is very good in the whole interval of the figure.

For growing asymmetry t_w^{MAX} is smaller, as expected. The ageing dynamics is interrupted earlier and a stationary dynamics dominates the scene. For $\epsilon = 0.4$ the proposed scaling of the autocorrelation still works well with an exponent $\beta = 0.32$ for $2^5 \leq t_w \leq 2^9$ as can be seen in figure 5. For $t_w \geq 2^{10}$ a stationary regime clearly sets in with a characteristic stretched exponential relaxation. The scaling in the ageing region is shown in figure 6.

Already for $\epsilon = 0.5$ the complete dynamics is no longer slow, and the ageing regime is completely suppressed. One could guess that the stretched exponential relaxation would also change to a simpler exponential one at some $\epsilon < 1$ but our results up to $\epsilon = 0.7$ still show the stretched exponential behaviour.

For small values of $\epsilon = 0.1$ and $\epsilon = 0.2$ the dynamics very much resembles that of the symmetric model, at least up to the timescales which we were able to simulate. For $\epsilon = 0.1$ the proposed scaling for the ageing regime works well in the whole time window from $2^9 \leq t_w \leq 2^{14}$ with an exponent $\beta = 0.90$, as one can see from the linear fit in figure 7. In figure 8 we show that in this case there are two well-defined regimes with a behaviour very similar to that of the symmetric case (see figure 5 of [9]). Our scaling works very well in the long time regime when $\tau \geq t_w^\beta$. The other

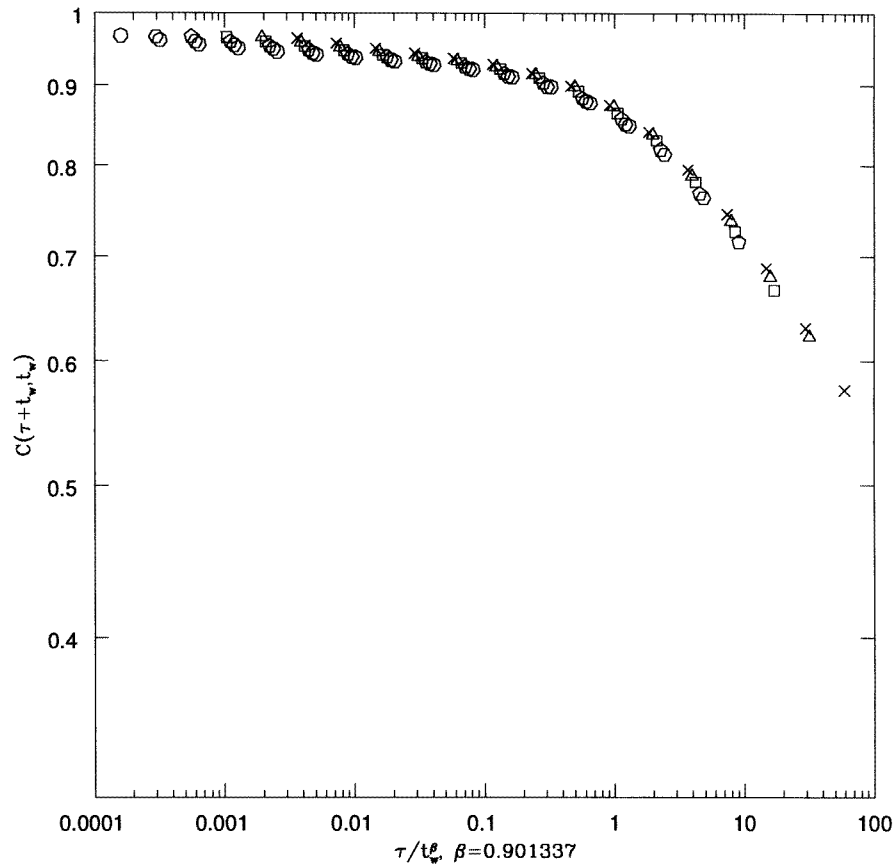


Figure 8. Scaling of the autocorrelation for $\epsilon = 0.1$ and $t_w = 2^9 \dots 2^{14}$.

corresponds to the quasiequilibrium one where the autocorrelation depends only on the time difference τ .

For $\epsilon = 0.2$ the division in two well-defined regimes begins to break down and the crossover region is larger (see figure 9). Nevertheless the ageing scaling works well in the whole time window explored and a good data collapse is obtained with an exponent $\beta = 0.72$. In our simulations, for these two values of $\epsilon = 0.1$ and 0.2 , it is very difficult to see a maximum waiting time which signals the interruption of the ageing. Here longer runs would be needed.

4. Conclusions

Finite-dimensional disordered systems with quenched asymmetric couplings behave very much like the Hamiltonian systems if the asymmetry is small. We have shown that two kinds of behaviour can be observed: for small asymmetry a *typical ageing*, and for large asymmetry an interrupted ageing. We have been able to qualify in good detail the large asymmetry phase, where reasonable correlation times make possible a good description of the dynamical behaviour. We have suggested that a stretched exponential behaviour

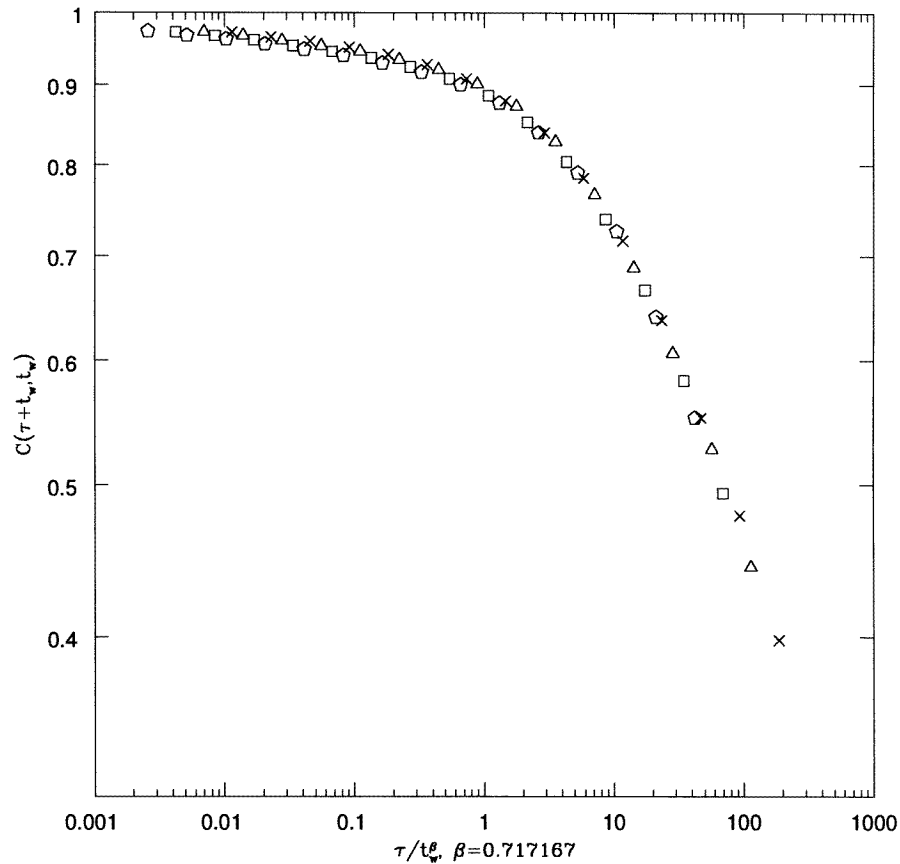


Figure 9. Scaling of the autocorrelation for $\epsilon = 0.2$ and $t_w = 2^8, 2^{10}, 2^{12}, 2^{14}$.

can be explicative of many observed features. A more detailed analysis is needed for a full understanding of the small asymmetry phase, where it is very difficult to observe any difference from the pure case.

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