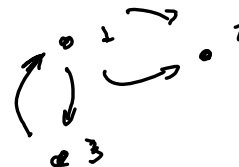


## CUIDADO COM A BASE

$$|P_k\rangle = |\psi_0\rangle \underbrace{\langle \phi_0 | P_0 \rangle} + \sum_{n=1}^{N-1} \lambda_n^L |\phi_n\rangle \langle \psi_n | P_0 \rangle$$

Condição inicial:  a base dos estados

$$\begin{aligned} |P_0\rangle &= P_0 |0\rangle + P_1 |1\rangle + \dots + P_N |N\rangle \\ &= \sum_n P_n |n\rangle = \sum_n \langle n | P_0 \rangle |n\rangle \\ &= \sum_n |n\rangle \langle n | P_0 \rangle \\ &\neq P_0 |\phi_0\rangle + P_1 |\phi_1\rangle + \dots + P_N |\phi_N\rangle \end{aligned}$$

Porque as bases são diferentes!

# Reversibilidade microscópica

Cadeia de Markov,  $T$ : matriz regular

No regime estacionário:

$$\vec{p}_\infty = \hat{T} \vec{p}_\infty \quad \xrightarrow{\text{componentes}}$$

$$p_\infty(n) = \sum_m T(n, m) p_\infty(m)$$

$$|p_\infty\rangle = \hat{T} |p_\infty\rangle$$


$$p_\infty(n) \times 1 = \sum_m T(n, m) p_\infty(m)$$

$$p_\infty(n) \sum_m T(n, m) = \sum_m T(n, m) p_\infty(m)$$

$$\sum_m T(n, m) p_\infty(m) = \sum_m T(n, m) p_\infty(m)$$

$$\boxed{\sum_m [T(m, n) P_\infty(n) - T(n, m) P_\infty(m)] = 0}$$

Vale para qualquer estado estacionário

→ Balanceamento GLOBAL.

→ Considere uma trajetória:  $n_0 \rightarrow n_1 \rightarrow \dots \rightarrow n_{t-1} \rightarrow n_t$

Probabilidade:

$$P(n_0, n_1, \dots, n_t) = T(n_t, n_{t-1}) \dots T(n_1, n_0) P(n_0)$$

→ Considere a trajetória inversa:  $n_t \rightarrow n_{t-1} \rightarrow \dots \rightarrow n_1 \rightarrow n_0$

$$\begin{aligned} P(n_t, n_{t-1}, \dots, n_1, n_0) &= T(n_0, n_1) T(n_1, n_2) \dots T(n_{t-1}, n_t) P(n_t) \\ &= \tilde{T}(n_t, n_{t-1}) \dots \tilde{T}(n_1, n_0) P(n_0) \end{aligned}$$

Basta que

$$\boxed{\tilde{T}(n, m) = T(m, n) \frac{P(n)}{P(m)}} \geq 0$$

porque todas as termos à direita são  $\geq 0$

$$(P(m) > 0)$$

→ trajetória inversa também é markoviana?

Ex:  $P(n_0, n_1) = T(n_1, n_0) P(n_0)$

$$\begin{aligned} P(n_1, n_0) &= T(n_0, n_1) P(n_1) \\ &= T(n_0, n_1) \frac{P(n_0) P(n_1)}{P(n_0)} \end{aligned}$$

$$= \left[ T(n_0, n_1) \frac{P(n_1)}{P(n_0)} \right] P(n_0) = \tilde{T}(n_1, n_0) P(n_0)$$

Vale  $\sum_n \tilde{T}(n, m) = 1$  ?

$$\sum_n \tilde{T}(n, m) = \sum_n T(m, n) \frac{P(n)}{P(m)} = \frac{1}{P(m)} \sum_n T(m, n) P(n)$$

$$= \frac{1}{P(m)} P(m) = 1 \quad \text{ok,}$$

Reversibilidade microscópica

• Se  $\tilde{T}(n, m) = T(n, m)$   
 def.  $\left\{ \begin{array}{l} T(m, n) \frac{P(n)}{P(m)} = T(n, m) \end{array} \right.$

$$\boxed{T(m, n) P(n) = T(n, m) P(m)} \quad (A)$$

↑ Transição entre 2 estados  $m$  e  $n$

→ Balanceamento DETALHADO

↳ Cada parcela do balanceamento global se anula.

- Matrizes estocásticas que possuem balanceamento DETALHADO têm todos os autovalores reais.

$$\text{Seja } \hat{T}(m, n) = \frac{1}{X(m)} T(m, n) X(n)$$

$$\text{onde } X(m) = \sqrt{P_{\infty}(m)}$$

$$\text{De } \textcircled{A} \quad T(m, n) P_{\infty}(n) = T(n, m) P_{\infty}(m)$$

$$\frac{1}{X(m)X(n)} T(m, n) P_{\infty}(n) = \frac{1}{X(m)X(n)} T(n, m) P_{\infty}(m)$$

$$\frac{X(n)}{X(m)} T(m, n) = \frac{X(m)}{X(n)} T(n, m)$$

$$\boxed{\hat{T}(m, n) = \hat{T}(n, m)}$$

↗ Simétrica  
(hermitiana,  
pois elementos são  
reais)

$$T_{\text{orig}} |\psi_k\rangle = \lambda_k |\psi_k\rangle$$

↳ Em componentes:

$$\sum_n T(m, n) \psi_k(n) = \lambda_k \psi_k(m)$$

$$\frac{1}{\chi(m)} \sum_n T(m, n) \psi_k(n) = \lambda_k \frac{\psi_k(m)}{\chi(m)}$$

$$\sum_n \left( \frac{1}{\chi(m)} T(m, n) \frac{\chi(n)}{\chi(n)} \right) \psi_k(n) = \lambda_k \frac{\psi_k(m)}{\chi(m)}$$

$$\sum_n \hat{T}(m, n) \frac{\psi_k(n)}{\chi(n)} = \lambda_k \underbrace{\begin{pmatrix} \frac{\psi_k(m)}{\chi(m)} \\ \frac{\psi_k(n)}{\chi(n)} \end{pmatrix}}_{\hat{T} \frac{d_k}{1}} \text{ Autovetor}$$



→ Os autovalores de  $\hat{T}$  são os  
mesmos de  $T$ , mas com autovetores  
distintos.