Movimento Browniano

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Histórico

Até o final do século XIX:

- Estudo de equações diferenciais determinísticas;
- modelagem de fenômenos naturais por estes tipos de equações

➛ Se todas as condições iniciais fossem conhecidas, o futuro poderia ser determinado!

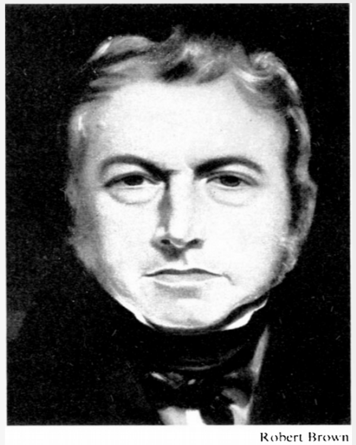
- Física nova com o advendo da Mecânica quântica
- Descoberta de caos, mesmo em mecânica clássica: pequenos erros nas condições iniciais são rapidamente amplificados

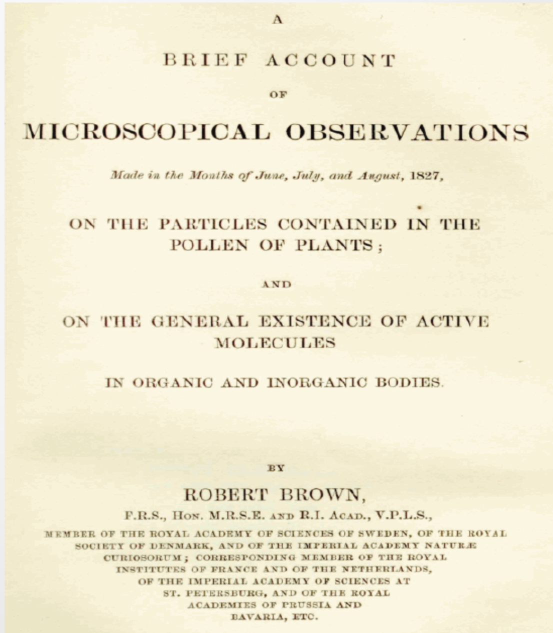
➛ Sugirmento de teoria fenomenológica de fenômenos flutuantes: modelos com predições limitada

- Histórico do Movimento Browniano
  - Brown, Einstein, Sutherland, Perrin, Langevin
  - Número de Avogadro
  - Equação de Langevin
  - Equação de Fürth
- Aplicação em migração celular
  - Dados experimentais
  - Eq. de Fürth modificada
  - Modelos estocásticos para migração

Movimento Browniano - Observação

- Robert Brown (1828), *“A brief account of microscopical observations made in the months of june, july, and august, 1827, on the* ***particles contained in the pollen of plants****; and on the general existence of active molecules in organic and inorganic bodies"* (partículas do pólen ~ 0.1 a 1μm, H_2O ~2.7 x 10^{-10} m)




A

BRIEF ACCOUNT

OF

MICROSCOPICAL OBSERVATIONS

Made in the Months of June, July, and August, 1827,

ON THE PARTICLES CONTAINED IN THE POLLEN OF PLANTS;

AND

ON THE GENERAL EXISTENCE OF ACTIVE MOLECULES

IN ORGANIC AND INORGANIC BODIES.

BY

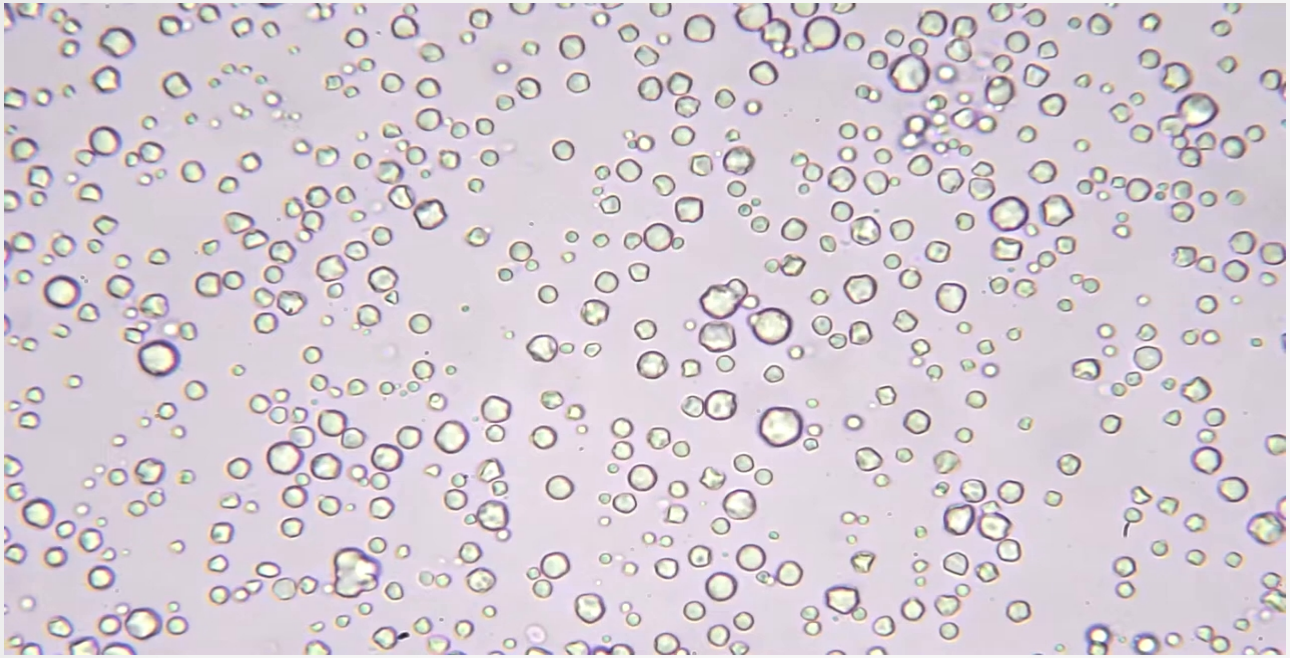
ROBERT BROWN,

F.R.S., HON. M.R.S.E. AND R.I. ACAD., V.P.L.S.,

MEMBER OF THE ROYAL ACADEMY OF SCIENCES OF SWEDEN, OF THE ROYAL SOCIETY OF DENMARK, AND OF THE IMPERIAL ACADEMY NATURÆ CURIOSORUM; CORRESPONDING MEMBER OF THE ROYAL INSTITUTES OF FRANCE AND OF THE NETHERLANDS, OF THE IMPERIAL ACADEMY OF SCIENCES AT ST. PETERSBURG, AND OF THE ROYAL ACADEMIES OF PRUSSIA AND BAVARIA, ETC.

https://www.youtube.com/watch?v=Xscn-QSmFo4

Movimento Browniano - Observação



Vídeo de Paul Baker: pequenos glóbulos de gordura de leite muito diluído

https://www.youtube.com/watch?v=NSe3TFLYcO8

Equipment : Brunel SP22 microscope. Sony Alpha 6000 camera attached via an adapter. 40x objective, 10x eyepiece.

Movimento Browniano - Teoria

Albert Einstein (1905)

- Evidência da existência de átomos/moléculas
- Estimativa do tamanho de átomos/moléculas
- Estimativa do número de Avogadro
- Evidência de flutuações microscópicas nas propriedades da matéria
- Caminhantes aleatórios na natureza

I

ON THE MOVEMENT OF SMALL PARTICLES SUSPENDED IN A STATIONARY LIQUID DEMANDED BY THE MOLECULAR-KINETIC THEORY OF HEAT

II

ON THE THEORY OF THE BROWNIAN MOVEMENT

(From the *Annalen der Physik* (4), **19**, 1906, pp. 371-381)

III

A NEW DETERMINATION OF MOLECULAR DIMENSIONS

(From the *Annalen der Physik* (4), **19**, 1906, pp. 289-306. Corrections, *ibid.*, **34**, 1911, pp. 591-592.) (23)

Determinação do número de Avogadro

§ 5. FORMULA FOR THE MEAN DISPLACEMENT OF SUSPENDED PARTICLES. A NEW METHOD OF DETERMINING THE REAL SIZE OF THE ATOM

In § 3 we found for the coefficient of diffusion D of a material suspended in a liquid in the form of small spheres of radius P—

$$D = \frac{RT}{N} \cdot \frac{1}{6\pi kP}.$$

Further, we found in § 4 for the mean value of the displacement of the particles in the direction of the X-axis in time t—

$$\lambda_x = \sqrt{2Dt}.$$

By eliminating D we obtain

$$\lambda_x = \sqrt{t} \cdot \sqrt{\frac{RT}{N} \frac{1}{3\pi kP}}.$$

This equation shows how λ_x depends on T, k, and P.

We will calculate how great λ_x is for one second, if N is taken equal to $6 \cdot 10^{23}$ in accordance with the kinetic theory of gases, water at 17° C. is chosen as the liquid ($k = 1{\cdot}35 \cdot 10^{-2}$), and the diameter of the particles ·001 mm. We get

$$\lambda_x = 8 \cdot 10^{-5} \text{ cm.} = 0{\cdot}8\mu.$$

The mean displacement in one minute would be, therefore, about 6μ.

On the other hand, the relation found can be used for the determination of N. We obtain

$$N = \frac{1}{\lambda_x^2} \cdot \frac{RT}{3\pi kP}.$$

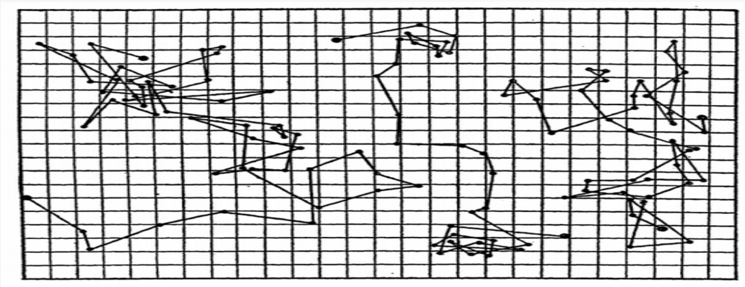
It is to be hoped that some enquirer may succeed shortly in solving the problem suggested here, which is so important in connection with the theory of Heat. (13)

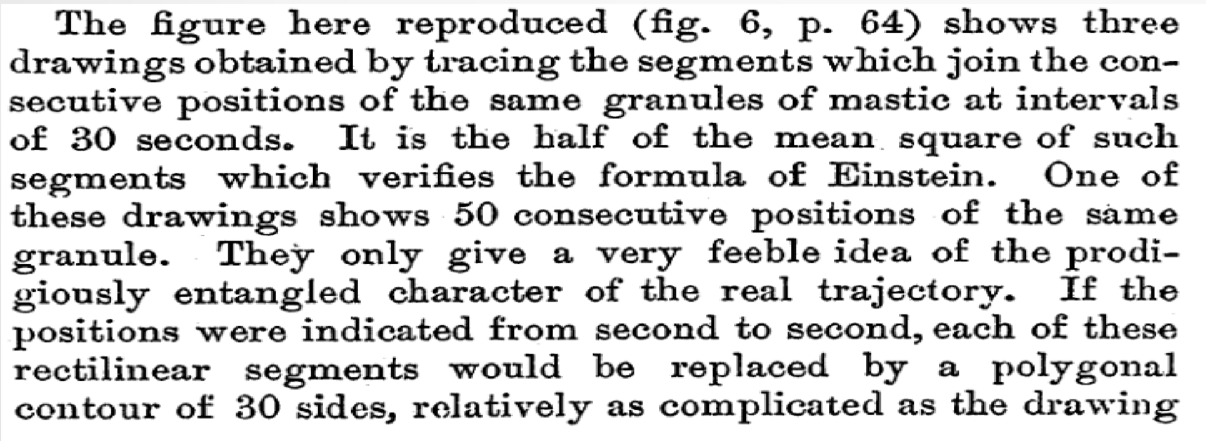
Berne, *May*, 1905.

Movimento Browniano - Experimento

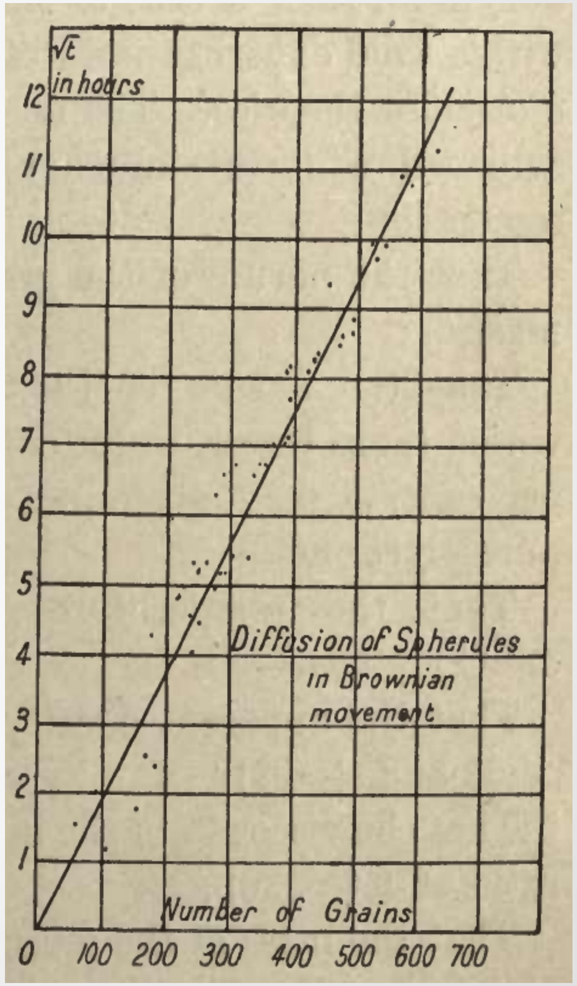
Jean Baptiste Perrin (1908) 

The Nobel Prize in Physics 1926 was awarded to Jean Baptiste Perrin "for his work on the discontinuous structure of matter, and especially for his discovery of sedimentation equilibrium".





The figure here reproduced (fig. 6, p. 64) shows three drawings obtained by tracing the segments which join the consecutive positions of the same granules of mastic at intervals of 30 seconds. It is the half of the mean square of such segments which verifies the formula of Einstein. One of these drawings shows 50 consecutive positions of the same granule. They only give a very feeble idea of the prodigiously entangled character of the real trajectory. If the positions were indicated from second to second, each of these rectilinear segments would be replaced by a polygonal contour of 30 sides, relatively as complicated as the drawing



Livros: Brownian movement and molecular reality, 1909 / The Atoms, 1913, J. B. Perrin

William Sutherland

LXXV. *A Dynamical Theory of Diffusion for Non-Electrolytes and the Molecular Mass of Albumin.* By WILLIAM SUTHERLAND ‡.

IN a paper communicated to the Australian Association for the Advancement of Science at Dunedin, 1904, on the Measurement of Large Molecular Masses, a purely dynamical theory of diffusion was outlined, with the aim of getting a formula for calculating from the data of diffusion those large molecular masses for which the ordinary methods fail. The formula obtained made the velocity of diffusion of a substance through a liquid vary inversely as the radius a of its molecule and inversely as the viscosity of the liquid. On

$$D = \frac{RT}{6\pi\eta a C}\,\frac{1+3\eta/\beta a}{1+2\eta/\beta a}$$

C: número de Avogadro

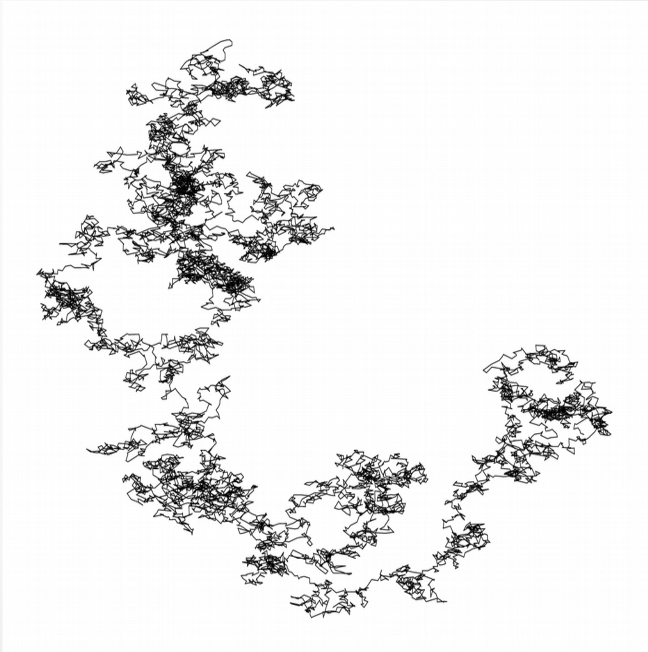
Phil. Mag. S. 6. Vol. 9. No. 54. *June* 1905.

Equação de Langevin

Paul Langevin (1908), *"Sur la théorie du mouvement brownien", C. R. Acad. Sci. (Paris) 146, 530 (1908)*

$$\frac{d}{dt}\vec{v} = -\frac{1}{P}\vec{v} + \frac{\sqrt{4D}}{P}\vec{\eta}(t)$$

- $\vec{v}$: velocidade
- P^{-1} : viscosidade
- D : coeficiente de difusão
- η : ruído branco estacionário



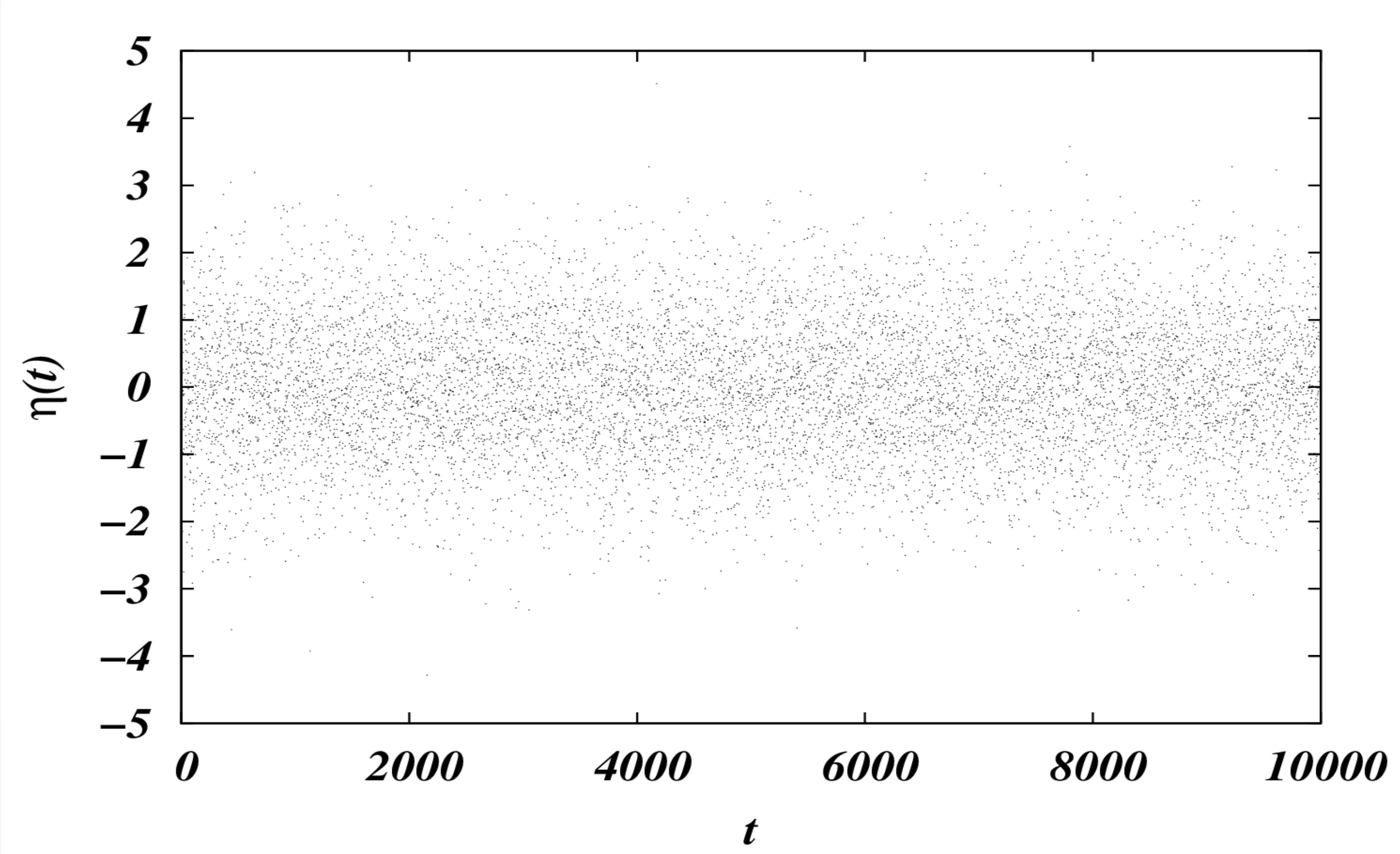
Langevin 1D

$$\frac{d}{dt}v = -\frac{1}{P}v + \frac{\sqrt{2D}}{P}\eta(t)$$

- v : velocidade
- P^{-1} : viscosidade
- D : coeficiente de difusão
- η : ruído branco gaussiano estacionário

Com essas condições do ruído, v é um processo de *Ornstein-Uhlenbeck*

Ruído branco e gaussiano



Ruído branco não significa necessariamente GAUSSIANO!

Ruído branco – autocorrelação

Para uma variável discreta:

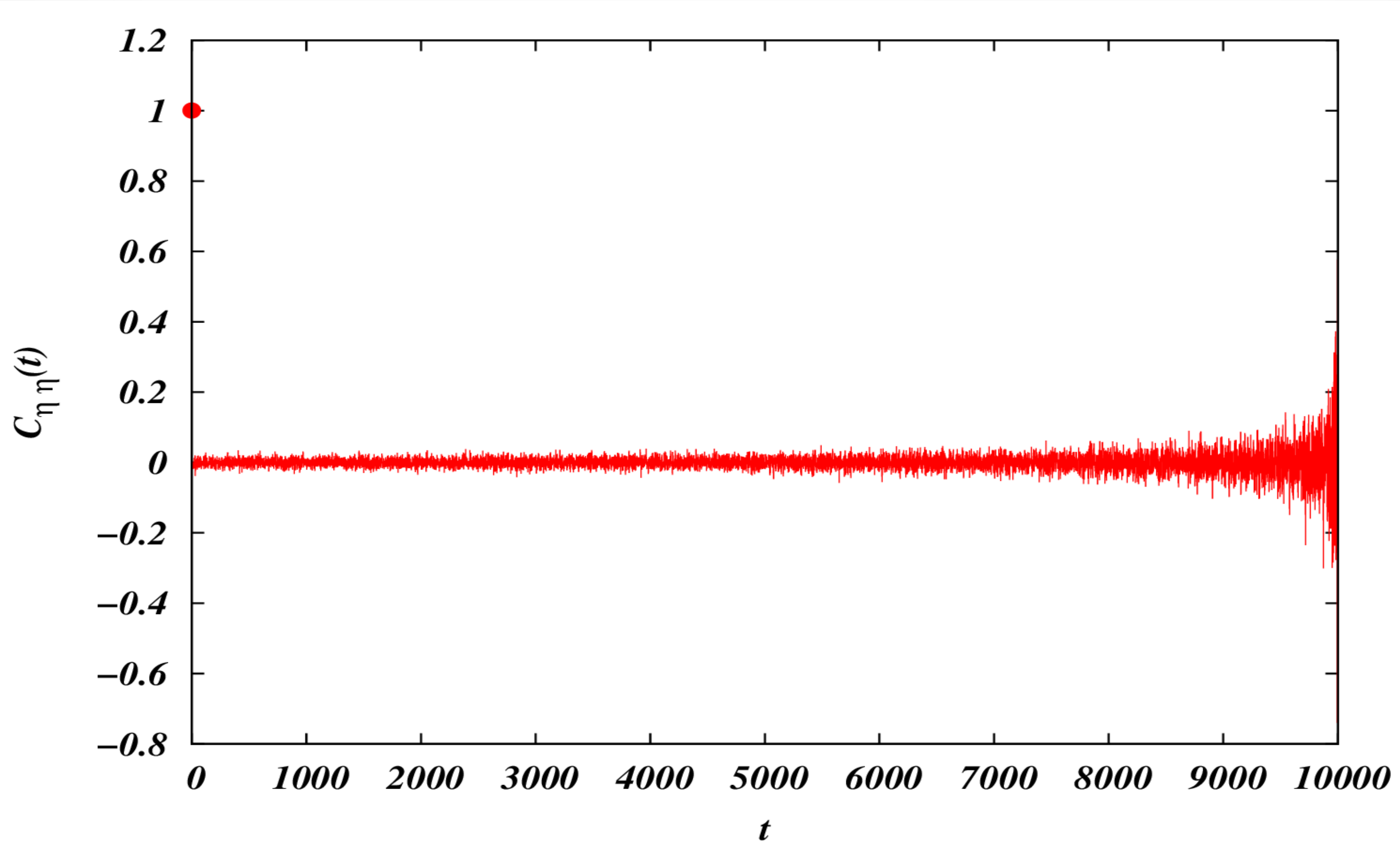
$$C_{\eta\eta}(\Delta t) = \langle \eta(t)\eta(t+\Delta t)\rangle = \frac{1}{N}\sum_{i=0}^{N} \eta(t_i)\eta(t_i+\Delta t)$$

Ruído branco estacionário:

$$\langle \eta(t)\eta(t+\Delta t)\rangle = \langle \eta(t)\eta(0)\rangle = \delta(t)$$

Ruído branco – autocorrelação

$$\langle \eta(t)\eta(t+\Delta t)\rangle = \langle \eta(t)\eta(0)\rangle = \delta(t)$$



Reinhold Fürth (1920)

Przibam: Movimento Browniano

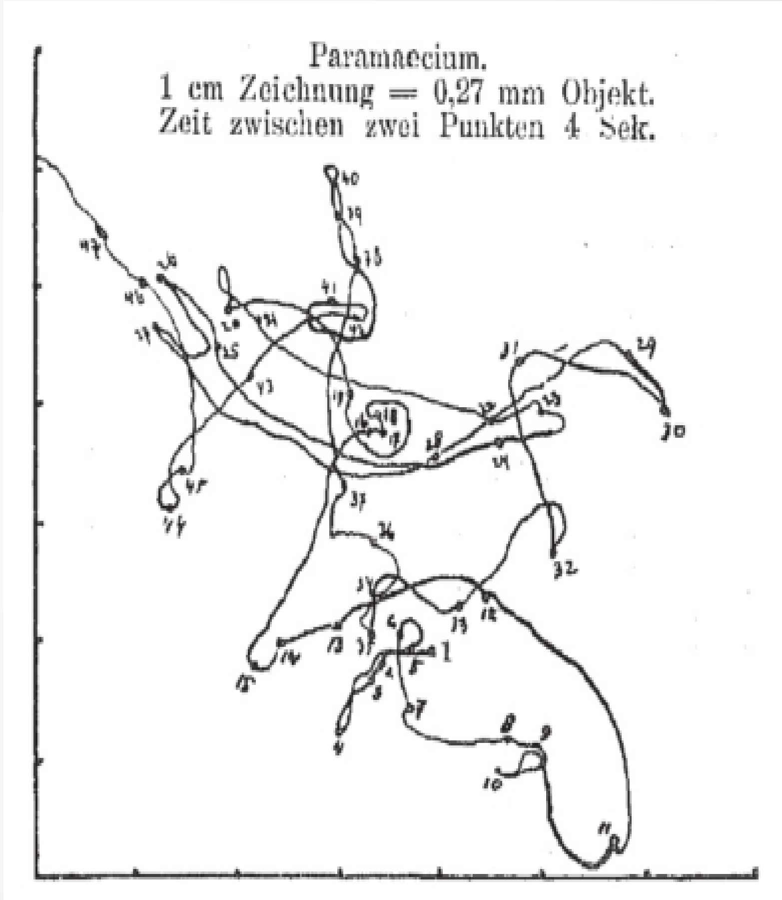
$$\langle \mathbf{d}(t)^2 \rangle = 2 n_{dim} D t$$

com $D > k_B T / 6 \pi \eta a$ (movimento ativo!)

Protozoários não obedecem à equação acima

- Movimento com **persistência**

$$\langle \mathbf{d}(t)^2 \rangle = 2 n_{dim} D \left[t - P \left(1 - e^{-t/P} \right) \right]$$



D. Selmeczi *et al,* Eur. Phys. J. Special Topics, 157, 1 (2008)
K. Przibram, Pflgers Arch. Physiol. 153, 401 (1913)
R. Fürth, Ann. Phys. 53, 177 (1917)
R. Fürth, Z. Physik 2, 244 (1920)