

Crossover exponents for the Potts model with quadratic symmetry breaking

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The effect of quadratic symmetry breaking (QSB) on two representations for the Potts vectors in a continuum-field model are studied to two-loop order in renormalized perturbation theory in $d=6-\epsilon$ dimensions, in extension of an earlier group-theoretical analysis by Wallace and Young. The explicit dependence of the crossover exponent ϕ that corresponds to QSB that destroys the equivalence between *pairs* of Potts vectors is obtained as a function of d and n for the p -state model with $p=n+1$. It is shown that this exponent follows from the calculation of vertex functions in a representation due to Wallace and Young, whereas a second crossover exponent $\bar{\phi}$, that can be identified with the critical exponent β , and which corresponds to QSB that favors a *single* Potts vector against the others, follows from a calculation using the representation of the Potts vectors due to Priest and Lubensky.

I. INTRODUCTION

The study of the critical properties of the p -state Potts model¹ has already been of interest for some time.² The model on a lattice with isotropic ferromagnetic nearest-neighbor exchange interaction can be described by the Hamiltonian

$$H = -J \sum_{\langle \vec{r} \vec{r}' \rangle} \vec{S}(\vec{r}) \cdot \vec{S}(\vec{r}'), \quad (1.1)$$

where the classical "spins" $\vec{S}(\vec{r})$ can be in p states (orientations) given by the Potts vectors \vec{e}^α , $\alpha=1,2,\dots,p$, which define the vertices of an n -dimensional hyper-tetrahedron, $n=p-1$, and satisfy the relations³

$$\begin{aligned} \sum_{\alpha=1}^p e_i^\alpha &= 0, \\ \sum_{\alpha=1}^p e_i^\alpha e_j^\alpha &= p \delta_{ij}, \\ \sum_{i=1}^{p-1} e_i^\alpha e_i^\beta &= p \delta_{\alpha\beta} - 1. \end{aligned} \quad (1.2)$$

The model has the discrete symmetry of the permutation group S_{n+1} .^{4,5}

The relevance of symmetry-breaking perturbations for critical phenomena is well known,⁶ and there is considerable interest in studying the effects of an anisotropic (in spin space) exchange interaction in the original lattice model. Wallace and Young⁴ (WY) showed by means of general group-theoretical arguments on the continuum field version⁷ of the Potts model that there are two crossover exponents associated with quadratic symmetry breaking (QSB) corresponding to the two nontrivial irreducible representations $(n,1)$ and $(n-1,2)$ of the group S_{n+1} . Combining this result with the tensorial structure of two-point vertex functions with insertions of operators that correspond to these irreducible representations, together with scaling arguments, they predicted that one of the

crossover exponents takes the value $\phi=1$ in the limit $n \rightarrow 0$,⁸ to all orders in perturbation theory, while the other exponent, $\bar{\phi}$, must be equal to the critical exponent β for any n . Since the first crossover exponent does not seem to be related directly to other known exponents, the explicit dependence of $\phi(d,n)$ on d and on n requires a detailed calculation by means of any of the momentum-space renormalization-group (RG) procedures,^{9,10} which was not done in Ref. 4.

WY also showed that their results are independent of a choice for a representation of the vectors \vec{e}^α . Different representations amount to distinct orientations of the vectors \vec{e}^α with respect to an orthogonal coordinate system in order-parameter space (i.e., the components of the fields ϕ_i). Two representations that are of particular physical interest have been used in the past. One, introduced by Priest and Lubensky (PL),¹¹ is essentially the same (up to a normalizing factor) as that of Zia and Wallace.⁷ The other is the WY representation which applies to the Potts model with $p=2^m$ states, m being an integer.⁴ For these particular values of p the hypertetrahedron defined by the Potts vectors can be embedded into a hypercube with the vectors \vec{e}^α going from the center to alternate vertices and the coordinate axes being perpendicular to the faces, as shown in Fig. 1 for $p=4$.

These two representations are equivalent for the symmetric theory. Indeed, the only properties of \vec{e}^α needed in that case are the relations given by Eq. (1.2). In other words, the symmetric theory is invariant under rotations of the coordinates which transform one representation into the other. Since crossover exponents correspond to QSB about the symmetric theory, it is reasonable to expect that they are independent of the representation, in accordance with the results of WY.

An important question that was not considered in the work of Ref. 4 is how to proceed with a practical calculation of a crossover exponent that corresponds to a specific break in quadratic symmetry. The main purpose of our work is to study the effects of QSB on the two representa-

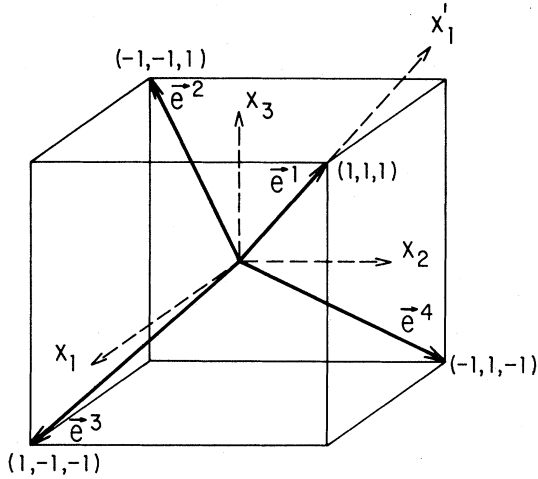


FIG. 1. Representation of the Potts vectors \vec{e}^α for the 4-state model with (a) the field components ϕ_i along the x_i axes, $i=1,2,3$, in the Wallace and Young representation and (b) the component ϕ_1 along the x'_1 axis and the others along mutually orthogonal directions, not drawn for clarity, in the representation of Priest and Lubensky.

tions for the Potts model and to relate the results to the group-theoretical analysis. Since there is a one-to-one correspondence between QSB and an irreducible representation of S_{n+1} , and on the other hand between the latter and a crossover exponent, one can relate either ϕ or $\bar{\phi}$ to a particular break in quadratic symmetry. However, to calculate ϕ or $\bar{\phi}$ one needs a choice for the representation of the \vec{e}^α , and it will be shown here that with diagonal QSB the crossover exponent ϕ is obtained if the vertex functions with appropriate insertion are calculated in the WY representation, while the exponent $\bar{\phi}=\beta$ follows from a calculation in the PL representation. The exact results of WY can then be used to assert that the expressions for $\phi(d,n)$ and $\bar{\phi}(d,n)$ hold for all n .

The RG approach used here consists of renormalized perturbation theory with dimensional regularization and minimal subtraction of poles,^{10,12} to two-loop order in $d=6-\epsilon$ dimensions. In Sec. II we introduce the relevant vertex functions with insertions that correspond to QSB in the continuum field Potts model and discuss the connection between each of the representations for the Potts vectors and a specific break in quadratic symmetry. The crossover exponents are obtained from the renormalization functions in Sec. III and the results are discussed and related to the group-theoretical arguments in Sec. IV.

II. VERTEX FUNCTIONS AND SYMMETRY BREAKING

In the continuum ϕ^3 -field theory corresponding to Eq. (1.1) the bare effective Hamiltonian becomes, in a standard way,³

$$\mathcal{H}_0 = \int d^d x \left[\frac{1}{2} m_0^2 \phi^2 + \frac{1}{2} (\vec{\nabla} \phi)^2 + \frac{1}{3!} g_{30} \sum_{i,j,k} d_{ijk} \phi_i \phi_j \phi_k + O(\phi^4) \right], \quad (2.1)$$

where ϕ is a real field with components ϕ_i , $i=1,2,\dots,n$, such that $n=p-1$, m_0 is the bare mass, and the tensorial coefficients

$$d_{ijk} = \sum_{\alpha=1}^p e_i^\alpha e_j^\alpha e_k^\alpha \quad (2.2)$$

yield the invariance of the trilinear term under the group S_{n+1} . The quartic term, which is necessary to stabilize the theory, will not be needed here since it is irrelevant in the disordered phase to which the present work is restricted under a RG transformation in $d=6-\epsilon$ dimensions. We also restrict the present work to a single trilinear coupling, g_{30} , although recent results have shown that the RG procedure generates trilinear symmetry breaking even for vanishingly small QSB,¹³ and that this behavior is enhanced in the presence of a finite break in quadratic symmetry.¹⁴ However, the results reported here should still correspond to the usual physical realizations of the 3- and 4-state Potts model.¹⁵

To study the crossover induced by QSB we follow earlier works, adding to Eq. (2.1) an anisotropy term¹⁶

$$\mathcal{H}_g = -\frac{1}{2} g \int d^d x B(x) \quad (2.3)$$

that favors the ordering into m “longitudinal” components if $g > 0$, with

$$B(x) = \frac{1}{n} [(n-m)\phi_L^2(x) - m\phi_T^2(x)], \quad (2.4)$$

where

$$\phi_L^2(x) = \sum_{i=1}^m \phi_i^2(x), \quad \phi_T^2(x) = \sum_{i=m+1}^n \phi_i^2(x) \quad (2.5)$$

and

$$\phi^2(x) = \phi_L^2(x) + \phi_T^2(x). \quad (2.6)$$

For simplicity, we restrict ourselves in the following to $m=1$. The effect of \mathcal{H}_g is to add a “mass” term to the remaining $(n-1)$ transverse components.

The bare one-particle irreducible (IPI) two-point longitudinal vertex function with one B insertion can then be written as

$$\Gamma_{11B}^{(2)}(u_0) = \Gamma_{11,1}^{(2)}(u_0) - \frac{1}{n} \Gamma_{11}^{(2,1)}(u_0) \quad (2.7)$$

in accordance with Eq. (2.4) for $m=1$, where $u_0 \equiv \kappa^{-\epsilon/2} g_{30}$ is the bare dimensionless coupling constant in which κ is an arbitrary momentum-scale parameter, and $\Gamma_{11,1}^{(2)}$, $\Gamma_{11}^{(2,1)}$ are the IPI two-point vertex functions with one ϕ_1^2 insertion and a full ϕ^2 insertion, respectively. It can also be seen that Eq. (2.7) is the two-point vertex function with an insertion of the operator $\{\phi_1^2\} = \phi_1^2 - \phi^2/n$ belonging to the irreducible representation $(n-1,2)$ of WY. While $\Gamma_{11}^{(2,1)}$ is just the $\Gamma_{\phi^2}^{(2,1)}$ of the symmetric theory which is independent of the representation for the vectors \vec{e}^α , the vertex function $\Gamma_{11,1}^{(2)}$ is representation dependent with the explicit dependence needed below.⁴

The vertex functions may be formally expanded as

$$\begin{aligned} \Gamma_{11}^{(2,1)}(u_0) &= 1 + C_1 u_0^2 + C_2 u_0^4, \\ \Gamma_{11,1}^{(2)}(u_0) &= 1 + D_1 u_0^2 + D_2 u_0^4, \\ \Gamma_{11B}^{(2)}(u_0) &= \frac{n-1}{n} (1 + F_1 u_0^2 + F_2 u_0^4), \end{aligned} \tag{2.8}$$

where

$$F_i = \frac{nD_i - C_i}{n-1}. \tag{2.9}$$

The coefficients C_i can be taken from Amit's work,³ while the D_i are given by the i -loop diagrams shown in Fig. 2. The j th diagram of the i th loop yields a contribution $D_{ij} = \hat{D}_{ij} L_{ij}$ in which the tensorial coefficients \hat{D}_{ij} need to be calculated while the momentum-space integrals L_{ij} , to two-loop order, can also be taken from Ref. 3. For the former we obtain

$$\begin{aligned} \hat{D}_1 &= (n+1)[d_{1111} - (n+1)], \\ \hat{D}_{21} = 2\hat{D}_{25} &= (n+1)^3[(n-3)d_{1111} + 2(n+1)], \\ \hat{D}_{22} &= 2(n+1)^3(n-2)[d_{1111} - (n+1)], \\ \hat{D}_{23} = 2\hat{D}_{24} &= (n+1)^3(n-1)[d_{1111} - (n+1)], \end{aligned} \tag{2.10}$$

where

$$d_{ijkl} = \sum_{\alpha=1}^p e_i^\alpha e_j^\alpha e_k^\alpha e_l^\alpha \tag{2.11}$$

now depends on the representation for the e_i^α . In the WY representation, $e_i^\alpha = \pm 1$ for any i , meaning

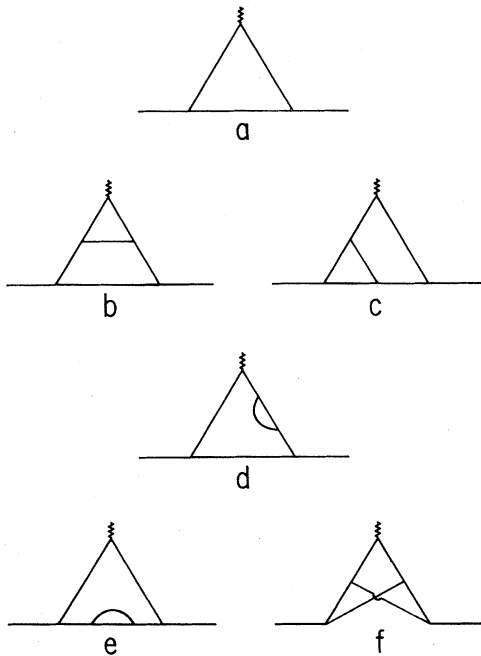


FIG. 2. One- and two-loop diagrams for the vertex functions $\Gamma_{11,1}^{(2)}$ or $\Gamma_{11}^{(2,1)}$ with a ϕ_1^2 insertion or a ϕ^2 insertion, respectively, in Eq. (2.7).

that none of the vectors is along one of the coordinate axis. In the PL representation, instead,

$$e_i^\alpha = \left[\frac{p(p-i)}{p-i+1} \right]^{1/2} \times \begin{cases} 0 & \text{if } \alpha < i, \\ 1 & \text{if } \alpha = i, \\ -\frac{1}{p-i} & \text{if } \alpha > i, \end{cases} \tag{2.12}$$

showing that the vector \vec{e}^1 has only one component different from zero lying along the $i=1$ axis, as shown in Fig. 1 for $p=4$, and this vector is therefore parallel to the field component ϕ_1 . Clearly, both representations satisfy the relations (1.2).

Equation (2.11) then yields

$$d_{1111}^{PL} = \frac{n^3+1}{n} \tag{2.13}$$

in the PL representation, while

$$d_{1111}^{WY} = n+1 \tag{2.14}$$

in the representation of WY. With these we obtain explicit expansion coefficients for $\Gamma_{11,1}^{(2)}$ and $\Gamma_{11B}^{(2)}$ in each of the two representations. These coefficients contain poles in $1/\epsilon$ and $1/\epsilon^2$ which must be cancelled by appropriate renormalization functions considered below, in order to yield renormalized vertex functions. We wish to point out first, however, that the two representations correspond to two different ways of implementing a break in quadratic symmetry.

Since \vec{e}^1 has only one nonzero component in the PL representation, QSB along a single field component that favors the $i=1$ direction implies a breakdown of the equivalence between \vec{e}^1 and all other vectors. This break in quadratic symmetry corresponds to the irreducible representation $(n,1)$ found by WY. In the WY representation for the 4-state model, there are two pairs of vectors going to diagonally opposite corners of two faces of the cube that are perpendicular to the direction of one of the fields. Then QSB along this direction, which can be chosen to be the $i=1$ axis, still maintains the permutation symmetry within each pair but breaks the equivalence between pairs. This break of quadratic symmetry belongs to the irreducible representation $(n-1,2)$. In the example of Fig. 1 for the 4-state model, QSB in the PL representation corresponds thus to a break in the (111) direction, while in the WY representation the break is along the (100) direction.

Renormalization of the bare two-point vertex function $\Gamma_{11B}^{(2)}$ with a B insertion requires a field renormalization through the function Z_ϕ and a renormalization of the insertion through another Z_B , together with coupling-constant renormalization. Thus

$$\begin{aligned} \Gamma_{11B \text{ ren}}^{(2)}(u) &= Z_\phi(u) Z_B(u) \Gamma_{11B}^{(2)}(u_0) \\ &\equiv \hat{Z}_B(u) \Gamma_{11B}^{(2)}(u_0), \end{aligned} \tag{2.15}$$

where the new \hat{Z}_B follows from combining the other Z 's, and u is the dimensionless renormalized coupling constant. Since the operator \mathcal{H}_g of Eq. (2.3) is a perturbation

$$u_0 = u + \frac{1}{4\epsilon}(n+1)^2(7-3n)u^3 + \frac{1}{32\epsilon^2}(n+1)^4[3(7-3n)^2 - \frac{1}{9}(125n^2 - 544n + 671)\epsilon]u^5. \quad (2.16)$$

The renormalization functions that make finite the vertex functions $\Gamma_{11B}^{(2)}$ in each of the representations are then obtained in expansion in powers of u as

$$\hat{Z}_B^{\text{PL}}(u) = 1 - \frac{1}{\epsilon}(n+1)^2(n-2)u^2 + \frac{1}{4\epsilon^2}(n+1)^4[(5n^2 - 21n + 22) - \frac{1}{12}(23n^2 - 99n + 118)\epsilon]u^4 \quad (2.17)$$

and

$$\hat{Z}_B^{\text{WY}}(u) = 1 + \frac{1}{\epsilon}(n+1)^2u^2 - \frac{1}{4\epsilon^2}(n+1)^4[3(n-3) + \frac{1}{12}(47-5n)\epsilon]u^4. \quad (2.18)$$

These will be used in the next section to obtain the crossover exponents.

III. CROSSOVER EXPONENTS

Defining

$$\hat{\gamma}_B(u) \equiv -\beta(u) \frac{\partial \ln \hat{Z}_B(u)}{\partial u}, \quad (3.1)$$

where $\beta(u)$ is the Wilson beta function,¹⁰ the crossover exponents follow from the fixed-point values $\hat{\gamma}_B^* \equiv \hat{\gamma}_B(u^*)$ through the relation

$$\phi = \nu(2 - \eta - \hat{\gamma}_B^*), \quad (3.2)$$

about the symmetric theory, the expansion of u_0 in powers of u may be taken from Amit's work, which yields³

obtained by integrating the RG equations for the vertex functions in standard way,^{10,16} in which ν and η are the usual exponents for the correlation length and the critical correlation function for the symmetric theory. These are given by³

$$\eta = \frac{(n-1)}{3(7-3n)} \epsilon \left[1 + \frac{43n^2 - 171n + 206}{9(7-3n)^2} \epsilon \right], \quad (3.3)$$

and

$$\nu = \frac{1}{2} - \frac{5(n-1)}{12(7-3n)} \epsilon \left[1 + \frac{134n^2 - 477n + 589}{5 \times 9(7-3n)^2} \epsilon \right], \quad (3.4)$$

while $\beta(u)$ can be obtained from Eq. (2.16) as

$$\beta(u) = -\frac{\epsilon}{2} \left[\frac{\partial \ln u_0}{\partial u} \right]^{-1} = -\frac{1}{2} \epsilon u \left[1 - \frac{1}{2\epsilon}(n+1)^2(7-3n)u^2 + \frac{1}{72\epsilon}(n+1)^4(125n^2 - 544n + 671)u^4 \right], \quad (3.5)$$

whose nontrivial root gives the fixed point

$$(u^*)^2 = \frac{2\epsilon}{(n+1)^2(7-3n)} \left[1 + \frac{125n^2 - 544n + 671}{18(7-3n)^2} \epsilon \right]. \quad (3.6)$$

Equations (3.1) and (3.5) then yield, with Eqs. (2.17) and (2.18),

$$\hat{\gamma}_B^{\text{PL}} = -(n+1)^2u^2[(n-2) + \frac{1}{24}(23n^2 - 99n + 118)(n+1)^2u^2], \quad (3.7)$$

and

$$\hat{\gamma}_B^{\text{WY}} = (n+1)^2u^2[1 + \frac{1}{24}(5n-47)(n+1)^2u^2], \quad (3.8)$$

and the crossover exponents that follow from Eqs. (3.2)–(3.4) are then given by

$$\phi = 1 - \frac{n}{7-3n} \epsilon - \frac{n(n-1)(133n-187)}{36(7-3n)^3} \epsilon^2, \quad (3.9)$$

in the WY representation, and

$$\bar{\phi} = 1 - \frac{1}{7-3n} \epsilon - \frac{(n-1)(79n-61)}{36(7-3n)^2} \epsilon^2 \quad (3.10)$$

in the representation of PL. We have identified here ϕ^{WY} with the exponent ϕ of WY on the grounds that QSB in the WY representation corresponds to the group-theoretical irreducible representation $(n-1, 2)$ that is associated with ϕ , in accordance with the discussion in Sec. II and with the work of Ref. 4. On the other hand, ϕ^{PL} is identified with $\bar{\phi}$, the exponent associated with the irreducible representation $(n, 1)$. It follows immediately that $\phi \rightarrow 1$ in the limit $n \rightarrow 0$, while $\bar{\phi}$ becomes the known expression for the critical exponent β , in accordance with Ref. 4.

Note that $n = 2^m - 1$, where m is an integer, in the representation of WY and, in principle, our calculation of $\phi = \phi^{\text{WY}}$ in Eq. (3.9) is restricted to these values of n . Here we may use, however, the group-theoretical argument of WY that demonstrates the independence of ϕ from the representation to assert that our result is valid

for all n . Similarly, the identification of $\bar{\phi}$ with β should be valid for all n .

Comparison of Eqs. (3.9) and (3.10) shows that $\phi = \bar{\phi}$ for $n=1$, at least to second order in ϵ . This, indeed, should be expected since the underlying symmetry of the Potts model which is responsible for the existence of two crossover exponents disappears for $n=1$ with the vanishing of the trilinear tensorial coefficients, and the model becomes Gaussian without the quartic terms.

IV. DISCUSSION AND CONCLUDING REMARKS

We have obtained in a RG calculation two crossover exponents for the Potts model that correspond to two distinct ways of breaking quadratic symmetry, in agreement with group-theoretical expectations. Although the exponent $\phi=1$ was already known in the percolation limit, the explicit form as a function of the number of states is, to our knowledge, a new result. In this context it is interesting to note that Coniglio¹⁷ obtained the result $\phi=1$, independent of n and of d , for the dilute p -state Potts model as $T \rightarrow 0$, but there is no obvious reason to believe that the critical behavior of this model is the same as that

of the usual Potts model considered in the present work.

Also, the outcome that $\bar{\phi}=\beta$ is a check on the relationship

$$\frac{1}{2} g_{30} \Gamma_{d_{mij}\phi_i\phi_j}^{kl} = d_{mkl} \Gamma^{(3)}, \quad (4.1)$$

used in Ref. 4, together with its scaling behavior, in which the two-point vertex Γ^{kl} has an insertion of the operator $d_{mij}\phi_i\phi_j$ and $\Gamma^{(3)}$ is the usual three-point vertex function. The operator $d_{mij}\phi_i\phi_j$ has only off-diagonal terms ($i \neq j$) in the WY representation, but Eq. (4.1) should not depend on whether $d_{mij}\phi_i\phi_j$ is diagonal or not. It can be shown that it becomes diagonal in the representation of PL and proportional to the B insertion of Sec. II, which is just the $\{\phi_1^2\}$ of Ref. 4. However, once the two-point vertex function has only diagonal insertions it is not clear to us, at least, that the derivation of the WY result yields two crossover exponents. The more explicit, although less general work, of the present paper shows more directly the existence of these two crossover exponents.

To see that $d_{mij}\phi_i\phi_j \propto \{\phi_1^2\} = \phi_1^2 - \phi^2/n$ (summation over i and j) in the PL representation, note that Eq. (2.12) yields

$$d_{ijk} = \left[\frac{p^3}{(p-i)(p-i+1)} \right]^{1/2} \times \begin{cases} -1 & \text{if } i=j > k \text{ and permutations,} \\ (p-i-1) & \text{if } i=j=k, \\ 0 & \text{otherwise.} \end{cases} \quad (4.2)$$

With this it can easily be checked that

$$d_{1jk}\phi_j\phi_k = \sqrt{n}(n+1) \left[\phi_1^2 - \frac{1}{n}\phi^2 \right], \quad (4.3)$$

and the vertex function $\Gamma_{1\bar{B}}^{(2)}$, with $\bar{B} \equiv d_{ijk}\phi_j\phi_k$, will be proportional to the $\Gamma_{1B}^{(2)}$ of Eq. (2.7). The choice of $i=1$ is imposed by the fact that the tensorial coefficient d_{ijk} of the quadratic insertion yields an overall factor d_{i11} when introduced into diagrams for $\Gamma_{11}^{(2)}$, and this factor is different from zero only if $i=1$.

Problems associated with the ground-state instability in continuum-field theories with trilinear interactions have been known for some time.¹⁸ Owing to the presence of instanton solutions the perturbation expansion for the continuum $(n+1)$ -state Potts model may be meaningless for general n , and one may question the interest in calculating critical exponents, for what is believed to be a second-order transition since the presence of instantons seems to signal a first-order transition.¹⁹ So far it has only been shown that a meaningful perturbation expansion is obtained for $n=0$, the percolation problem.²⁰ Actually, the classical argument showing the presence of instanton solutions breaks down as n goes to zero and one cannot ex-

clude a crossover to a different instanton solution, at some low n , which still allows a second-order transition. This may be related to the instability of the large Q (order parameter) minimum found by Pytte²¹ for $n < 1$, excluding a first-order transition in this case. Following this author we assume that the $(n+1)$ -state Potts model has a second-order transition for $n < 1$ (Ref. 22) and our calculated crossover exponents apply to this case. They could also eventually describe the spinodal point (the lower stability limit of ϕ^3 theory) for $1 < n < \frac{7}{3}$, as suggested by Pytte,²¹ but that seems uncertain for the moment in the absence of a study of the effect of instanton solutions applied to this case.

Our paper shows how to calculate the crossover exponents ϕ and $\bar{\phi}$ for other situations, e.g., with long-range isotropic²³ or dipolar interactions, that will be published elsewhere. It should also be of use for crossover in random Potts models that will be considered in future work.

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