## LETTER TO THE EDITOR

# Critical exponents of a Heisenberg ferromagnet in the reaction field approximation†

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**Abstract.** The critical exponents $\alpha$, $\beta$ and $\gamma$ of a Heisenberg ferromagnet are calculated in the reaction field approximation, the results showing agreement with the spherical model.

The reaction field approximation (RFA) is a mean field theory for spin systems which takes into account the correlations between spins through a parameter determined in consistency with the fluctuation theorems of statistical mechanics. It was initially developed for Ising ferromagnets (Brout and Thomas 1967), using the concept of 'reaction field' (Onsager 1936), and later extended to Heisenberg systems in both paramagnetic (Scherer and Aveline 1976) and ordered (Gusmão and Scherer 1979) phases. Summarising the results presented in these two later references, it is possible to describe the spontaneous magnetisation $M$, uniform susceptibility $\chi$ and magnetic specific heat ($C_H$) of a Heisenberg (isotropic) ferromagnet, in the RFA, by the following set of equations:

$$M = \frac{\sum_{m=-S}^{S} m \exp[J(0)\, Mm/k_B T G(1)]}{\sum_{m=-S}^{S} \exp[J(0)\, Mm/k_B T G(1)]} \tag{1}$$

$$\chi = \frac{1/J(0)}{s-1} \tag{2}$$

$$C_H = \tfrac{3}{2} k_B \{1 + [G(s)]^2/G'(s)\} \qquad (T > T_c) \tag{3}$$

$$G(s) = \theta/T \qquad (T > T_c) \tag{4}$$

where $S$ is the spin value, $J(0)$ is the zero-wavevector Fourier component of the exchange interaction, $k_B$ is Boltzmann's constant, $\theta$ is the Curie–Weiss temperature, $G(s)$ is defined by

$$G(s) \equiv \frac{1}{N} \sum_k \frac{1}{s - J(k)/J(0)} \tag{5}$$

$N$ being the number of spins in the system, and we use primes to indicate derivatives with respect to the argument.‡

With equations (1) to (4) we are able to obtain the critical exponents $\alpha$, $\beta$ and $\gamma$,

‡ The parameter $s$, determined through equations (4) and (5), depends on the correlation between spins (Scherer and Aveline 1976).

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defined by

$$C_{\mathrm{H}} = A + B(T - T_{\mathrm{c}})^{-\alpha} \qquad T > T_{\mathrm{c}} \tag{6}$$

$$M = C(T_{\mathrm{c}} - T)^{\beta} \qquad T < T_{\mathrm{c}} \tag{7}$$

$$\chi = D(T - T_{\mathrm{c}})^{-\gamma} \qquad T > T_{\mathrm{c}} \tag{8}$$

where $A$, $B$, $C$ and $D$ are constants and $T_{\mathrm{c}}$ is the critical temperature for transition between the paramagnetic and ferromagnetic phases.

Equation (1) has the same form as the correspondent equation in the molecular field approximation (MFA), except for the renormalisation factor $G(1)$. So the exponent $\beta$ will be the same in both approximations, namely $\beta = \frac{1}{2}$.

By equation (2) we see that the behaviour of $\chi$ for temperatures close to $T_{\mathrm{c}}$ will be governed by the dependence of $s$ on $T$ in this region. Expanding $s$ in powers of $T - T_{\mathrm{c}}$ and using the fact that $s(T_{\mathrm{c}}) = 1$, we have

$$s - 1 = (\partial s/\partial T)_{T_{\mathrm{c}}}(T - T_{\mathrm{c}}) + (\partial^2 s/\partial T^2)_{T_{\mathrm{c}}}(T - T_{\mathrm{c}})^2 + \ldots. \tag{9}$$

Transforming the derivative of $s$ with respect to $T$ as follows:

$$\frac{\partial s}{\partial T} = \frac{\partial G(s)/\partial T}{\partial G(s)/\partial s} = \frac{1}{G'(s)}\frac{\partial G(s)}{\partial T} \tag{10}$$

and using equation (5), we get

$$\frac{\partial G(s)}{\partial T} = -\frac{\theta}{T^2} = -\frac{1}{\theta}[G(s)]^2 \tag{11}$$

which implies that

$$\frac{\partial s}{\partial T} = -\frac{1}{\theta}\frac{[G(s)]^2}{G'(s)}. \tag{12}$$

We shall see later that all derivatives of $G(s)$ with respect to $s$ are divergent at $s = 1$ ($T = T_{\mathrm{c}}$), while $G(s)$ is finite. Then $(\partial s/\partial T)_{T_{\mathrm{c}}} = 0$, being necessary to work out the second term in the right-hand side of equation (9).

From equations (11) and (12) it is straightforward to show that

$$\frac{\partial^2 s}{\partial T^2} = \frac{2[G(s)]^3}{\theta^2[G'(s)]^2} - [G(s)]^4\frac{G''(s)}{[G'(s)]^3}. \tag{13}$$

The first term in equation (13) is again null at $s = 1$, but the second is finite, as we shall prove.

Equation (5), with the usual transformation

$$\sum_{\mathbf{k}} \rightarrow \frac{V}{(2\pi)^3}\int \mathrm{d}^3k \tag{14}$$

may be written in the form

$$G(s) = \frac{V}{(2\pi)^3}\int \mathrm{d}^3k \frac{1}{s - J(\mathbf{k})/J(0)} \tag{15}$$

$V$ being the volume of the system.

For nearest-neighbour interaction and in the limit of small $\mathbf{k}$,

$$1 - J(\mathbf{k})/J(0) \approx k^2. \tag{16}$$

It suffices to consider this limit, for the divergence occurs in the $k = 0$ value of the integrand, as shown by the following equations.

$$G(1) = \frac{V}{(2\pi)^3} \int d^3k \frac{1}{1 - J(k)/J(0)} \approx \int_0^{k_{max}} k^2 \, dk \left(\frac{1}{k^2}\right) \approx 1 \tag{17}$$

$$G'(1) = -\frac{V}{(2\pi)^3} \int d^3k \frac{1}{[1 - J(k)/J(0)]^2} \approx \int_0^{k_{max}} k^2 \, dk \frac{1}{k^4} \approx \lim_{k \to 0} \frac{1}{k} \tag{18}$$

$$G''(1) = \frac{2V}{(2\pi)^3} \int d^3k \frac{1}{[1 - J(k)/J(0)]^3} \approx \int_0^{k_{max}} k^2 \, dk \left(\frac{1}{k^6}\right) \approx \lim_{k \to 0} \left(\frac{1}{k^3}\right) \tag{19}$$

where $k_{max}$ is the limit of the first Brillouin zone. Equations (18) and (19) show that the divergences of $G'(1)$ and $G''(1)$ cancel out in the quotient $[G'(1)]^3/G''(1)$, appearing in equation (13). Then equations (2), (8) and (9) lead to

$$\chi \approx (T - T_c)^{-2} \tag{20}$$

which gives the value $\gamma = 2$, in contrast with the value $\gamma = 1$ obtained by MFA.

Now, expanding the term $[G(s)]^2/G'(s)$ of equation (3), we have

$$\frac{[G(s)]^2}{G'(s)} = \frac{[G(1)]^2}{G'(1)} + \left[\frac{\partial}{\partial T} \frac{[G(s)]^2}{G'(s)}\right]_{T_c} (T - T_c) + \dots \tag{21}$$

With an analysis similar to the above, we can easily prove that the second term in equation (21) is finite, which implies that

$$C_H = \tfrac{3}{2}k_B + \text{const}(T - T_c). \tag{22}$$

Equations (6) and (22) give $\alpha = -1$, while the result of MFA is $\alpha = 0$.

Notice that in both approximations the exponents satisfy the scaling law $\alpha + 2\beta + \gamma = 2$.

The values $\alpha = -1, \beta = \tfrac{1}{2}, \gamma = 2$, obtained by RFA, are the same as those of the spherical model. This fact reinforces the similarity between these two approaches, already pointed out by Brout and Thomas (1967).

**References**

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