AIP | Physics of Plasmas

Halo formation and emittance growth in the transport of spherically symmetric mismatched bunched beams

Thales M. Corrêa da Silva, Renato Pakter, Felipe B. Rizzato, and Yan Levin

Citation: Physics of Plasmas (1994-present) **22**, 023102 (2015); doi: 10.1063/1.4907198
View online: http://dx.doi.org/10.1063/1.4907198
View Table of Contents: http://scitation.aip.org/content/aip/journal/pop/22/2?ver=pdfcov
Published by the AIP Publishing

PHYSICS OF PLASMAS **22**, 023102 (2015)

Halo formation and emittance growth in the transport of spherically symmetric mismatched bunched beams

Thales M. Corrêa da Silva,[a)] Renato Pakter, Felipe B. Rizzato, and Yan Levin
Instituto de Física, Universidade Federal do Rio Grande do Sul, Caixa Postal 15051, 91501-970 Porto Alegre, Rio Grande do Sul, Brazil

(Received 14 November 2014; accepted 15 January 2015; published online 2 February 2015)

The effect of an initial envelope mismatch on the transport of bunched spherically symmetric beams is investigated. A particle-core model is used to estimate the maximum radius that halo particles can reach. The theory is used to obtain an empirical formula that provides the halo size as a function of system parameters. Taking into account, the incompressibility property of the Vlasov dynamics and the resulting Landau damping, an explicit form for the final stationary distribution attained by the beam is proposed. The distribution is fully self-consistent, presenting no free fitting parameters. The theory is used to predict the relevant beam transport properties, such as the final particle density distribution, the emittance growth, and the fraction of particles that will be expelled to form halo. The theoretical results are compared to the explicit N-particle dynamics simulations, showing a good agreement. © *2015 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4907198]

I. INTRODUCTION

Despite the existence of various known equilibrium solutions for the transport of intense beams,[1,2] in practice, it is impossible to create and inject a beam that fully satisfies one of such equilibria. As a consequence, the beam distribution will start to change as it propagates, seeking a more stable stationary state.[3] Generally, this process is very detrimental because it diminishes the beam quality by producing emittance growth and halo formation. Since this, in general, cannot be prevented, it is crucial to have a method to quantify how much the beam will deteriorate, allowing to accommodate this information in the machine design. This, however, requires a precise knowledge of the final stationary state that will be achieved by the relaxed beam.

One might expect that the beam will relax to thermodynamic equilibrium with a Boltzmann-Gibbs distribution.[4,5] In general, however, this is not the case. The difficulty is that unlike systems with short range forces, beam particles interact through a long-range electromagnetic field. In the thermodynamic limit, where the number of particles in the beam goes to infinity, $N \to \infty$, while keeping the total charge constant, the transport becomes collisionless and is exactly described by the Vlasov equation, which has an infinite number of stationary solutions.[1,2] In fact, any distribution function that depends only on the constants of motion of the single particle dynamics is a stationary solution of the Vlasov equation. The problem is then to choose the appropriate stationary solution out of an infinite set, which is a very difficult task. Unlike the thermodynamic equilibrium of short range interacting systems, the stationary state of the beam will depend explicitly on the initial particle distribution. In this respect, different phenomena, such as, envelope mismatch,[6–17] off-axis motion,[13,18–22] beam distribution nonuniformities,[23–30] and forces due to the surrounding conductors,[31–33] have been investigated as the precursors of the beam relaxation. Among these phenomena, the one that has attracted most attention is the envelope mismatch, which arises from the unbalance between the external focusing force and the defocussing space-charge repulsion. The envelope mismatch can lead to large amplitude oscillations. When this happens, some of the beam particles can become resonantly excited, gaining large amounts of energy from the coherent density oscillations.[6] The resulting particle evaporation will lead to the formation of a halo that will surround the central core. The evaporative cooling produced by the halo formation dampens out the oscillation of the core leading to a stationary particle distribution. The resulting core-halo distribution is composed of a dense core of low energy particles and a low density halo of hot particles. Models based on the core-halo distribution have been successfully used to predict the final stationary state and the corresponding emittance growth and halo formation.[14,15,34–36] The analysis done so far, however, was performed only for infinitely long, unbunched beams.

In this paper, we will investigate the effect of the envelope mismatch on the transport of a bunched beam. In particular, we will concentrate on the case, where the external focusing is isotropic and the beam has a spherical symmetry. This assumption is used to simplify the calculations, but is also relevant for systems of practical interest[37] and is expected to give a fair qualitative insight to the more general anisotropic focusing problem. We will first use a reduced particle-core model to determine the maximum radius attained by the halo particles that are resonantly excited by the bulk beam oscillations. An empirical law that relates the halo size with the relevant parameters of the system will then be obtained. Finally using the proposed core-halo distribution, we will predict the stationary state to which a bunched beam will evolve and calculate the corresponding emittance growth. All the theoretical results will be tested against the N-body dynamics simulations.

[a)]thalestr@gmail.com

The paper is organized as follows. In Sec. II, we introduce the model for the spherically symmetric bunched beam and the equations that describe its evolution. In Sec. III, we derive the corresponding particle-core model and use it to determine an empirical formula that provides the halo size as a function of the relevant parameters of the system. Based on the dynamical scenario that leads to the relaxation of an initially mismatched beam, in Sec. IV, we propose a theoretical model for the final stationary state attained by the beam. In Sec. V, we present the predictions of the model and compare them to N-particle dynamics simulations. Finally, in Sec. VI, we present the conclusions of the paper.

II. MODEL

We consider a nonrelativistic spherically symmetric bunched charged particle beam, whose center is propagating with a constant axial velocity V_b along the z-axis of a laboratory frame. The beam particles are subject to an external isotropic focusing force that points towards the center of the beam and that tends to balance the repulsive Coulomb self-repulsion. The dynamics of each particle is governed by[2]

$$\frac{d^2\mathbf{r}}{ds^2} = -\kappa\,\mathbf{r} - \nabla\psi, \tag{1}$$

where $s = V_b t$ is the beam center axial position that plays the role of a scaled time variable, $\mathbf{r} = (x, y, \tilde{z})$ is the displacement vector from the beam center, $\tilde{z} = z - s$ is the axial displacement, and the parameter κ measures the strength of focusing force in the smooth-focusing approximation.[2] In Eq. (1), ψ is the normalized self-field potential that satisfies the Poisson Equation

$$\nabla^2\psi = -\frac{4\pi K}{N} n(\mathbf{r}, s), \tag{2}$$

where $n(\mathbf{r}, s)$ is the beam density, $K = q^2N/mV_b^2$ is a parameter that measures the beam intensity, and $N = \int n d\mathbf{r}$ is the constant number of particles in the bunch.

For the sake of simplicity, let us assume that the beam is injected at $z = 0$ with an uncorrelated uniform distribution in both position and velocity. This distribution corresponds to a spherically symmetric waterbag in phase space given by

$$f_0(\mathbf{r}, \mathbf{v}) = \eta_0\,\Theta(r_m - r)\Theta(v_m - v), \tag{3}$$

where $\eta_0 = 9N/16\pi^2 r_m^3 v_m^3$ is the normalization constant that measures the density of the initial distribution in the phase space, $\Theta(x)$ is the Heaviside step function, $v = |d\mathbf{r}/ds|$ is the magnitude of the velocity, and r_m and v_m are the maximum values of r and v for the initial beam distribution. It is worth mentioning that the theory that will be presented here to determine the final stationary state can be generalized to more complex initial distributions by approximating them by a number of waterbag distributions.[34] Moreover, studies performed for unbunched beams have shown that even a single waterbag distribution can already provide a very good estimate of the stationary distribution and of emittance growth for more complex initial conditions.[15,35] Taking into account, the dynamical equation for each particle, Eq. (1), and the fact that the beam transport is essentially collisionless, the beam distribution function will evolve from f_0 according to the Vlasov equation

$$\frac{\partial f}{\partial s} + \mathbf{v}\cdot\nabla f + (-\kappa\,\mathbf{r} - \nabla\psi)\cdot\nabla_v f = 0. \tag{4}$$

By integrating the distribution function over the velocity space, one obtains the beam density $n(\mathbf{r}, s) = \int f d\mathbf{v}$, which is then substituted in Eq. (2) to obtain a closed set of equations for the beam evolution. The Vlasov equation (4) states that the total derivative of $f(\mathbf{r}, \mathbf{v}, s)$ with respect to the time variable s vanishes. This means that the particle distribution evolves in the phase space as an incompressible fluid. Therefore, the maximum phase space density throughout the evolution cannot exceed η_0, which is the value of the initial distribution in Eq. (3). This is the fundamental feature of the beam dynamics that will be used to construct the distribution function for the final stationary state.

Besides the number of particles in the bunch N, another quantity that is conserved along the transport is the average energy per particle. It is given by[2]

$$E = \frac{\langle v^2\rangle}{2} + \frac{\kappa\langle r^2\rangle}{2} + \frac{1}{2K}\int_0^\infty \left(\frac{\partial\psi}{\partial r}\right)^2 r^2 dr, \tag{5}$$

where $\langle\cdots\rangle = \int \cdots f d\mathbf{v} d\mathbf{r}$ represents the average over the distribution, and one readily recognizes the contributions from the kinetic, focusing force, and self-field energies, respectively. For the injected beam of Eq. (3), the average energy per particle is readily calculated to be

$$E_0 = \frac{3v_m^2}{10} + \frac{3\kappa r_m^2}{10} + \frac{3K}{5r_m}, \tag{6}$$

so that $E = E_0$ throughout the transport.

III. PARTICLE-CORE MODEL AND HALO SIZE

The main mechanism for the halo formation in mismatched beams is a resonant coupling between the single particle orbits and the bulk oscillations of the beam.[6] A widely used tool to estimate the size (maximum radius) of the halo is the, so-called, particle-core model, in which a test particle trajectory is driven by the bulk (core) oscillations described by the envelope equation. For the spherically symmetric beam considered here, the envelope of the beam is related to its RMS size by $r_b = [5\langle r^2\rangle/3]^{1/2}$, where the "5/3" factor is introduced so that $r_b = r_m$ at $s = 0$ for the initial distribution of Eq. (3). Taking two derivatives of r_b with respect to s, using Eq. (1), and conveniently rearranging the terms, we can write

$$\frac{d^2 r_b}{ds^2} = -\kappa\, r_b - \frac{5}{3}\frac{\langle\mathbf{r}\cdot\nabla\psi\rangle}{r_b} + \frac{\epsilon^2}{r_b^3}, \tag{7}$$

where

$$\epsilon = \frac{5}{3}\left[\langle r^2\rangle\langle v^2\rangle - \langle\mathbf{r}\cdot\mathbf{v}\rangle^2\right]^{1/2}, \tag{8}$$

is the emittance of the beam. Substituting the initial distribution of Eq. (3) in (8), we readily find that the emittance of the injected beam is $\epsilon_0 = r_m v_m$. In general, the emittance is a function of s and, therefore, changes as the beam propagates. However, in order to obtain a closed equation for the envelope evolution, in the particle-core model it is assumed that the emittance is a conserved quantity.

In the case of axisymmetric unbunched beams, the self-field contribution to the envelope equation [second term on the rhs of Eq. (7)], can be computed exactly because it is independent of the specific form of the beam density.[2] This, however, is not the case for the bunched beams and a specific density has to be assumed in order to evaluate this term. To be consistent with the initial distribution, we will assume for the purpose of deriving the envelope equation that the beam core density remains uniform with particles distributed up to $r = r_b$, along the transport. Using such density profile in the Poisson equation (2), the self-field potential due to the core, ψ_{core}, can be readily computed. Its gradient reads

$$\nabla\psi_{core} = \begin{cases} -K\mathbf{r}/r_b^3(s) & \text{if } r \leq r_b(s), \\ -K\mathbf{r}/r^3 & \text{if } r > r_b(s). \end{cases} \tag{9}$$

Substituting $\nabla\psi = \nabla\psi_{core}$ in (7), we obtain the desired closed equation for the envelope

$$\frac{d^2 r_b}{ds^2} = -\kappa\, r_b + \frac{K}{r_b^2} + \frac{\epsilon_0^2}{r_b^3}. \tag{10}$$

It is worth noting that two main assumptions have been used to derive Eq. (10): the conservation of emittance and of the beam core uniformity. Although these may seem as very restrictive approximations, they are well justified if one only wants to describe the trajectory of the particles that will reach the outermost portion of the halo, which is exactly the aim of the particle-core model. As a matter of fact, such particles are expected to be resonantly driven in the very beginning of the beam evolution, when the envelope oscillations are at their maximum amplitude and have not yet been severely damped by the emittance growth. Moreover, because these particles reach very large radii, they spend most of their time away from the beam core and are very little affected by its evolution.

It is clear from Eq. (10) that there is a competition between the external focusing force and the defocusing forces due to space charge and emittance. Matched beams are those for which these forces cancel each other and the envelope remains constant along the transport. Equating $d^2r_b/ds^2 = 0$ in Eq. (10), we obtain an algebraic equation that defines the matched beam envelope r_{bm}

$$\kappa r_{bm}^4 - K r_{bm} - \epsilon_0^2 = 0. \tag{11}$$

Despite being a fourth-order polynomial equation, for any given set of parameters κ, K, and ϵ_0, Eq. (11) presents only one physical solution corresponding to a real positive r_{bm}. In general, however, the initial envelope $r_b(0) = r_m$ will differ from r_{bm}. In such cases, the envelope will start to oscillate according to Eq. (10) and may resonantly couple to single particle trajectories. To investigate this, in the particle-core model, we numerically integrate the trajectories of test particles that evolve according to Eq. (1), but where the self-field gradient $\nabla\psi$ is substituted by the modeled core field $\nabla\psi_{core}$ of Eq. (9). This leads to a *low-dimensional* dynamical system that can be analyzed with the aid of stroboscopic plots,[8,38] where the test particle phase space positions are plotted at the periodicity of the mismatched envelope oscillation. Note that by conveniently scaling r_b and s to $(\epsilon_0^2/\kappa)^{1/4}$ and $\kappa^{-1/2}$, respectively, the system is completely determined by two dimensionless parameters: a mismatch parameter $\mu \equiv r_b(0)/r_{bm}$ and a beam intensity parameter $\zeta \equiv K/(\kappa\epsilon_0^6)^{1/4}$. An example of a stroboscopic plot is presented in Fig. 1 for $\zeta = 1$ and $\mu = 1.6$. In the figure, we see the formation of a large resonant island with a stable fixed point located at $r/r_{bm} \approx 2.0$ and $dr/ds = 0$. The separatrix of this island (thicker blue dots) is responsible for the resonant excitation of particles just outside the beam core to the halo.[6] Once these particles have been excited, their trajectories become nearly stationary because they are very insensitive to the evolution of the core that occurs as the beam propagates. Therefore, we can estimate the halo size r_h in the full N-particle system by computing the maximum radius of the separatrix. In the case of Fig. 1, we find $r/r_{bm} \approx 3.7$.

Although the particle-core model allows for the estimate of the halo size without the need to solve the full N-particle dynamics, it still requires a numerical integration of coupled nonlinear differential equations. Given the relevance of the halo size for practical purposes, it is desirable to have an explicit expression for r_h as a function of the parameters of the system. An empirical method to accomplish this was proposed in Ref. 8 for the case of axisymmetric unbunched beams. Based on a dimensional analysis, it was observed that the dependence of the halo size on the system parameters is of the form $r_h/r_{bm} = f(\mu)$, where the function $f(\mu)$ is determined empirically. Adopting the same strategy for the case of spherically symmetric bunched beams, we use the particle-core model to calculate r_h for different values of

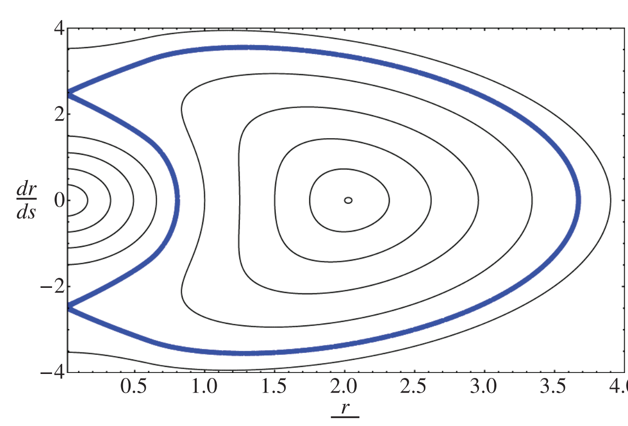

FIG. 1. A stroboscopic plot of test-particle trajectories obtained using the particle-core model for the choice of parameters $\zeta = 1$ and $\mu = 1.6$. The trajectories are plotted each time the envelope reaches a minimum. The thick blue dots correspond to the separatrix that is responsible for the evaporation of particles that form halo. The maximum radius achieved by the most energetic test particle is $r \approx 3.7 r_{bm}$, and serves as an estimate for the halo size r_h.

the dimensionless parameters ζ and μ, and plot the ratio r_h/r_{bm} vs. μ. The results are presented in Fig. 2. We see that all the data for various values of ζ collapse onto the same curve that is well described by a straight line

$$\frac{r_h}{r_{bm}} = A\mu + B, \tag{12}$$

where the coefficients are $A = 1.784$ and $B = 0.7931$. This simple equation provides us with a very accurate estimate of the extent of the halo for bunched beams. Note that although ζ does not explicitly appear in Eq. (12), the halo radius r_h does depend on the beam intensity, initial emittance, and focusing strength through the matched beam envelope r_{bm} given by Eq. (11).

IV. FINAL STATIONARY STATE

As discussed in the previous sections, the main mechanism for the relaxation of the initially mismatched beams is the resonant coupling between the envelope oscillations and the single particle trajectories. Using the particle-core model of Sec. III, we saw that the beam particles can gain significant energy from the envelope oscillations, leading to the formation of halo. However, this simple model neglected the fact that evaporation of particles must also affect the dynamics of the core. From energy conservation, one can envision that as some of the particles gain energy, the core oscillation have to dampen out. In fact, as the halo is being formed, the particles that remain in the core have to lose energy and should condense to the lowest energy states. In principle, one might expect that, in this process, the core will eventually collapse to form a dense condensate at the lowest energy point of the phase space. The incompressibility requirement imposed by the Vlasov dynamics, however, prevents this from taking place—the density in the phase space cannot exceed that of the initial distribution. Hence, the lowest energy core consistent with the initial condition, Eq. (3), must be a waterbag distribution in energy with the phase space density η_0, which extends from the minimum energy up to the *Fermi energy* ε_F that is yet unknown. We refer to the Fermi energy because the core distribution resembles that of a fully degenerate Fermi gas. The incompressibility of the phase space imposed by the Vlasov dynamics plays the role of the exclusion principle. Taking all these into account, we propose that the final stationary state of a bunched beam has a distribution function of the form

$$f_s = \eta_0[\Theta(\varepsilon_F - \varepsilon) + \chi\Theta(\varepsilon_h - \varepsilon)\Theta(\varepsilon - \varepsilon_F)], \tag{13}$$

where

$$\varepsilon(r, v) = \frac{v^2}{2} + \frac{\kappa r^2}{2} + \psi_s(r), \tag{14}$$

is the single particle energy and ψ_s is the corresponding self-field potential. In Fig. 3, we illustrate the phase space distribution described by f_s. The first term inside the square bracket of Eq. (13) represents the core distribution of low energy particles with the phase space density η_0. The second is the halo contribution, which extends from ε_F to the halo energy ε_h, which corresponds to the maximum energy that a particle can gain through the resonant coupling with the mismatched envelope oscillations. From the knowledge of the halo size r_h given by Eq. (12), this energy can be readily computed as $\varepsilon_h = \varepsilon(r_h, 0) = \kappa r_h^2/2 + \psi_s(r_h)$. Based on the previous results obtained for various long range interacting systems,[36] we assume that the halo distribution is also uniform, but with a much lower phase space density $\eta_0\chi$, with $\chi \ll 1$. Hence, the stationary distribution of Eq. (13) has only two yet undetermined parameters, namely, ε_F and χ. These are computed by imposing the conservation of the number of particles in the bunch $\int f_s d\mathbf{r}d\mathbf{v} = N$ and of the average energy per particle obtained by calculating E in Eq. (5) for the distribution $f_s(r,v)$ and requiring it to be equal to the initial energy E_0 of Eq. (6). We now have a self-consistent theory that predicts the stationary state to which a bunched beam will relax. Note that $f_s(r,v)$ only depends on the phase space coordinates through the single particle constant of

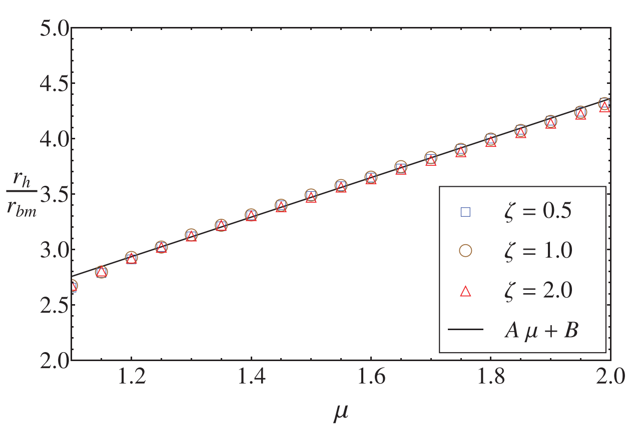

FIG. 2. Ratio between the halo size and the matched radius vs. the mismatch parameter as obtained from the particle-core model for different values of ζ (symbols). We see that the dependence is well approximated by the linear function of Eq. (12), shown by the solid line, irrespective of the value of ζ.

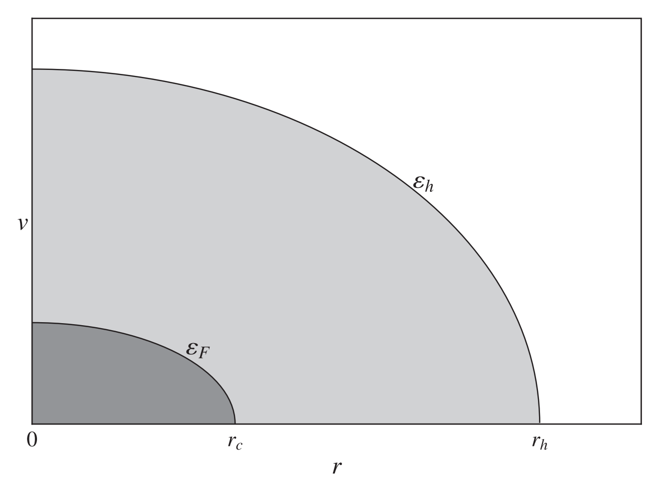

FIG. 3. Schematic of the phase space corresponding to the stationary distribution of Eq. (13). The core particles are distributed up to the Fermi energy ε_F with phase space density η_0 of the initial distribution, Eq. (3). The halo is also uniformly distributed in the phase space, but with a much lower density $\eta_0\chi$ with $\chi \ll 1$. It extends from ε_F to the halo energy ε_h. The maximum radius attained by the core particles, r_c, and the halo particles, r_h, are directly related to ε_F and ε_h, respectively.

motion $\varepsilon(r, v)$ and is, therefore, a stationary solution of the Vlasov equation.

To explicitly calculate the particle distribution of a fully relaxed beam, we need to obtain the stationary self-field potential. To this end, we integrate $f_s(r, v)$ with respect to velocity to obtain an expression for the stationary beam density as function of ψ_s. It reads

$$n_s(r) = \frac{2^{\frac{7}{2}}\pi\eta_0}{3}\left[\chi\Theta(r_h - r)\left(\varepsilon_h - \frac{\kappa r^2}{2} - \psi_s(r)\right)^{3/2} + (1-\chi)\Theta(r_c - r)\left(\varepsilon_F - \frac{\kappa r^2}{2} - \psi_s(r)\right)^{3/2}\right], \quad (15)$$

where r_c is the maximum radius attained by the core particles (see Fig. 3), which is related to the Fermi energy by $\varepsilon_F = \varepsilon(r_c, 0) = \kappa r_c^2/2 + \psi_s(r_c)$. Substituting this into Poisson Equation (2), we obtain a closed ordinary nonlinear differential equation for the stationary self-field potential

$$\frac{d^2\psi_s}{dr^2} + \frac{2}{r}\frac{d\psi_s}{dr} = -\frac{4\pi K}{N} n_s(r), \quad (16)$$

which has to be solved numerically subject to the boundary conditions $d\psi_s/dr(r = 0) = 0$ and $\psi_s(r \to \infty) = 0$. Once the solution to Eq. (16) is known, we can substitute it back in Eq. (13) and compute the corresponding number of particles and the average energy per particle. The problem then reduces to finding the correct values of ε_F and χ that satisfy the conservation of these quantities. To solve this problem, we used an iterative Newton-Raphson method.

V. NUMERICAL RESULTS

In order to test the theory, we performed N-particle dynamics simulations of the beam transport. A spherical symmetry of the charge distribution allows us to use the Gauss law, so that the field at a certain point of radius r is determined only by the number of particles within this radius.[7] This method not only leads to a significant speed up of the simulations, but also avoids the effects of collisions between the individual particles, in a closer agreement with the Vlasov dynamics. In the simulations, $N = 5000$ particles are launched according to the initial distribution of Eq. (3). As the simulation evolves, we continuously monitor the emittance of the beam. When its growth saturates, we consider that the beam has reached a stationary state.

In Fig. 4, we compare the stationary density distribution obtained using N-particle dynamics simulations (dots) with the density distribution predicted by the theory (solid curve), for $\mu = 1.6$ and two different values of ζ. In panel (a), we consider a case of *small* space charge, with $\zeta = 0.1$. The figure shows a good agreement between the simulation and theory. Overall, both the core and the halo are well described by the theory. This is particularly the case for larger r, which correspond to the more energetic halo particles that are the most responsible for the emittance growth

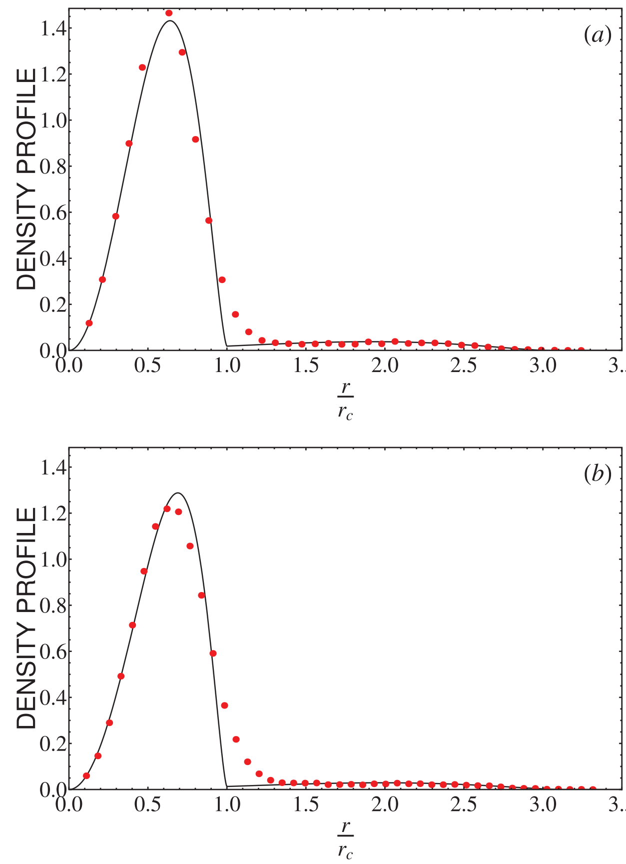

FIG. 4. Stationary state beam density distribution obtained using the theory (solid curve) and the N-particle dynamics simulations (symbols). The parameters are $\eta = 1.6$, $\zeta = 0.1$ in (a) and $\eta = 1.6$, $\zeta = 1.0$ in (b).

and for possible particle losses during the transport. The only significant discrepancy is at the border of the core ($r/r_c = 1.0$), which is not as sharp as predicted by the theory. This suggests that the core particles do not completely *freeze* into the lowest energy states, but can be excited somewhat above the Fermi energy.[15] In panel (b), we consider a more space-charge dominated beam, with $\zeta = 1.0$. Again the agreement is quite good; however, there is an increased discrepancy near the core border, as compared to the lower space charge case of panel (a). Nevertheless, the particle distributions both in the core and the halo are still quite well described by the theory.

Generally, the degradation of a mismatched beam is measured by the emittance growth, which occurs as the beam relaxes. Since the core-halo stationary distribution $f_s(r, v)$ carries all the information about the final stationary state, we can readily compute its associated emittance by using Eq. (13) to calculate the averages in Eq. (8). Taking the ratio of the final and initial emittance, we can theoretically predict the emittance growth during the relaxation process. In Fig. 5, we present the theoretical prediction of the emittance growth (solid curve) as a function of the mismatch parameter μ for $\zeta = 0.1$ [panel (a)] and $\zeta = 1.0$ [panel (b)]. In order to test the theory, we have run several N-particle simulations for different parameter values and have computed the final emittance growth. The results are shown by the symbols in Fig. 5. A strikingly good agreement is found between the theory and the simulations. The good agreement

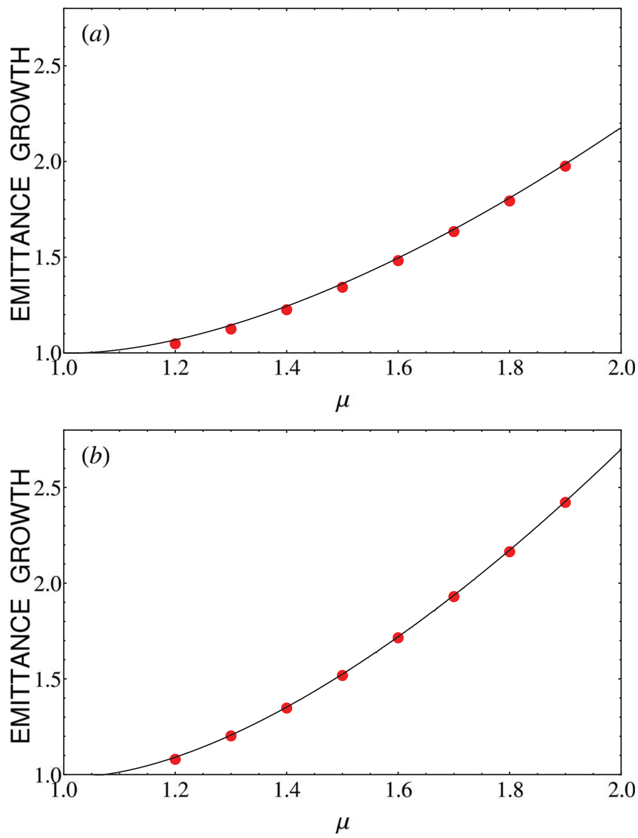

FIG. 5. Emittance growth ϵ/ϵ_0 vs. the mismatch parameter for $\zeta = 0.1$ (a) and $\zeta = 1.0$ (b). The solid curves are the predictions of the theory, whereas the symbols are the results of the N-particle dynamics simulations.

between the theory and the simulations is a consequence of the fact that the emittance growth depends mostly on the halo particles, which are very well described by the present theory, as shown in Fig. 4.

Even though the halo particles play such a crucial role for the beam transport; in practice, it is not easy to explicitly define the "halo."[17] In this regard, the proposed stationary distribution, Eq. (13), presents an interesting feature that the core and the halo contributions are clearly distinguishable. In particular, the halo particles are those whose single particle energies lie in the range between ε_F and ε_h. Therefore, we can have a simple access to the full characterization of halo properties. For instance, we can estimate the fraction of particles that evaporate to the halo during the relaxation process. In our model, this can be determined by computing

$$\frac{N_h}{N} = \eta_0 \chi \int \Theta(\varepsilon_h - \varepsilon)\Theta(\varepsilon - \varepsilon_F) d^3r d^3v. \tag{17}$$

In Fig. 6, we show the results obtained as a function of the mismatch parameter μ for both $\zeta = 0.1$ (solid line) and for $\zeta = 1.0$ (dashed line). As expected, the halo fraction increases with the mismatch μ. However, something less intuitive is the fact that N_h/N decreases with the increase of space charge. This trend has already been observed in the case of unbunched beams.[35] Due to the difficulties mentioned above, we restrict our analysis of the halo fraction to the model, which has been proven very accurate against

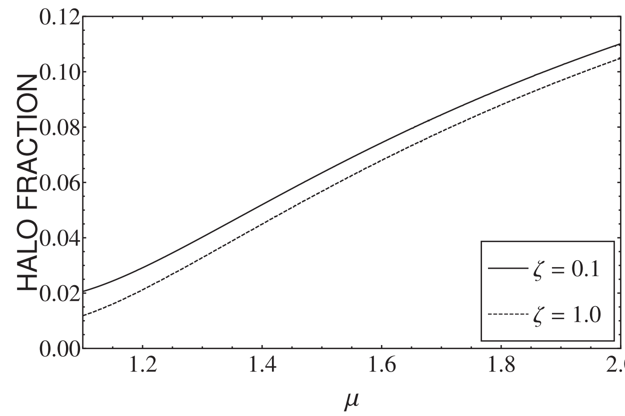

FIG. 6. Fraction of particles in the halo vs. the mismatch parameter μ, as obtained using the theoretical stationary distribution, Eq. (13), for different values of the beam intensity parameter ζ.

numerical results for beam density and emittance growth (see Figs. 4 and 5).

VI. CONCLUSIONS

We have investigated the effect of an initial envelope mismatch on the transport of bunched beams. For now, we have concentrated on the case of spherically symmetric beams. Using the particle-core model, we first studied the resonant coupling between the beam particle trajectories and the envelope oscillations. This low dimensional model allowed us to estimate the maximum distance from the center of the core that a resonant particle can reach. This then defined the halo radius r_h, for which an empirical formula was derived. Taking into account, the constraints imposed on the beam evolution by the Vlasov dynamics, we then proposed a form for the final stationary distribution to which a bunched beam will relax. This core-halo distribution is fully self-consistent, presenting no free fitting parameters. It also contains all the information about the final stationary state, and can be used to predict relevant properties of the beam transport such as the final density profile, the emittance growth, and the fraction of particles that will be ejected from the core to form halo. The theory was tested against N-particle dynamics simulations, showing a good agreement.

ACKNOWLEDGMENTS

This work was partially supported by CNPq and FAPERGS, Brazil, and by the US-AFOSR under Grant No. FA9550-12-1-0438.

[1] M. Reiser, *Theory and Design of Charged Particle Beams* (Wiley-Interscience, New York, 1994).
[2] R. C. Davidson and H. Qin, *Physics of Intense Charged Particle Beams in High Energy Accelerators* (World Scientific, Singapore, 2001).
[3] M. Reiser, J. Appl. Phys. **70**, 1919 (1991).
[4] N. Brown and M. Reiser, Phys. Plasmas **2**, 965 (1995).
[5] E. A. Startsev and S. M. Lund, Phys. Plasmas **15**, 043101 (2008).
[6] R. L. Gluckstern, Phys. Rev. Lett. **73**, 1247 (1994).
[7] H. Okamoto and M. Ikegami, Phys. Rev. E **55**, 4694 (1997).
[8] T. P. Wangler, K. R. Crandall, R. Ryne, and T. S. Wang, Phys. Rev. Spec. Top. Accel. Beams **1**, 084201 (1998).
[9] C. Chen and R. Pakter, Phys. Plasmas **7**, 2203 (2000); R. Pakter and C. Chen, IEEE Trans. Plasma Sci. **28**, 502 (2000).

[10]J. Qiang and R. D. Ryne, Phys. Rev. Spec. Top. Accel. Beams **3**, 064201 (2000).
[11]M. Comunian, A. Pisent, A. Bazzani, G. Turchetti, and S. Rambaldi, Phys. Rev. Spec. Top. Accel. Beams **4**, 124201 (2001).
[12]C. K. Allen, K. C. D. Chan, P. L. Colestock, K. R. Crandall, R. W. Garnett, J. D. Gilpatrick, W. Lysenko, J. Qiang, J. D. Schneider, M. E. Schulze, R. L. Sheffield, H. V. Smith, and T. P. Wangler, Phys. Rev. Lett. **89**, 214802 (2002).
[13]K. Fiuza, F. B. Rizzato, and R. Pakter, Phys. Plasmas **13**, 023101 (2006).
[14]R. P. Nunes, R. Pakter, and F. B. Rizzato, Phys. Plasmas **14**, 023104 (2007); J. Appl. Phys. **104**, 013302 (2008).
[15]Y. Levin, R. Pakter, and T. N. Teles, Phys. Rev. Lett. **100**, 040604 (2008).
[16]M. A. Dorf, R. C. Davidson, and E. A. Startsev, Phys. Plasmas **18**, 043109 (2011).
[17]P. A. P. Nghiem, N. Chauvin, W. Simeoni, Jr., and D. Uriot, Appl. Phys. Lett. **104**, 074109 (2014).
[18]M. Hess and C. Chen, Phys. Plasmas **7**, 5206 (2000); Phys. Lett. A **295**, 305 (2002); M. Hess and C. Chen, Phys. Rev. Spec. Top. Accel. Beams **7**, 092002 (2004).
[19]J. S. Moraes, R. Pakter, and F. B. Rizzato, Phys. Rev. Lett. **93**, 244801 (2004); Phys. Plasmas **12**, 023104 (2005).
[20]M. Hess, IEEE Trans. Plasma Sci. **36**, 729 (2008).
[21]L. C. Martins, F. B. Rizzato, and R. Pakter, J. Appl. Phys. **106**, 043305 (2009).
[22]A. Goswami, P. Sing Babu, and V. S. Pandit, Phys. Plasmas **20**, 103119 (2013).
[23]A. Anderson, Part. Accel. **21**, 197 (1987).
[24]S. Bernal, R. A. Kishek, M. Reiser, and I. Haber, Phys. Rev. Lett. **82**, 4002 (1999).
[25]S. G. Anderson and J. B. Rosenzweig, Phys. Rev. Spec. Top. Accel. Beams **3**, 094201 (2000).
[26]S. M. Lund, D. P. Grote, and R. C. Davidson, Nucl. Instrum. Methods Phys. Res., Sect. A **544**, 472 (2005).
[27]F. B. Rizzato, R. Pakter, and Y. Levin, Phys. Plasmas **14**, 110701 (2007).
[28]R. P. Nunes, R. Pakter, F. B. Rizzato, A. Endler, and E. G. Souza, Phys. Plasmas **16**, 033107 (2009).
[29]E. G. Souza, A. Endler, R. Pakter, F. B. Rizzato, and R. P. Nunes, Appl. Phys. Lett. **96**, 141503 (2010).
[30]E. G. Souza, A. Endler, F. B. Rizzato, and R. Pakter, Phys. Rev. Lett. **109**, 075003 (2012).
[31]B. L. Qian, J. Zhou, and C. Chen, Phys. Rev. Spec. Top. Accel. Beams **6**, 014201 (2003).
[32]J. Zhou, B. L. Qian, and C. Chen, Phys. Plasmas **10**, 4203 (2003).
[33]R. Pakter, Y. Levin, and F. B. Rizzato, Appl. Phys. Lett. **91**, 251503 (2007).
[34]T. N. Teles, R. Pakter, and Y. Levin, Appl. Phys. Lett. **95**, 173501 (2009).
[35]T. N. Teles, R. Pakter, and Y. Levin, Phys. Rev. Spec. Top. Accel. Beams **13**, 114202 (2010).
[36]Y. Levin, R. Pakter, F. B. Rizzato, T. N. Teles, and F. P. da C. Benetti, Phys. Rep. **535**, 1 (2014).
[37]P. A. P. Nghiem, N. Chauvin, M. Comunian, O. Delferriere, R. Duperrier, A. Mosnier, C. Oliver, and D. Uriot, Nucl. Instrum. Methods Phys. Res., Sect. A **654**, 63 (2011).
[38]R. Pakter, G. Corso, T. S. Caetano, D. Dillenburg, and F. B. Rizzato, Phys. Plasmas **1**, 4099 (1994).