Comment on "Thermostatistics of Overdamped Motion of Interacting Particles"

In a recent Letter [1] Andrade *et al.* argued that overdamped particles in contact with a reservoir at $T = 0$ "obey Tsallis statistics with entropic index $\nu = 2$." To justify this, Andrade *et al.* performed a simulation of classical particles in two dimensions, interacting through a pair potential $V(r) = qG(\mathbf{x_1}, \mathbf{x_2})$, where $G(\mathbf{x_1}, \mathbf{x_2}) = qK_0(|\mathbf{x_1} - \mathbf{x_2}|/\lambda)$, and K_0 is a modified Bessel function of order zero. This potential models a vortex-vortex interaction in a type II superconductor, where q is the vortex strength and λ is the effective London penetration length. The particles were confined in a 1D parabolic trap with a potential $W(x) = \frac{\alpha}{2}x^2$, while a periodic boundary condition was used in the y direction (periodicity L_y). The simulation was performed using overdamped molecular dynamics (MD) with the thermostat set at $T = 0$. The resulting stationary density profile was then fitted to a distribution that maximizes the Tsallis entropy of index $\nu = 2$.

In this Comment we will show that the density distribution observed by Andrade *et al.* has nothing to do with Tsallis statistics, but is simply a consequence of classical thermodynamics—particles in contact with a reservoir at $T = 0$ lose their kinetic energy and collapse to the ground state in which the net force on each particle vanishes.

From now on we will measure all lengths in units of λ. We will work in thermodynamic limit $N \to \infty$, at fixed total vortex strength, $q^2N = 1$. We note first that the potential produced by a particle located at $\mathbf{x_1}$ satisfies $\nabla^2 G(\mathbf{x}, \mathbf{x}_1) - G(\mathbf{x}, \mathbf{x_1}) = -2\pi q\delta(\mathbf{x} - \mathbf{x_1})$. Taking into account the periodicity in the y direction, this equation can be solved exactly to yield

$$G(\mathbf{x}; \mathbf{x_1}) = \frac{\pi q}{L_y} \sum_{m=-\infty}^{\infty} e^{(2\pi mi/L_y)(y-y_1)} \frac{e^{-\gamma_m|x-x_1|}}{\gamma_m}, \qquad (1)$$

where $\gamma_m = \sqrt{1 + 4\pi^2 m^2/L_y^2}$. Furthermore, symmetry requires that at equilibrium the total potential inside the system is a function of the x coordinate only, $\varphi(x)$. The force balance on each particle then reduces to $q\varphi'(x) = -\alpha x$. The potential must also satisfy the inhomogeneous Helmholtz equation, $\varphi''(x) - \varphi(x) = -2\pi q\rho(x)$, from which we conclude that the particle density $\rho(x)$ is a restricted parabolic function,

$$\rho(x) = \frac{\alpha}{4\pi q^2}(x_m^2 - x^2)\Theta(x_0^2 - x^2), \qquad (2)$$

where Θ is the Heaviside step function. Note that contrary to the prediction based on Tsallis statistics [1], the density does not go to zero smoothly, instead it extends up to $\pm x_0$, after which it drops discontinuously to zero. The value of x_m is determined by the density normalization, $N = \int \rho(x)dxdy$, $x_m = \sqrt{\frac{2\pi}{\alpha L_y x_0} - \frac{x_0^2}{3}}$, and the extent of the density distribution by the force balance, $q\varphi'(x_0) = -\alpha x_0$. Using the Green function (1), the force balance on a particle at $x = x_0$ requires $\frac{1}{4}\int_{-x_0}^{x_0}(x_m^2 - x^2)e^{-(x_0-x)}dx = x_0$, which reduces to the equation for x_0, $-2 + 2x_0 - x_0^2 + x_m^2 + e^{-2x_0}(2 + 2x_0 + x_0^2 - x_m^2) = 4x_0$. The density discontinuity is a general property of confined particles at $T = 0$, completely missed by Tsallis statistics [2].

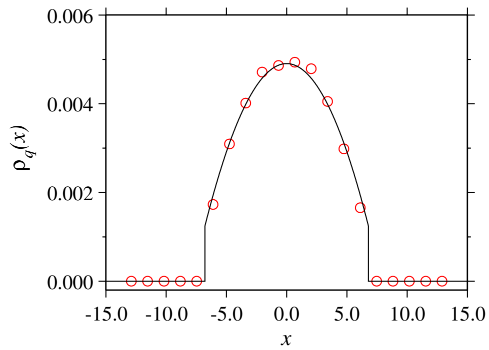

FIG. 1 (color online). The vortex strength density distribution $\rho_q(x) \equiv q^2\rho(x)$ for $\alpha = 10^{-3}$ and $L_y = 20$. Circles are the result of our MD simulation with $N = 1000$ particles, while the solid curve is the prediction of the present theory. There are no adjustable parameters.

This constitutes the exact solution for the particle distribution at $T = 0$. In Fig. 1 we compare it with the MD simulation. As expected, an excellent agreement is found between the theory and the simulation, without any fitting parameters. We conclude, therefore, that the density distribution of particles in contact with a reservoir at $T = 0$ has nothing to do with Tsallis statistics, and everything to do with Newton's second law. Of course, for finite temperature, the density distribution will be described by the usual Boltzmann-Gibbs statistical mechanics [2].

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[1] J. S. Andrade, Jr. *et al.*, Phys. Rev. Lett. **105**, 260601 (2010).
[2] Y. Levin and R. Pakter, arXiv:1105.1316.

