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Wealth redistribution in our small world

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Abstract

We present a simplified model for the exploitation of resources by interacting agents, in an economy with small-world properties. It is shown that Gaussian distributions of wealth, with some cutoff at a poverty line are present for all values of the parameters, while the frequency of maxima and minima strongly depends on the connectivity and the disorder of the lattice. Finally, we compare a system where the commercial links are frozen with an economy where agents can choose their commercial partners at each time step.
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We live in a "small world". Indeed, it has been found experimentally that any two people in the world chosen at random are connected to each other by a short chain of intermediate acquaintances [1]. In general, any network in which the length of such chains is much shorter than the number of people in the network is said to exhibit the *small-world effect*. The simplest model of a complex network consists of a system of N agents where each one has, on average, $2k$ acquaintances. Such a network is called a *random graph*. Random graphs were first studied by Paul Erdös and Alfréd Rényi. According to their model, one starts with N nodes and connect every pair of

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nodes with probability p, creating a graph with approximately $pN(N-1)/2$ edges distributed randomly (for a complete review we refer to Ref. [2]). In some sense, a random graph is the opposite of a completely ordered lattice, the simplest example of which, a one-dimensional lattice, can be thought as a set of agents arranged on a straight line. Random graphs show small-world behavior but do not present clustering. Clustering implies that there is some remaining order in the connections between agents. To take into account clustering, a model for small-world networks has been proposed [3]. In it, the network is modelled as a low-dimensional ordered lattice in which some degree of randomness is introduced to account for the small-world effect. To fix ideas, one assumes N agents arranged on a one dimensional regular lattice, and each agent connected to its $2k$ nearest neighbors. Then, with probability p, one link is disconnected and reconnected to some other agent, chosen at random from the rest of the lattice. The resulting small-world lattice exhibits a clustering that is bigger than that of a random graph and that decreases when increasing p [2,3].

Just as any other human activity, economics takes place in a small world: Together with production factors, social interconnections pervade, and thus influence, all economic activities. The purpose of this paper is to explore one particular aspect of this influence, namely wealth redistribution and the consequences of the small-world property on it. We will model our *economy* as a one-dimensional lattice, every site of which represents an agent. Agents with closer geographical ties will be neighbors on the lattice. For simplicity sake we assume periodic boundary conditions. Each agent i will be characterized by some wealth parameter, w_i, representing its welfare. The exact choice of this parameter is not straightforward. For instance, when thinking of countries in the world economy, the GDP or GNP could be a reasonable choice. In the case of companies, equity or share price are reasonable candidates. To make sure that no agent has an unfair advantage, we choose an initial configuration where the wealth is distributed randomly among agents, with values between 0 and 1. We assume that each agent interacts with its $2k$ nearest neighbors (k to each side) and that, with probability p, each of these $2k$ links is disconnected and reconnected to some other agent chosen at random from the rest of the lattice. The algorithm employed makes sure that self-connections or double-connections between the same sites are forbidden. Also, complete disconnection of one portion of the lattice is avoided. We will consider two cases: In case I, the rewiring of the lattice is performed at the beginning of the simulation ($t=0$). This represents a limit situation in which the *commercial environment* of each agent does not change with time and also where the commercial links are symmetric. In case II, we rewire the lattice at each time step. This is meant to represent a situation in which the agent faces an ever-changing and non-symmetric (business) environment [4].

In the marketplace, all agents strive to improve their situation. In particular, the poorest agent is the one feeling the strongest pressure to move up the ladder. We model this process via an extremal dynamics [5–7]: at each time step, the poorest agent, i.e., the one with the minimum wealth, will take some action trying to improve its economic state [5]. Since the outcome of any such measure is uncertain, this outcome is simulated as a random change in the wealth parameter of this agent; the new value is taken at random from a uniform distribution between 0 and 1. We also assume that

whatever wealth is gained (or lost) by the poorest agent, that will be at the expense of its $2k$ neighbors and we assume it to be equally divided among them. This restriction means that our model is a conservative exchange market model (CEMM) [5,8,9]. There are several motivations for such a description: On the one hand, it can be argued that resources are material objects, and consequently they cannot be created or destroyed by means of exchanging them. On the other hand, even if the use of the CEMM implies that our economy is a zero-sum game, something that may seem at odds with usual economic orthodoxy, we seek to describe a reference behavior, i.e., a behavior against which any non-zero sum system could be compared. Moreover, our results also hold for systems in which the total amount of wealth increases uniformly and smoothly in time (no jumps). Last but not least, our model is parameter free. We would like to remark that, apart from conservation, we do not impose any limit on the wealth evolution, so any negative value is in principle possible, corresponding to an agent having debt rather than wealth. Finally, default is not taken into account, that is, any agent may stay for ever in debt, albeit with a very low probability.

When performing numerical simulations the system arrives, after a transient, at a self-organized critical state [7]. Despite the fact that at any given time step the new assets of the poorest site are chosen at random between zero and one, almost all agents self organize with wealths above a certain threshold and, beyond that threshold, the distribution of wealths is exponential, i.e., there are exponentially few rich agents while the mass of them remain in what we call a *middle class*. Furthermore, the distribution of avalanches follows a power law with a exponent belonging to the Bak–Sneppen universality class [6]. Typical wealth's landscapes are shown in Fig. 1 for a case I (static links) economy and in Fig. 2 for a case II (dynamic links) economy. The common aspect of all those figures is that all distributions are characterized by a threshold $w_c(p,k)$, the *poverty line*, and an exponential decrease above it. Indeed above the poverty line, each one of the distributions can be fitted to a Gaussian tail, or Gibbs distribution, of the form $f(w) \approx \exp^{-w^2/2\sigma^2}$, where $\sigma = \sigma(p,k)$. For fixed k it is possible to write the "width" of the Gaussian as: $\sigma(p) = \sigma_0 + \eta p^{\alpha}$. So $\sigma(p)$ is a kind of effective temperature that increases with the disorder.

In Fig. 3, we show a plot of $\sigma(p)$ for different values of k, as well as the fit to a power of p for both cases. Several conclusions can be drawn from these data. First and foremost, in the absence of disorder the higher the coordination number the broader the distribution. Indeed, σ_0 is a monotonously increasing function of k, having similar values in both cases I and II (This is obvious since for $p = 0.0$, there is no difference between cases I and II). Secondly, while in the first case (I) σ exhibits an only moderate increase with increasing p (indeed is almost constant for $k = 1$), in case II there is a crossing of the curves: for high values of p, the wealth distribution is broader for lower values of k. That means that in the later case the effect of disorder is "screened" when the connectivity is higher. This is particularly evident for $k = 1$. While in case I the distribution is almost independent of disorder, in case II there is a strong dependence on the probability p, so that for $p = 1$ (in case II) the distribution is almost flat with a very low poverty line. Indeed this is the extreme case, as for higher values of k the distribution is less sensitive to a change of the parameters. A possible explanation of the differences between cases I and II is that in the former the

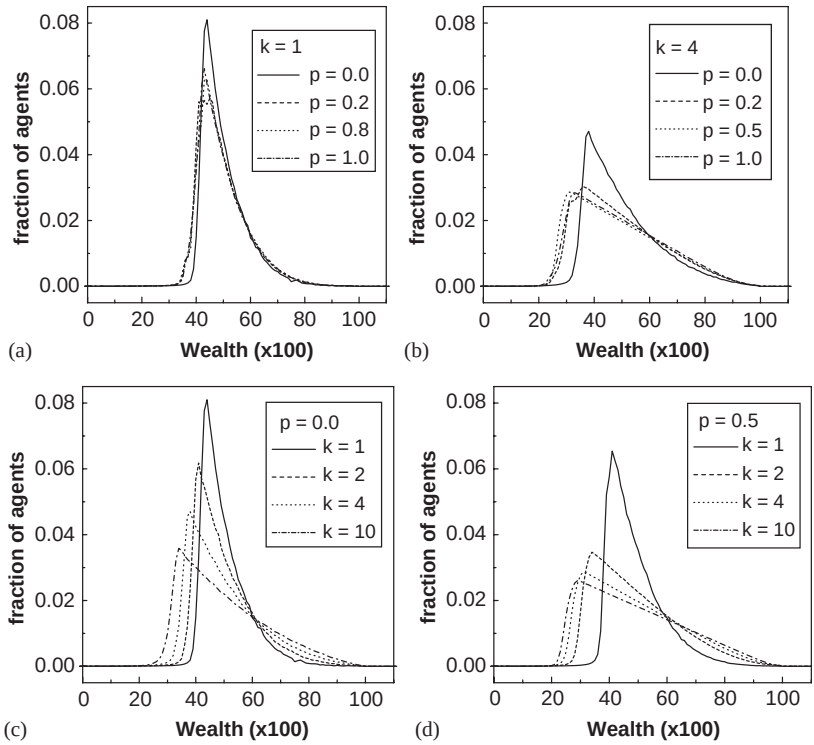

Fig. 1. Wealth distributions for case I economy: distributions for different values of p and $k = 1$ (a) and $k = 4$ (b); distributions for $p = 0$ (c) and $p = 0.5$ (d) and different values of k.

"environment" of each site is frozen, so we are dealing with lattice with symmetric and static bonds and avalanches remain localized in non-local "clusters"; on the other hand, in case II the bonds change all the time, there is no possibility of clustering of agents; actually, since the connections are re-build at all the time steps, there is no fixed lattice.

Finally, if one looks at the number of time steps an agent spends as the richest one, we verify that, for all values of k and p, the system is not ergodic [5]. The system is in a non-equilibrium situation in which some particular agent stays as the wealthiest one during most of the time. This *disparity* tends to disappear when the coordination number and/or the disorder increase. So, it seems that globalization and the dynamics of exchanges produce a more equitable situation, but at the same time the poverty line is lower, thus increasing the number and the poverty of the poor agents but also the number of rich agents, both coming from the middle class.

We have presented here a model of wealth redistribution, based on conservative exchanges, that takes into account the underlying small-world structure of our societies. The model exhibits a Gaussian-like distribution of wealth irrespective of the number of neighbors considered and the degree of disorder. Indeed it seems that when the

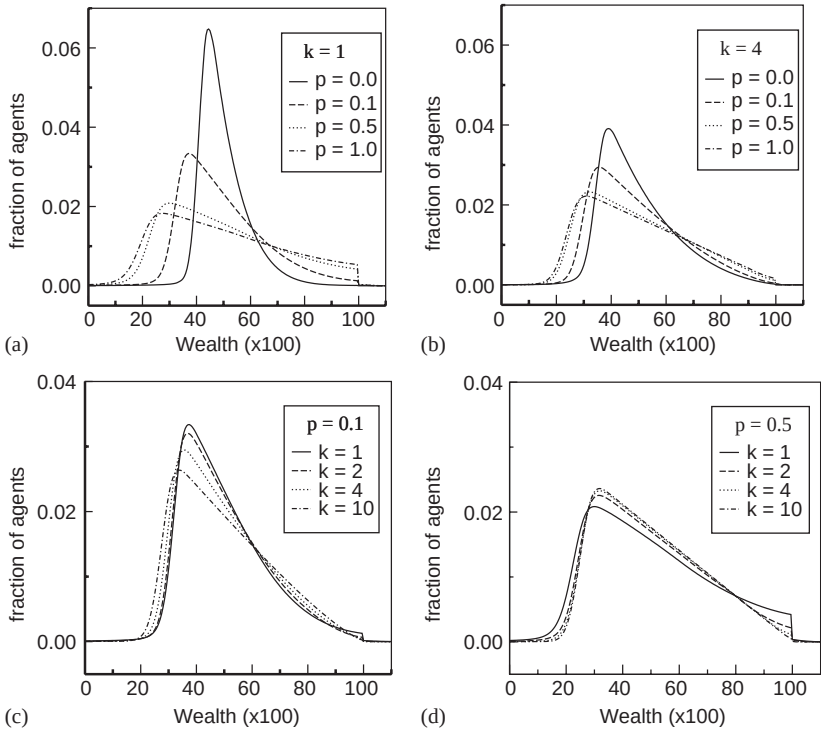

Fig. 2. Wealth distributions for case II economy: distributions for different values of p and $k = 1$ (a) and $k = 4$ (b); distributions for $p = 0.1$ (c) and $p = 0.5$ (d) and different values of k.

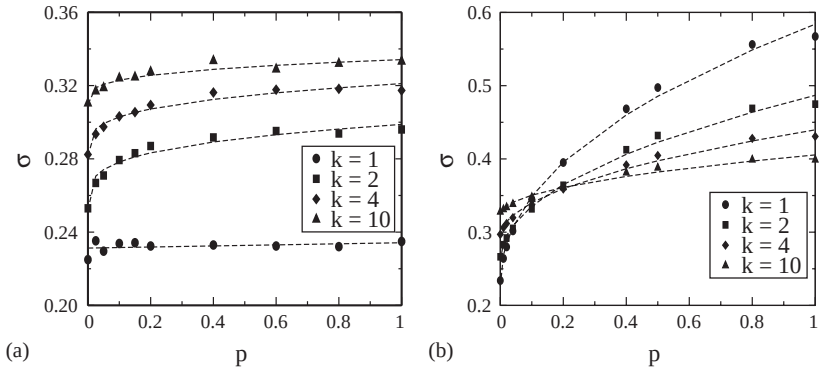

Fig. 3. Behavior of $\sigma(p)$ for several k values: (a) case I economy; (b) case II economy.

commercial environment is fixed in time, the system is more sensitive to the coordination number than to the level of disorder. On the other hand, when the commercial environment changes, the level of disorder becomes very important and may even reverse the *natural* tendency of the wealth distribution to broaden with increasing

coordination number. It has long been suggested that wealth distribution exhibits power law behavior, at least in some strata of society [10]. However, analysis of GNP per capita data seems to indicate an exponential rather than power law distribution, at least for some particular years [11]. In our model two agents may exchange wealth irrespective of their actual resources at the time of the exchange and, consequently, no power laws (Pareto) are observed. For them to appear, the effect of interest rates, taxes, savings and other mechanisms should be included. The analysis presented here is equivalent to the transition between a kind of feudal world $k = 1, p = 0.0$ [5] to a globalized one. Globalization, understood as increase and variability in connectivity to non-geographical neighbors, described particularly by the version II of our model, increases the number of rich agents but at the same time decreases the wealth of the poorest ones. These conclusions may, we hope, be of interest to the debate over the goods and evils of globalization.

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