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Correlation between risk aversion and wealth distribution

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Abstract

Different models of capital exchange among economic agents have been recently proposed trying to explain the emergence of Pareto's wealth power-law distribution. One important factor to be considered is the existence of risk aversion. In this paper, we study a model where agents possess different levels of risk aversion, going from a uniform to a random distribution. In all cases the risk aversion level for a given agent is constant during the simulation. While for uniform and constant risk aversion the system self-organizes in a distribution that goes from an unfair "one takes all" distribution to a Gaussian one, a random risk aversion can produce distributions going from exponential to log-normal and power-law. Besides, interesting correlations between wealth and risk aversion are found.

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Probably one of the most important contributions to the study of the distribution of personal income and wealth was made at the end of the XIXth century by Italian economist Vilfredo Pareto. In his book "Cours d'Economie Politique" [1], he presented the statistical analysis of the income distribution of different European regions and countries. He concluded that the income distribution follows a rather universal law,

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characterized by a logarithmic pattern, described by the formula: $\log N \propto \alpha \log w$, where N is the number of income earners with an income higher than w and the exponent α is named Pareto index. This income distribution is a power-law and Pareto determined the exponent to be $-1.9 \leqslant \alpha \leqslant -1.2$. Analysis of current economic data seems to indicate that Pareto's law is valid for the high income strata of society, while for middle and low income classes the distribution appears to be a log-normal (Gibrat) distribution. Data for Japan [2–4], the United Kingdom and the United States of America [5–7] confirm this idea. Also, we have verified from the 2002 Gross National Income (GNI) data of 179 countries [8], that the GNI of the richer countries can be fitted with a power-law, while for the poorest ones, the best fit is an exponential or log-normal distribution.

A great deal of effort has been devoted to obtain the power-law distribution of the wealthiest strata [9–15]. In previous articles we have proposed a Conservative Exchange Market Model (CEMM) [16,17] with extremal dynamics of the kind of self-organized criticality (SOC) theories [18,19]. The obtained distribution was a Gibbs-exponential type and the results were in good agreement with the distribution of some welfare states such as Sweden [20]. Other authors [10–12] have proposed models in which agents save a fraction of their capital, and only the rest may be exchanged. In the language of economics this *saved part of the resources* is a measure of the agent risk aversion. Following these ideas, we present here a family of models that combine the risk aversion ingredients with Monte Carlo dynamics and extremal dynamics. We found different interesting shapes for the wealth distribution, and in some particular cases a power-law profile is obtained.

Let us consider a set of economic agents characterized by a risk aversion factor $\beta(i)$, so that $[1-\beta(i)]$ is the percentage of wealth the i-agent is willing to risk. An agent with $\beta(i)=0$ is a radical one who risks all his assets while, on the other hand, $\beta(i)=1$ characterizes a totally conservative agent who simply does not play the game. The dynamics of the system is as follows: one chooses two partners that exchange resources; the choice of the two agents may be carried out using extremal dynamics as in previous calculations [16,17], or a Monte Carlo method as in Refs. [10–13]. In the first case we start by determining the site with the minimum wealth, and then we choose at random the other partner of the exchange. In the second case both agents are chosen at random. When considering the case of extremal—minimum—dynamics we model the situation where the poorest agent will try to do something to improve its situation, or else, some external regulator (the government, for example) will act in order to favor the handicapped. In that case one expects a more equitable wealth distribution. The second case is best adapted to represent a kind of stock market, where the transactions occur independently of the fortune of the agents. In both cases, we prescribe that no agent can win more than he puts at stake, so the value that will be exchange is the minimum value of the available resources of each agent, i.e., $\mathrm{d}w = \min[(1-\beta_1)w_1; (1-\beta_2)w_2]$. Finally, we introduce a probability $p \geqslant 0.5$ of favoring the poorer of the two partners, because *a stable society requires that the poor have an advantage in transactions with the wealthy and are protected by particular rights and marketing freedom* [12]. Increasing the probability of favoring the poorer agent is a way to simulate the action of the state or of some type of regulatory policy that tries to redistribute the resources

[17,20]. We consider two cases: (a) a fixed probability p going from 0.5 to 1 and (b) a random value of p, making use of the expression proposed in Refs. [11,12]

$$p = \tfrac{1}{2} + f \times \frac{w_1 - w_2}{w_1 + w_2} \tag{1}$$

w_1 being the wealth of the richer partner and w_2 that of the poorer one, f is a factor going from 0 (equal probability for each agent) to $\frac{1}{2}$. We consider the number of agents N ranging from 10^4 to 10^6 and a number of average exchanges going from 10^3 to 10^4 per agent. In addition to the two different types of dynamics for the system we present here results for (a) β and p uniform (b) β and p random. The discussion of the first case, although rather idealized, is important in order to have a clear idea of the effect of risk aversion and of the probability of having or not a better treatment for the disfavored layers of the population. The second case is a more realistic vision of a heterogeneous society. In all cases we determine the wealth distribution, $P(w)$, as the number of agents with wealth between w and $w + \Delta w$, so it is not the cumulative distribution defined by Pareto, but rather its derivative.

(a) *Uniform β and p*

In this rather hypothetical situation all agents have the same risk aversion parameter β, and all transactions have the same probability p of favoring the poorer agent. Both parameters are constant during the simulation. Let us first present the results for the Monte Carlo simulation, where both agents are chosen at random. The results are summarized in the diagram depicted in Fig. 1. The different regions correspond to different types of resulting wealth distributions. Wealth distributions in Region I are very narrow and Gaussian-like, so we call this region *Utopian socialism* because almost all agents have the same income with a small dispersion. Region II has Gaussian-like distributions too but skewed to higher values of wealth, therefore we named it

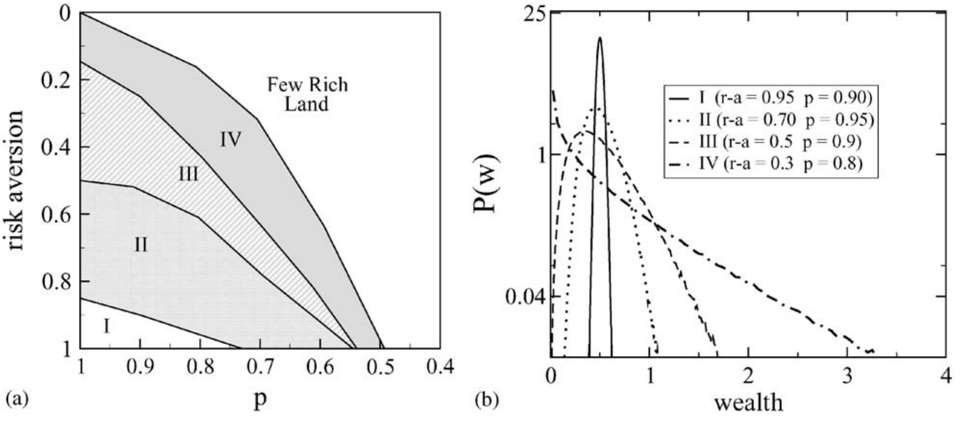

Fig. 1. (a,b) Model with uniform β and p, for $N = 10^5$ and 10^3 transaction per agent. Region I corresponds to a very narrow wealth distribution, *utopian socialism*, region II and III present skewed Gaussians and region IV corresponds to an exponential distribution. Outside these regions there is no true wealth distribution because in the *few rich land* one or few agents concentrate all the available resources while the others have strictly zero wealth.

liberal socialism. Next region (III) has hybrid wealth distribution, Gaussian-like for low wealth values, and exponential for high wealth values, and we call it *moderated capitalism*. In the last region (IV) wealth distribution are exponentials with a tendency to power-laws, so we call this region of *ruthless capitalism*. The different type of wealth distribution in the four regions can be seen in Fig. 1 for some typical values of the parameters β and p. Outside Region IV there is a region of parameters that we call the *few rich land* as the outcome of the dynamics ultimately favors just one (or a very few) agent which concentrates all the available resources. In this later case, since no more exchanges are possible, the system freezes: a very greed economy carries in itself its own destruction. Obviously the $\beta = 1$ line is of no interest, and the same is true for the $p \leqslant 0.5$ region which is always in the few rich land. Simple as it is, this model captures the essence of general economic exchanges, considering the resulting wealth distribution corresponding to different economic policies old and present. It is amusing that just playing with the two numerical parameters of the model very different behaviors are obtained. *Utopian socialism*, for example, exhibits slight economical differences between agents and this is due to the combined force of high values of β and p, which means a repressed market and a strong intervention favoring the poorer. The gradual liberalization of the market, through lower risk aversion (less controlled market) and less state intervention in the social sense (lower values of p), gives rise to more liberal economies with higher inequalities in the income.

We have also performed simulations for this case, but using extremal dynamics of the type described in Refs. [16,17]. That means that one of the partners is the agent with minimum wealth, while the second one is chosen at random. The results are rather odd. For low values of p, the dynamics of the system freezes with no subsequent economic activity, because the agent with minimum wealth has no resources to exchange, so the system proceeds to zero activity. One possible way to overcome this situation should be to consider a different asset transfer rule. On the other hand, for $0.7 \leqslant p \leqslant 1$, the minimum dynamics generates an exponential distribution, where almost all the agents lie in the middle class. However, for some values of $\beta \geqslant 0.7$ and $p \approx 1$ the middle class is split into two income regions with a gap in between. This is probably because of some kind of anti-resonance combined effect of the rules of the dynamics and the conservation constraint.

(b) *Random β and p*

In a more realistic approach to the risk preferences in the population, we consider a disordered risk aversion parameter throughout the system. Each agent is assigned a value of β_i, drawn at random from a uniform distribution on the interval $(0, 1)$. We consider only quenched disorder, where each agent maintains his risk aversion despite the outcome of the exchange. Simulations have been carried out for different values of the probability p and also for the complete range of the asymmetry parameter f in Eq. (1).

Some typical distribution curves are shown in the left panel of Fig. 2. For $f = 0$, i.e., when trades do not favor either of the partners, the distribution of wealth becomes, slowly but steadily, a delta function at $w = 0$, with the wealth concentrated in one or a few agents, and the rest owing effectively nothing (these results are not shown in Fig. 2). This can happen, even though each agent risks only part of his capital at each

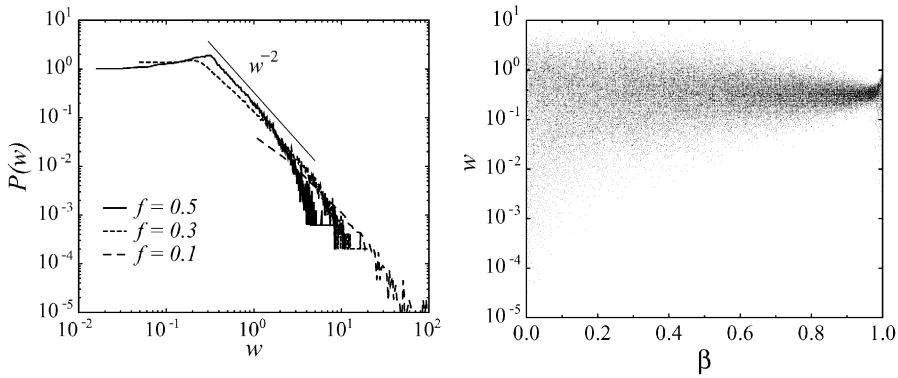

Fig. 2. Left panel: Wealth distribution for random β and Monte Carlo dynamics, the distribution is calculated for $N = 10^5$ and 10^4 exchanges per agent on average. Results are shown for three values of the asymmetry parameter, as shown in the legend. A $P(w) \propto w^{-2}$ curve is also shown to guide the eye. Right panel: Correlation between wealth and saving parameter for $N = 10^5$, $f = 0.5$.

interaction, because there is no restriction in the amount he can loose in successive exchanges. The situation is a multiplicative process with $w = 0$ as an absorbing point. On the contrary, when the externally imposed asymmetry favors, statistically, the poorer agent, we observe the emergence of a distribution characterized by three regimes. There is a peak in the distribution that separates a poor class to the left, from a middle class to the right: see for example the full line in Fig. 2, corresponding to $f = 0.5$; for $f = 0.1$ the peak correspond to a very small value of the wealth and it is not included in the figure. The middle class follows a power-law distribution of the form $P(w) \sim w^{-\alpha}$, with α depending on the value of f, and $\alpha \approx 2$ for $f = 0.5$. This value of α correspond to a Pareto exponent equal to 1. Finally, there is a transition from this power-law behavior to a Gaussian-like tail encompassing the wealthier agents. This Gaussian tail is not an effect of the finite size of the system, as has been verified for system sizes up to 10^6. Also one can observe that there is a finite number of very poor people ($w \simeq 0$), contrasting with the previous case (a). We have also represented on the right panel of Fig. 2 the correlation between wealth and risk aversion. One observes that the range of wealth variation is up to five magnitude orders. Besides that, on average, the higher values of income are consistent with a high risk aversion, while the highest individual wealth corresponds to a risk-loving agent. But also the lowest incomes belong to risky agents, as expected.

When considering extremal (minimum) dynamics, the results are quite different as they are shown on Fig. 3. It is possible to see that there is a minimum threshold, or *poverty line*, and also that the distribution is very narrow compared to the Monte Carlo case. One observes that just a few people lie below the poverty line and its value is around 0.38 for $f = 0.5$. Moreover, the high income region behaves in a Gaussian-like way, following a law of the type $P(w) \approx \exp[-a(w - w_o)^2]$ with $a \approx 1.1$ and $w_o \approx 0.7$ for $f = 0.5$. The poverty line is well seen too on the right panel of Fig. 3, all the agents are above 0.38 and it is clear that a low risk aversion favors, on average, a higher wealth.

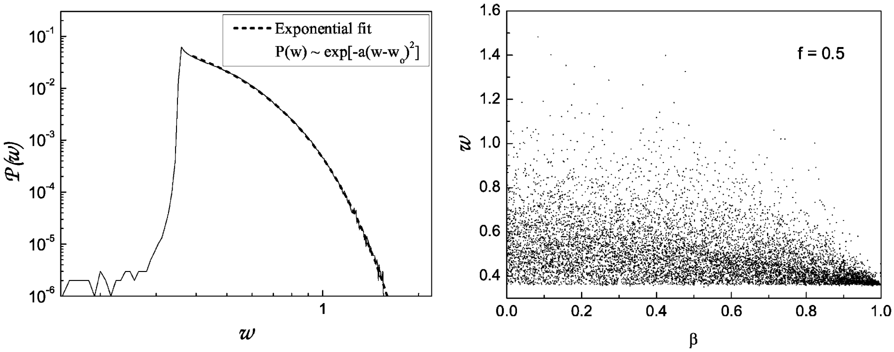

Fig. 3. Left panel: Wealth distribution for extremal dynamics and random β, for $N = 10^5$ and 10^4 exchanges per agent, in average, with $f = 0.5$. The high income region is fitted by a kind of Gibbs distribution with $a \approx 1.1$ and $w_o \approx 0.7$. Right panel: Correlation between wealth and saving (β) for the same values of N and f.

To conclude: taking into account risk aversion (or saving, as defined by other authors [10–12]) generates a rich variety of wealth distributions, when combined with different choices of trading rules. For some particular values of the exchange probability p and a random choice of β a power-law profile is obtained. Here we have compared in detail an extremal and Monte Carlo dynamics for constant and random risk aversion and a simple exchange bias. Extremal (minimum) dynamics provides a more equitable society, in the sense proposed in the classical work by Rawls [21]: *no redistribution of resources within...a state can occur unless it benefits the least well-off*, and this is clear because of the existence of a poverty line and the emergence of a wealth distribution with a large middle class. Monte Carlo dynamics seems to better reproduce a capitalist society: there are very many people with almost zero income and one can observe a power-law distribution for the higher layers of the social spectra. Of course, in the real world, agents can change their risk strategy by considering his own success or failure in increasing his wealth. This possibility will be discussed in a forthcoming article.

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