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Inequalities of wealth distribution in a conservative economy

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Abstract

We analyze a conservative market model for the competition among economic agents in a society with conserved total wealth. A minimum dynamics ensures that the poorest agent has a chance to improve its economic welfare. After a transient, the system self-organizes into a critical state where the wealth distribution have a minimum threshold, with almost no agent below this poverty line. Also, very few extremely rich agents are stable in time. Above the poverty line the distribution follows an exponential behavior. The local solution exhibits a low Gini index, while the mean field solution of the model generates a wealth distribution similar to welfare states like Sweden.

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1. Introduction

The study of the distribution of the income of workers, companies and countries started a little more than a century ago when Italian economist Vilfredo Pareto investigated data of personal income for different European countries and found a power-law distribution that seems not to depend on different economic conditions of the countries. In 1887, in his book *Cours d'Economie Politique* [1] he asserted that in all countries and times the distribution of income and wealth follows a power-law behavior where

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the cumulative probability $P(w)$ of people whose income is at least w is given by $P(w) \propto w^{-\alpha}$, where the exponent α is named Pareto index. This universal power law is now called Pareto law and it was shown by Pareto himself that the exponent α for several countries was $1.2 \leqslant \alpha \leqslant 1.9$. However, recent data suggest that Pareto's distribution does not agree with observed data over the middle range of income, while it is generally accepted that it provides a good fit to the distribution of high range of income. Many other distributions of income were proposed, supported by international empirical data, and the distribution behaves in a log-normal, exponential or Levy way [2] in the middle income range. For instance, there are data from Japan and USA that are fitted by a log-normal distribution in middle range plus a power law for high income one [3], data from United Kingdom income and wealth fitted by an exponential law (middle range) and a power law (high range) [4] and data from Brazil [5] worker's income and enterprise capital that are fitted by a log-normal plus a power law for high range of income. [1]

One of the goals of the study of wealth distribution is to determine the social inequalities implied in a given model of economic exchanges. Many ways to quantitatively measure the fairness of a given income distribution have been proposed in the economic science. Among them, the Gini index is well known and frequently used. In brief, for any given wealth density function $p(x)$, with mean ρ and a given number $v \in [0,1]$, $P(\leqslant v) = \int_0^v p(x)\,\mathrm{d}x$ represents the cumulative fraction of population with wealth below v. The Lorenz function, $L(v)$, is defined as the fraction of the total wealth which is attributed to the poorest $100v$ percentage of the society. It is easy to show that $L(v) = \frac{1}{\rho}\int_0^v xp(x)\,\mathrm{d}x$ and the Gini index can be then defined as $G = 2\int_0^1 (v - L(v))\,\mathrm{d}v$ [6]. It assumes values between 0 percent (perfect equality) and 100 percent (perfect inequality).

Recently, we developed a simple parameter free model for the competition among different economic agents within a conservative economy. Results for the wealth distribution has been published elsewhere [7] and also considering the effect of a small world network [8]. Within this model we obtained an exponential Gibbs-style distribution for the wealth, with a finite lower threshold or poverty line, that is characteristic of models with extremal dynamics. The idea of a society that take measures in order to improve the situation of the most impoverished is compatible with the propositions of John Rawls, in his book "A Theory of Justice" [9], directed towards an inventive way of securing equality of opportunity as one of the basic principles of justice. He asserts that *no redistribution of resources within…a state can occur unless it benefits the least well-off* and this should be the only way to prevent the stronger (or richer) from overpowering the weaker (or poorer).

In the following, we review the assumptions and principal results of the model and we present some new calculations concerning the time evolution of the system. The determination of the Gini index and the comparison with wealth distribution of several countries will be presented in Section 2 along with the conclusions.

The Conservative Exchange Market Model (CEMM) [7] is a simple macroeconomic model that consists of a one-dimensional lattice with N sites and periodic boundary

[1] As 500 maiores sociedades anônimas do Brasil, Fundação Getulio Vargas, www.fgv.br/IBRE/CEAE/arq/x500M.xls

conditions, where every site represents an economic agent (individuals, industries or countries). To each agent it is assigned some wealth-parameter that represents its welfare, like the GNI for countries or salary for individuals. We choose an initial configuration where the wealth is a number between 0 and 1 distributed randomly among agents. The dynamics of the system is supported on the idea that some measure should be taken to modify the situation of the poorest agent. In this context, we model this process by a minimum dynamics [10]: at each time step, the poorest agent, i.e., the one with the minimum wealth, will take some action, trying to improve its economic state. Since the outcome of any such measure is uncertain, the minimum suffers a random change in its wealth [7]. We assume that whatever wealth is gained (or lost) by the poorest agent it will be at the expense of its neighbors and we assume that it is equally debited (or credited) between its two nearest neighbors on the lattice, making the total wealth constant.

Numerical simulations on this model showed that, after a relatively long transient, the system arrives at a self-organized critical (SOC) state with a stationary wealth distribution (Fig. 1 of Ref. [7]) in which almost all agents are beyond a certain threshold. We have also considered the case of random exchanges (annealed or mean field version of the model). This corresponds to a situation in which the agents with which the exchange takes place are chosen at random and not based on geographical proximity. In the nearest-neighbor version of the model, one founds a minimum wealth or poverty line that is $\eta_T \approx 0.4$ and above this threshold the distribution of agents is an exponential $P(w) \approx \exp(-w^2/2\sigma^2)$, with $\sigma = 0.228$ [8]. In the mean-field case the model exhibits a lower threshold, $\eta_T \approx 0.2$, and beyond it also an exponential distribution with a higher value of $\sigma = 0.567$ [8].

The temporal evolution, in the SOC state, of the position of the system's minimum and maximum wealth is quite different. While the site of minimum wealth is changing continuously, generating avalanches of all sizes among neighbors, the wealthiest site is stable over relatively long periods of time [7] as it is represented by the lines in the Fig. 1. Indeed, even when affected by an avalanche the maximum frequently recover

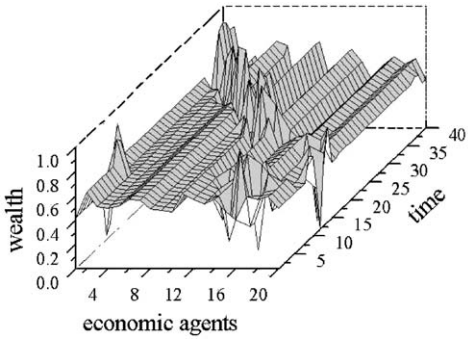

Fig. 1. Snapshot of avalanche propagation for the local case. Note that inside the avalanches the maximum can go up to very high values, but just for one or two steps, while the sites with relative maximum wealth are very stable in time.

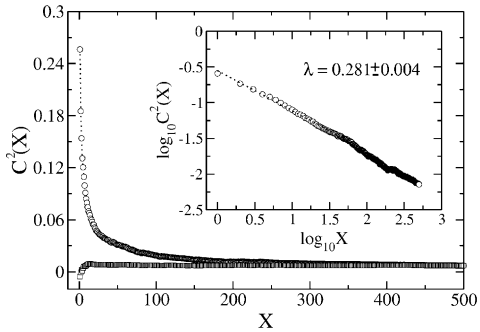

Fig. 2. Spatial correlation for the maximum wealth site (upper curve and inset) and for the average wealth site (lower curve) for 1001 agents averaging over ten seeds and 5×10^3 time steps.

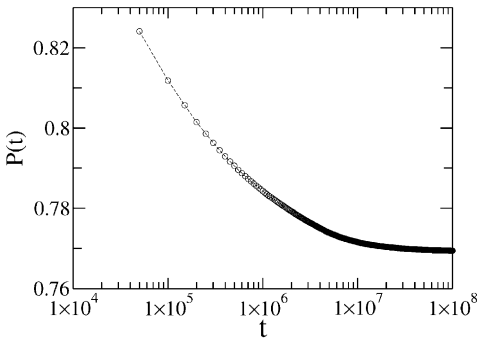

Fig. 3. Probability of an agent becoming wealthier in a time step for a market with $N = 1000$ agents.

its status after a short time. Not only the absolute maximum is stable, but also a privileged group, whose wealth is around the same value of the maximum, remains in its prosperous position for quite a while. This group is also privileged by having high wealth values, well above the market average. Those results are confirmed by the determination of the time average of spatial correlation, $C^2(X = |x - x_0|) = \langle w(X)^2 \rangle_X - \langle w \rangle^2 \approx X^{-2\lambda}$, being x_0 the site chosen as origin. The behavior of the correlation function is very different for the maximum wealth site compared with an average wealth site, as it is visualized in Fig. 2. One can verify that, even when considering long periods of time compared with the number of agents, the time average of the correlation function for the maximum site does not approach the average value. This result confirms the stability of the maximum site.

On the other hand, in spite of the stability of the richer agents, one observes that the probability that the minimum asset agent improves its situation in a time step is relatively high. Fig. 3 shows that this probability is very high at the beginning of the simulation, then it decreases during the transient to finally converge to a finite value,

$p \approx 0.77$. We remark that the probability is well above 0.5, so this minimum dynamic has the effect of improve, in the average, the situation of the poorer agents.

2. Income inequality

Inequalities are related to social and economic phenomena and its origin is a subject of much debate. The fraction of rich and poor people in a country depends on the inequalities of the income distribution. The standard way to determine the inequality of a wealth distribution is with the help of quintile tables that show the percentage of national income received by equal percentiles of individuals or households ranked by their per capita income levels. For example in a welfare state as Sweden the richest 20 percent of the population receives about 4 times more than the poorest quintile, while in a country like Brazil, with low social protection, the average worker's salaries are very low, so the richest quintile receives more than 25 times more than the poorest one. Table 1 gives the quintile data for a few countries, while full data can be obtained in the World Bank site [11,12].

To measure income inequality in a country and to make a comparison among countries more accurately, economists use Lorenz curves and Gini indexes. A Lorenz curve plots the cumulative percentages of total income against the cumulative percentages of recipients. In Fig. 4 we have represented the Lorenz curves for Brazil, Sweden and Spain. For comparison, we have plotted in the same figure the Lorenz curve of an hypothetical absolutely equalitarian income distribution and this is the straight line with slope equal 1. This figure shows that Brazil's Lorenz curve strongly deviates from the hypothetical line of absolute equality while that for Sweden lies much nearer and that for Spain in the middle of both. This means that of these three countries, Brazil has the highest income inequality and Sweden the lowest. We also constructed Lorenz curves for the local and global cases of CEMM model (Fig. 4). Clearly we see that the global case have a Lorenz curve very similar to that of Sweden, while the local case is close to perfect equality. The Gini index is even more convenient than a Lorenz curve when the task is to compare income inequality among many countries; it is calculated as the area between a Lorenz curve and the hypothetical line of absolute equality. It assumes values between 0 percent (perfect equality) and 100 percent. Of course, real Gini indexes are always in between. We calculated the Gini index for the local and

Table 1
Percentage of income in each population quintile for some selected countries [12]

	USA (1997)	Sweden (1992)	Spain (1990)	Brazil (1998)
Poorest 20%	5.2	9.6	7.5	2.2
Fourth 20%	10.5	14.5	12.6	5.4
Third 20%	15.6	18.1	17.0	10.1
Second 20%	22.4	23.2	22.6	18.3
Richest 20%	46.4	34.5	40.3	64.1
Gini index	40.8	25.0	32.5	60.7

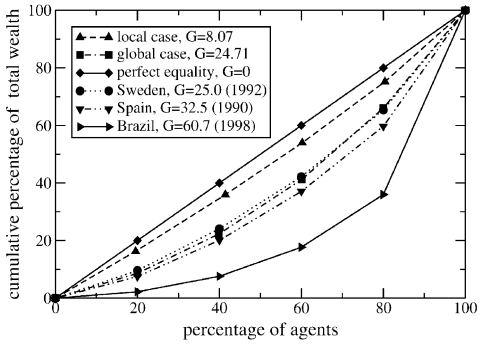

Fig. 4. The Lorenz curves and the Gini index for ideal, local and the global case. Also represented are the World Bank data for Brazil, Sweden and Spain [12].

global cases. For the local case, the Gini index is 8.07, close to perfect equality, while its value is 24.71 for the global one (Fig. 4). This last value is very close to that of Sweden (25.0), and also close to values of other European developed countries.

The analysis of the Gini index for the present model shows that economic inequalities increase when going from a local to a global (mean field) description. Moreover, the poverty line in global case is well below that of the local case [7]. It is not evident that in real economies globalization necessarily increases inequalities. It is true that in the case of some Third World countries recent data show evidences of a deterioration in the fairness of income distribution [11] in the last ten years. But it is possible that austerity policies designed to guarantee the reimbursement of the external debt deserve the liability for this situation. Coming back to the model, one remarks that the local version generates a wealth distribution with a very low Gini index, so very close to an equitable society. In the global case, the similarity of the Lorenz curve obtained from the model with that of Sweden is remarkable, even though the model is too simplified to describe all the aspects of a modern economy. Another interesting aspect is that the minimum dynamic favors the poorest agent, so promoting wealth redistribution in the sense defined by Rawls [9]. It seems that a minimum dynamics, as that proposed in the Bak–Sneppen model [10], when applied to economics, is a way to ensure to the poorest agent a chance to improve its situation. We conclude that the model, in spite of its simplicity, is able to reproduce some properties of modern economies, particularly the wealth distribution of welfare societies, and it indicates a way to improve some extreme inequality situations in other countries with high Gini indexes.

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