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Economic exchanges in a stratified society: End of the middle class?

M.F. Laguna[a], S. Risau Gusman[b], J.R. Iglesias[b,*]

[a]The Abdus Salam International Center for Theoretical Physics, Strada Costiera, 11 (34014) Trieste, Italy
[b]Instituto de Física, UFRGS, Caixa Postal 15051, 91501-970, Porto Alegre, RS, Brazil

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Abstract

We study the effect of the social stratification on the wealth distribution on a system of interacting economic agents that are constrained to interact only within their own economic class. The economical mobility of the agents is related to its success in exchange transactions. Different wealth distributions are obtained as a function of the width of the economic class. We find a range of widths in which the society is divided in two classes separated by a deep gap that prevents further exchange between poor and rich agents. As a consequence, the middle wealth class is eliminated. The high values of the Gini indices obtained in these cases indicate a highly unequal society. On the other hand, lower and higher widths induce lower Gini indices and a fairer wealth distribution.

Keywords: Econophysics; Wealth distribution; Hierarchical systems; Pareto's law; Economic classes

Several models of capital exchange among economic agents have been recently proposed [1–6] trying to explain the emergence of power law distributions of wealth obtained by Pareto more than a century ago [7]. In his original work, Pareto analyzed the distribution of the income of workers and companies in different countries, and asserted that in all countries and times the distribution of income and

*Corresponding author. Tel.: +55 51 3316 6466; fax: +55 51 3316 7286.
E-mail addresses: mlaguna@ictp.trieste.it (M.F. Laguna), srisau@if.ufrgs.br (S.R. Gusman), iglesias@if.ufrgs.br (J.R. Iglesias).

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wealth follows a power law behavior, where the cumulative probability $P(w)$ of people whose income is at least w is given by $P(w) \propto w^{-\alpha}$, with $1.2 \leqslant \alpha \leqslant 1.9$ [7]. Recent international empirical data suggest that Pareto's distribution provides a good fit to the income distribution of different countries in the range of high income. Nevertheless, it does not agree with observed data over low and middle range of income, for which different distributions were proposed [5,8–12].

Most of the models consider an asymmetric probability that the poorer agent might be privileged in the exchanges. Increasing the probability of favoring the poorer agent is a way to simulate the action of the state or of some type of regulatory policy that tries to redistribute the resources [4,13–15]. Moreover, almost all these models consider exchanges of agents picked up either at random [16], or following an extremal dynamics [14,17]. The obtained distribution is a Gibbs-exponential type in most cases, the results being in good agreement with real distributions of welfare states [15]. Other authors have proposed models in which agents have a risk aversion [1,3,4,16]. The effect of this factor on the wealth distribution also gives rise to a Gibbs-exponential distribution in most cases and shows a power law behavior in some limits [3,16].

All those models have a common point: no correlations between the wealth of the agents and the probability of interaction between them are considered. However, the fact that people tend to strongly interact mainly with others of their own social and economic class, might be a determinant factor in the wealth distribution of a population. An example of that is the work of Inaoka et al. [18]. They analyzed the exchanges in Japanese banks and found that big banks have more interactions between them and with the others than the small ones. A work by the present authors considers this fact by including a correlation between the success of an agent in their economic exchanges and his degree of connectivity [19].

In this paper, we consider a society in which agents are constrained to interact with others that belong to the same economic class. We introduce a parameter that establish the maximum difference in wealth two agents can have in order to interact. This kind of approach was previously used to study the formation of a public opinion as the result of social interactions [20].

We consider a population of N interacting agents characterized by a wealth w_i and a risk aversion factor β_i. We chose as initial condition for both these parameters a uniform distribution between 0 and 1 [21].[1] For each agent i, the number $[1-\beta_i]$ measures the percentage of wealth he is willing to risk. We consider this percentage as an individual fixed parameter in the whole process. But while the value of β_i remains fixed, the value of $w_i = w_i(t)$ will change as a consequence of the interactions. At each time step t we first select the two agents that will exchange resources in the following way: we choose at random one agent i and, also at random, a second one j that belongs to the same economic class of agent i. It means that agent j is randomly chosen from the subset of the system for which $|w_i(t) - w_j(t)| < u$. The parameter u measures the "width" of the economic class,

[1] Taking as initial condition a fixed value of wealth for all the population gives the same long time distribution.

and determines the number of agents that can interact with agent i at the time t. We also establish that no agent can earn more than the amount he puts at stake. Then, the quantity to be exchanged is the minimum value of the available resources of both agents, i.e., $dw = \min[(1-\beta_i)w_i(t); (1-\beta_j)w_j(t)]$. Finally, following previous works we consider a probability $p \geqslant 0.5$ of favoring the poorer of the two partners [4,16],

$$p = \frac{1}{2} + f \times \frac{|w_i(t) - w_j(t)|}{w_i(t) + w_j(t)} , \tag{1}$$

where f is a factor going from 0 (equal probability for both agents) to $1/2$ (highest probability of favoring the poorer agent). Thus, in each interaction the poorer agent has probability p of earn a quantity dw, whereas the richer one has probability $1-p$.

When performing the simulation with these rules, after a transient the system arrives to a stationary state where the wealth has been redistributed. We present numerical simulations for a system of $N = 10^4$–10^5 agents, and several values of f and u. Stationary state analysis was made after $t = 10^5 N$ interactions.

We first describe the process of economic exchange between agents. As we stated before, at $t = 0$ each agent receives a wealth in the interval $[0, 1]$. As time evolves, exchanges between agents generate a redistribution of wealth that, although dependent on the values of f and u, presents some common features. The first one is that in all the cases the number of people with very low income increases. As the model is conservative, the resources of impoverished agents contribute to increase the wealth of other agents. For low values of f, a sharp peak appears for income equal to zero. This means that a significative fraction of the population had quickly lost all their resources. The exchange process is different for high values of f and u: a maximum appears for intermediate wealths and the distribution at long times seems to be fairer.

A better characterization of this model can be done by analyzing the stationary states obtained as average of several runs with different seeds and the same value of the parameters involved. Results are shown in Fig. 1 for a system of $N = 10^5$ agents and 100 runs for each family of parameters. For very small values of u a peak in $w = 0$ is obtained, as well as a rather flat distribution for higher values of wealth. This means that very narrow classes prevent any kind of redistribution. In other words, there is a irreversible and strong transfer of income from lower to higher classes. The high peak for income zero is present for low and intermediate values of u when the probability of favoring the poorer agent is low ($f = 0.1$ in the figures). Only for a very high value of u the distribution is more uniform, with a peak for intermediate values of w. For $f = 0.1$ and 0.3 the peak in $w = 0$ disappears for $u \gtrsim 100$, whereas for $f = 0.5$ it happens for $u \sim 1$. But more interestingly, for all the values of f studied we found an intermediate range of u in which a formation of two classes is obtained. One class corresponds to agents with a very small wealth, whereas the other is formed by rich agents. The two classes are separated by a gap that prohibits further wealth exchanges between the low and the high classes. This has as a consequence the elimination of the middle wealth class. The minimum of the gap happens for $w \sim u$, suggesting that the permitted range of interaction appears as a scale for the system. This polarization of the agents in two classes reminds of the

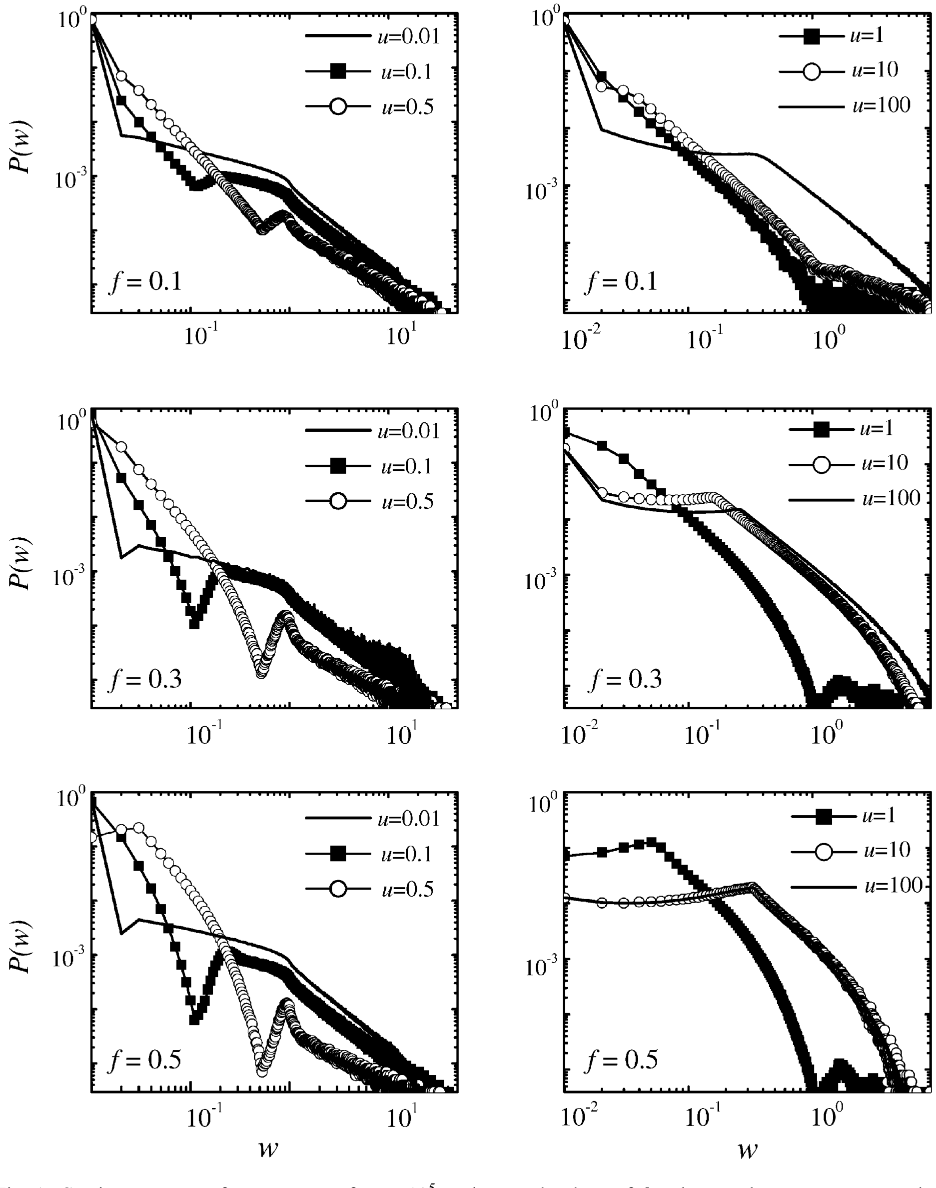

Fig. 1. Stationary states for a system of $N = 10^5$ and several values of f and u. Each curve correspond to an average of 100 runs. Note that the minimum of the gap corresponds to $w{\sim}u$.

polarization of opinions that has been observed, for example, in Ref. [20] (in spite of the fact that the rules to change opinions are different than the present ones to change the wealth). For big enough values of u no gap is observed, as very few agents can attain a high wealth. The obtained distribution is similar to the model without restrictions [16].

The gap between classes is also observed in the plots of the correlation between wealth and risk aversion of Fig. 2. We have represented a particular snapshot of a stationary state configuration. For low values of f ($f = 0.1$ in the plot), the poorest people have practically zero wealth and, consequently, they are not seen in the logarithmic plot. For $u = 0.05$, a gap appears in all the range of β, being much wider in the region $\beta < 0.5$. This means that agents with a low risk aversion can be only very rich or very poor in the stationary state (the last ones do not appear in the plot). For $u = 100$, we find a distribution in which agents with high values of beta are in the middle class, as expected for agents that do not risk their assets, while the richest and poorest agents are those with very low values of the risk aversion parameter β. The intermediate values of β assures agents in all the range of wealth, whereas the poorest agents have the lowest risk aversion (again, they do not appear in the plot).

The situation changes if the probability of favoring the poorer agent is high ($f = 0.5$ in the plot). For $u = 0.05$, the gap between the upper and low classes is present in all the range of β and can be clearly observed in the plot because the poorest agents have a low but finite wealth in the stationary state. Agents with the lowest values of β may be in the richest or poorest stripe of the population. A higher value of the threshold ($u = 100$ in the plot) does not present a gap. The richest and poorest agents are those with very low values of the risk aversion parameter β.

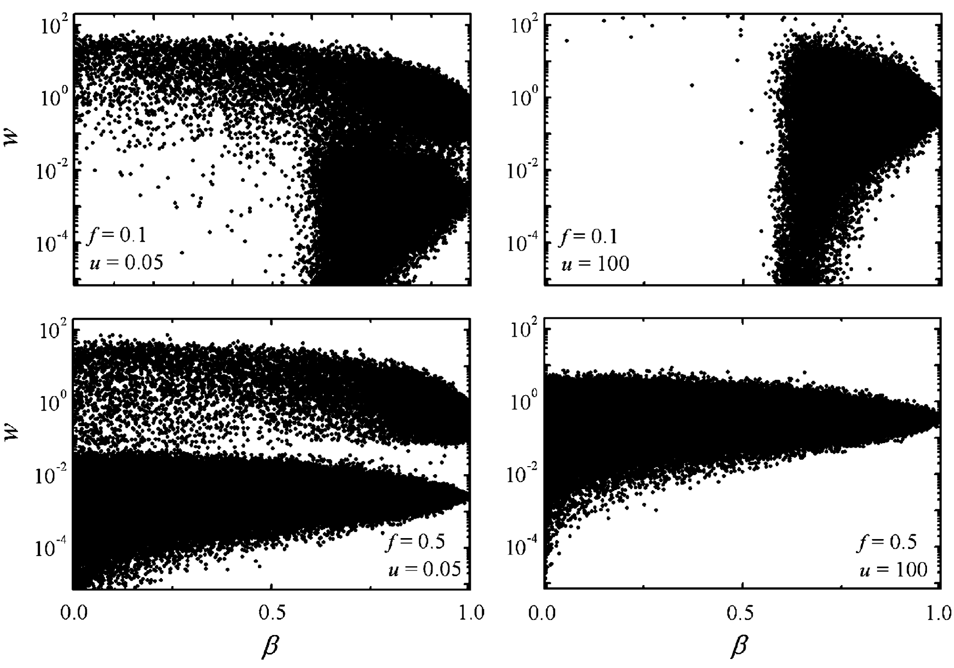

Fig. 2. Logarithmic plot of a snapshot of wealth, w, vs. risk aversion β for a population of $N = 10^5$ agents. Values of u and f are indicated in each panel. Each point corresponds to an agent. For $f = 0.1$, agents with very low wealth ($w \sim 0$) are not visible in the logarithmic plot.

Table 1
Gini coefficients for the three values of f treated in the article and several values of u

f–u	0.005	0.01	0.05	0.1	0.5	1	5	10	100
0.1	0.819	0.916	0.952	0.963	0.982	0.992	0.987	0.976	0.915
0.3	0.819	0.840	0.953	0.964	0.978	0.971	0.920	0.840	0.674
0.5	0.818	0.915	0.950	0.974	0.962	0.932	0.690	0.488	0.472

The columns in each case correspond to the different values of u, whereas the rows correspond to the different values of f.

Finally, in Table 1 we show the Gini coefficients for the parameters studied. As can be observed, the highest values correspond to the intermediate range of u, where the gap is present. Only for high values of f and u we obtain Gini indexes close to the ones obtained from data of different countries: while the Gini index that we obtain can go very close to 1, the highest values observed in real societies are of the order of 0.7.

We analyze the behavior of a society in which agents are constrained to interact with others who belong to the same economic class. We use a simple model that includes the existence of risk aversion and an asymmetric probability that the poorer agent might be privileged in the exchange. Moreover, we introduce a parameter u that defines the maximum range in wealth for two agents to interact. We studied the evolution of these systems and found different wealth distributions depending on the values of the parameters u and f. For all the values of f studied, we find a range of u in which the society is divided in two classes separated by a deep gap that prevents further exchange between them. This gap is related to the lack of opportunity of the poorer agents to ascend to a class economically higher. It is important to note that this "opportunity gap" is conceptually different to the poverty gap widely studied, which has to do with the existence of poor and rich agents. The main consequence of this wealth redistribution is the disappearance of the middle class. We remark that we tried to fit the obtained wealth distributions with Gibbs-like or power law functions. While for low values of u, a power law tail appears over a narrow band of wealth values, for high values of u, an exponential law provides a better fit. We also calculated the Gini coefficients of the stationary wealth distributions and find that low and high values of u present a fairer wealth distribution than intermediate values.

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