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Crime and punishment: the economic burden of impunity

M.B. Gordon[1], J.R. Iglesias[2,a], V. Semeshenko[1], and J.P. Nadal[3]

[1] Laboratoire TIMC-IMAG (UMR 5525), University of Grenoble I, Domaine de La Merci, Jean Roget, 38706 La Tronche, France
[2] Instituto de Física and Faculdade de Ciências Económicas, Universidade Federal do Rio Grande do Sul, 91501-970 Porto Alegre, Brazil
[3] Laboratoire de Physique Statistique (UMR8550), École Normale Supérieure, 24, rue Lhomond, 75231 Paris Cedex 05 and Centre d'Analyse et de Mathématique Sociales (CAMS) (UMR8557) École des Hautes Études en Sciences Sociales, 54, Bd. Raspail, 75270 Paris Cedex 06, France

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Abstract. Crime is an economically relevant activity. It may represent a mechanism of wealth distribution but also a social and economic burden because of the interference with regular legal activities and the cost of the law enforcement system. Sometimes it may be less costly for the society to allow for some level of criminality. However, a drawback of such a policy is that it may lead to a high increase of criminal activity, that may become hard to reduce later on. Here we investigate the level of law enforcement required to keep crime within acceptable limits. A sharp phase transition is observed as a function of the probability of punishment. We also analyze other consequences of criminality as the growth of the economy, the inequality in the wealth distribution (the Gini coefficient) and other relevant quantities under different scenarios of criminal activity and probabilities of apprehension.

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1 Introduction

Crime is a human activity probably older than the crudely called "oldest profession". Criminal activity may have many different causes: envy, like in the case of Cain and Abel [1], jealousy like in the opera Carmen, or financial gain like Jacob cheating Esau with the lentil pottage (and his father with the lamb skin) to obtain the birthright [2]. This is to say that crime can have many different causes, some of them "passional", some "logical" or "rational" [3].

All along history, organized societies have tried to prevent and to deter criminality through some kind of punishment. In all the societies and all the times punishment has been in some way proportional to the gravity of the offense. Methods ranged from the *lex talionis* "an eye for an eye", to fines, imprisonment, and even the death penalty. Most of the literature on crime considered criminals as deviant individuals. Usual explanations of why people offend use concepts like insanity, depravity, anomia, etc. In 18th Century England criminals were massively "exported" to Australia because it was thought that the criminal condition was hereditary and incurable: incapacitation was the solution.

In any case the increase of criminal activity in different countries has led some sectors of the population, as well as politicians, to ask for harder penalties. The understated idea is that a hard sentence, besides incapacitation of convicted criminals, would have a deterrent effect on other possible offenders and would also prevent recidivism. Yet, the deterrent effect of punishment is a polemic subject and law experts diverge with respect to whether offenders should be rehabilitated or simply punished (see for example the recent public discussion in France [4]). In many countries, incapacitation is the main reason for imprisonment of criminals. For instance the Argentine constitution states that prisons are not there for the punishment of the inmates but for the security of the society: criminals are isolated, not punished [5].

Although the idea that the decision of committing a crime results from a trade off between the expected profit and the risk of punishment dates back to the eighteenth and nineteenth centuries, it is only recently that crime modeling emerged as a field worth of being investigated.

In a review paper, Blumstein [6] traces back the recent interest in crime modeling to 1966, when the USA

[a] e-mail: roberto.iglesias@ufrgs.br

President's Commission on Law Enforcement and Administration of Justice created a Task Force on Science and Technology. Composed mainly by engineers and scientists, its aim was to introduce simulation modeling of the American criminal justice system. The model allowed to evaluate the resource requirements and costs associated to a criminal case, from arrest to release, by considering the flow through the justice system. For example, it estimated the opportunity of incarceration of convicted criminals and the length of the incarceration time. In his concluding remarks, Blumstein states that: "*We are still not fully clear on the degree to which the deterrent and incapacitation effects of incarceration are greater than any criminalization effects of the incarceration, and who in particular can be expected to have their (tendency to)*[1] *crime reduced and who might be made worse by the punishment*".

In a now classical article Becker [7] presents for the first time an economic analysis of costs and benefits of crime, with the aim of developing optimal policies to combat illegal behavior. Considering the social loss from offenses, which depends on their number and on the produced harm, the cost of apprehension and conviction and the probability of punishment per offense, the model tries to determine how many offenses should be permitted and how many offenders should go unpunished, through minimization of the social loss function.

Using a similar point of view, Ehrlich [8,9] develops an economic theory to explain participation in illegitimate activities. He assumes that a person's decision to participate in an illegal activity is motivated by the relation between cost and gain, or risks and benefits, arising from such activity. The model seemed to provide strong empirical evidence of the deterrent effectiveness of sanctions. However, according to Blumstein [6] the results "*... were sufficiently complex that the U.S. Justice Department called on the National Academy of Sciences to convene a panel to assess the validity of the Erlich results. The report of that panel highlighted the sensitivity of these econometric models to details of the model specification, to the particular time series of the data used, and to the sensitivity of the instrumental variable used for identification, and so called into question the validity of the results*". In fact, the issue of what constitutes an optimal crime control policy is still controversial.

Another controversial subject is the relationship between income inequality and victimization. Becker's economic model of crime would suggest that as income distribution becomes wider, the richer become increasingly attractive targets for the poorer. There are many reasons why this hypothesis may not be correct. For example, Deutsch et al. [10] consider the impact of wealth distribution on crime frequency and, contrary to the general consensus in the literature, conclude that variations of the wealth differences between "rich" and "poor" do not explain variations in the rates of crime. Bourguignon et al. [11] based on data from seven Colombian cities conclude that the average income of the population determines the expected gain of criminal activity, but that potential criminals belong to the segment of the population whose income is below 80% of the mean. More recently, Dahlberg and Gustavsson [12] pointed out that in crime statistics one should distinguish between permanent and transitory incomes. Disentangling these income components based on tax reports in Sweden, they find that an increase in inequality in permanent income (measured through the variance of the distribution) yields a positive and significant effect on crime rates, while an increase in the inequality due to a transitory income has no significant effect. Levitt [13] concludes from empirical data that, probably because rich people engage in behaviors that reduce their victimization, between 1970 and 1990 property crime victimization has become increasingly concentrated on the poor. Demougin and Schwager [14] conclude that in order to achieve a low level of criminality it will at some point become advantageous to redistribute the income.

As pointed out by many authors, the fact that strong police forces reduce crime is far from being demonstrated. Realizing that (at least in the USA) the number of police officers increases mostly in election years, Levitt [15] has studied the correlation between these variations (uncorrelated to crime) and variations in crime reduction. He finds that increases in the size of police forces substantially reduce violent crime, but have a small impact on property crime. However, these results have been criticized (see [16]) and the debate that followed [17] shows how difficult it is to assess the impact of police on crime.

Moreover, the social benefit of reducing crime is not necessarily larger than the cost of hiring additional police. For example, Freeman [18] estimates that the overall cost of crime in the US is of the order of 4 percent of the GDP, 2 percent lost to crime and 2 percent spent on controlling crime. This amounts to an average of about 54 000 dollars/year for each of the 5 million or so men incarcerated, put on probation or paroled.

In most models, punishment of crime has two distinct aspects: on one side there is the frequency at which illegal actions are punished (which corresponds to the punishment probability in the models), and on the other, the severity of the punishment. In a review paper, Eide [19] comments that although many empirical studies conclude that the probability of punishment has a preventive effect on crime, the results are ambiguous.

More recent reviews of the research literature consider the factors influencing crime trends [20] and present some recent modeling of crime and offending in England and Wales [3]. They note the above mentioned difficulties in estimating the parameters of the models and the cost of crime.

Most of the models in the literature follow Becker's economic approach. Ehrlich [9] presents a market model of crime assuming that individuals decisions are "rational": a person commits an offense if his expected utility exceeds the utility he could get with legal activities. At the "equilibrium" between the supply of crimes and the "demand" (or tolerance) to crime – reflected by the expenditures for protection and law enforcement – neither criminals, private individuals nor government can expect

[1] We added these words in parenthesis

to improve their benefits by changing their behaviors. In particular, the model is based on standard assumptions in economic models, with "well behaved" monotonic supply and demand curves, which cannot explain situations where social interactions are important [21,22]. Indeed, Glaeser et al. [23] attribute to social interactions the large variance in crime on different cities of the US.

Another kind of models [3,24] treat criminality as an epidemics problem, which spreads over the population due to contact of would-be criminals with "true" criminals (who have already committed crime). This kind of models incorporates effects due to social interactions, which introduce large nonlinearities in the level of crime associated to different combinations of the parameters. These may explain the wide differences reported in the empirical literature.

In this paper we focus on economic crimes, where the criminal agents try to obtain an economic advantage by means of the accomplished felony. No physical aggression or death of the victim will be considered, and on the side of punishment we consider the standard of most developed civilized countries, i.e. fines and imprisonment. We assume that [25] *"most criminal acts are not undertaken by deviant psychopathic individuals, but are more likely to be carried out by ordinary people reacting to a particular situation with a unique economic, social, environmental, cultural, spatial and temporal context. It is these reactionary responses to the opportunities for crime which attract more and more people to become involved in criminal activities rather than entrenched delinquency"*. We have two main purposes in mind when simulating this artificial society with criminal activity: (a) we want to determine what is the minimum percentage of crimes that should be punished in order to guarantee a relatively low level of criminal activity, and (b) we want to evaluate the economic cost of crime and punishment, measured both trough the economic growth and by the eventual inequality induced (or not) in the population. In Section 2 we describe the model, in Section 3 we explain its dynamics. We present the results of the simulations in Section 4 and leave the conclusions to Section 5.

2 Description of the model

We simulate a population of heterogeneous individuals. They earn different wages, have different tendencies for criminality, and modify their behaviors according to the risk of punishment. We are interested in the consequences of the punishment policy on the costs of crime, in the wealth distribution consistent with different levels of criminality, in the economic growth of the society as a function of time, and in the (possibly bad) consequences of allowing for some criminal activity in order to minimize the cost of the law enforcement apparatus. Inside this model society, agents have a very simplified economic activity: they earn monthly wages and may commit crimes. The latter, which consists of picking a fraction of the wealth from other agents, may or may not be punished with a given probability. Punished subjects not only spend some time in prison but are also subject to a financial penalty, i.e. a fine. We assume that both the probability and the severity of the punishment increase with the magnitude of the crime. We separate the detailed description of the model into four parts: (a) a description of the "ordinary" economic activity without crime, (b) crime, (c) punishment and (d) the economic consequences of both the crime and the charge of the repressive apparatus.

Economic activity.

We consider a model of society with a constant number of agents, N, who perceive a periodic (monthly) positive income W_i (that we call wage). This wage is different for each agent and may be the remuneration for some productive activity or a rent. For simplicity, wages are drawn at random from a finite support distribution $p_W(W_i)$ ($W_i \in [W_{min}, W_{max}]$) and remain constant during the simulated period. We assume that in this utopian society there is no unemployment, and the minimum wage is enough to provide for the minimum needs of each agent, i.e. a person perceiving the minimum wage will expend it completely within the month. People receiving higher wages spend this minimum plus a fraction of the surplus. The resources not spent during the month are saved and constitute the "assets" of each agent. Wages and possible booties of successful criminal activities constitute the only income source of the individuals. Besides the living expenses, the individual's wealth may decrease due to the plunders and to the taxes or fines related to conviction as we will see below.

Crime.

We assume that each agent has an inclination to abide by the law that may be psychological, ethical, or a reflection of his educational level and/or socio-economical environment. This inclination is represented by a honesty index H_i ($i \in [1, N]$), in a similar way as defined by Bourguignon et al. [11]. However, contrary to [11], here this index is not an intrinsic characteristic of the individuals: it evolves in time according to the risk of apprehension upon performing a crime. At the beginning of the simulations the H_i are drawn at random from a compact support distribution $p_H(H_i)$ with $H_i \in [H_{min}, H_{max}]$ without any correlation with the wages. This is justified by the lack of empirical evidence that poorer agents are more or less law abiders than the rich.

Encounters between criminals and victims may arise among individuals that share common daily trajectories or live in close neighborhoods, or just by chance. Generally, they may be represented by "social" connections which in this context are not meant to represent social closeness but rather the fact that the individuals may meet each other. In spite of the fact that social correlations between criminals and victims is an interesting subject, here, for simplicity, we consider the simplest situation in which every individual may be connected to every other one.

Notice that the nature of social interactions in our model is very different from the mimetic interactions considered by Glaeser et al. [23], or the social pressure introduced by Campbell et al. [24]. We study just the case of individual crimes. Neither illicit criminal associations ("gangs") nor collective victims (like in bank assaults) are considered. In future developments we plan to study the effect of more involved social interactions.

Finally, both victims and potential criminals are chosen among the individuals not in prison. In our simulations, the probability that the potential criminal actually commits an offense depends (albeit not exclusively) on both his honesty index and his monthly income. We assume that every month there is a number of possible criminal attempts. Among these attempts, some are successful, i.e. the criminal robs his victim of a (random) fraction of his assets. Here, we do not take into account the wealth of the victim, which might be an important incentive for the criminal's decision. Thus, our treatment should be interpreted as an oversimplification that includes relatively important crimes, like stealing a car, but also minor larcenies, like stealing a bag in public transportation. In order to include this wide spectrum of booties, we will consider that the felon robs a random amount S and, of course, that amount cannot be larger than the victim's wealth. The detailed calculation of this quantity will be described in Section 3.3.

Punishment.

In contrast with most models in the literature, criminals are caught and punished with a probability that depends on the magnitude of the loot. In our model, caught criminals are always punished. We neither consider the possibility of early release nor law enforcement system failures. When a criminal is catched, he goes to jail for a period of time that depends on the booty, and his wealth is cut down of an amount larger than the stolen one. However, only part of the booty is given back to the victim. The rest is considered as a contribution to the law enforcement system. As it stays, in the model only criminals and victims support the cost of the system's expenses. However, including a generalized tax to all the population is equivalent to decreasing globally the wages. Indeed, we studied the model for different wages distributions and verified that our results are qualitatively similar. Maintaining each criminal in prison bears a fixed monthly cost to the society, that we evaluate. Some additional simplifying assumptions are adopted concerning the inmates: agents in prison can neither commit offenses nor be victims; they do not earn wages and their needs are covered by the society using the fines collected through the punishments.

Lastly, conviction (or impunity) of the offender produces a deterrent (or deleterious) effect on the society. Each time a crime is punished, the honesty index of all the individuals but the criminal increases by a small amount, reflecting the fact that the population becomes more afraid of committing a felony, if only to avoid being punished. Nonetheless, the honesty level of the convicted offender remains unchanged, meaning that he will leave prison with the same honesty index he entered in. We are aware that this is a controversial point, since depending on the prison system, convicts may be "rehabilitated", i.e. their honesty index increased, or on the contrary suffer a further "deterioration" because of the cohabitation with hard criminals, their honesty index being concomitantly decreased. Here we prefer to leave the honesty of the offender unchanged.

If the crime is not punished, the honesty index of all the population decreases, because of a generalized feeling of impunity. We assume that this feeling is stronger for a successful offender: his honesty index is decreased even more than that of the others. In our model, the honesty index is not affected by the importance of the punishment. This assumption is an extreme simplification of the observation [6] that the crime rate is more sensitive to the risk of apprehension than to the severity of punishment.

We have studied different initial honesty index distributions, p_H, and also different dynamics, differing in the way one treats its lower bound H_{min}. In this study we describe in details the case where H_{min} is considered as an *absolute* minimum of the honesty, which corresponds thus to the honesty index of the most recalcitrant offenders. We add the hypothesis (certainly controversial) that there is no possible redemption for agents with this lowest honesty. In other words, when the honesty index of an agent decreases down to H_{min} it remains there for ever. We leave to forthcoming discussions possible variations of this scheme.

Economic consequences.

We study the month to month evolution of different quantities that characterize the system, and how they depend on the probability of punishment. We evaluate every month the total assets in the hands of the agents as well as the Gini coefficient, to determine the degree of inequality without discriminating between their different origins: wages or booties. We also calculate the total amount collected in the form of fines, as well as the amount needed to cover the living expenses of inmates.

3 Monthly dynamics

In the following paragraphs we describe in details the dynamics of the model. Starting from initial conditions of honesties and wages described in Section 3.1, we simulate the model for a fixed number of months.

Within each month, there is a variable number of criminal attempts correlated with the honesty level of the population, as will be explained in Section 3.2. However, not all the attempts end up with a crime: the potential criminal has to satisfy some reasonable conditions that we will explicit in Section 3.3. If there is crime, there is a transfer of the stolen value from the victim to the criminal, as detailed in Section 3.3. Some of the offenders will be detected and punished (Sect. 3.4). In turn, the honesty of the population evolves after each crime, according to the success or failure in crime repression (Sect. 3.5). Finally, we keep track of the individuals' wealth, taking into account the monthly incomes, the living expenses, the plunders, and the cost of imprisonment, as detailed in Section 3.6.

3.1 Initial conditions

The individuals have initial honesty indexes $H_i(0)$ and wages W_i drawn with uncorrelated probability density functions (pdfs) p_H and p_W respectively. We assume that the majority of the population is honest and also that the majority earn low salaries, i.e. p_H has a maximum at large H while p_W has its maximum at small W. The results described below have been obtained considering triangular distributions, because they are the simplest way of introducing the desired individual inhomogeneities. More precisely, p_W is a triangular distribution of wages, with $W_{min} = 1$ and $W_{max} = 100$ (in some arbitrary monetary unit), with its maximum at W_{min} and a mean value $\langle W \rangle = W_{min} + (W_{max} - W_{min})/3$. There is an initial number of "intrinsic" criminals $N_C(0)$ drawn at random, with $H_i = H_{min}$, according with the idea that there always have been and will be a finite number of not redemptible criminals in a society. In our simulations this number has been set to 5% of the population ($N_C(0) = 0.05N$). The initial honesties of the remaining $N - N_C(0)$ (susceptible) individuals are drawn at random from a triangular distribution: from $H_{min} = 0$ up to $H_{max} = 100$, with the maximum at H_{max}.

Individuals' initial endowments are arbitrarily set to one month wage: $K_i(0) = W_i$. This initial amount controls the time needed for the dynamics to fully develop. Smaller initial endowments result in transients dominated by the size of the possible loots, because the latter cannot exceed the victims' wealth.

3.2 Number of monthly attempts

It is not easy to evaluate the number of attempts of crime, which depends on variables like opportunities, needs, etc. Here we adopt a simple rule and assume that the number of attempts is determined by the number of agents not imprisoned and also by the average honesty level of the population. More precisely, the number of attempts will increase with the number of free agents, but will decrease if the average honesty level of the population is high. This is, of course, an arbitrary way of determining the number of tries, as in principle an agent with low honesty could try more than an offense a month, and so the number of attempts might be larger than the number of free agents. We have made simulations with more complicated formulas for the number of attempts, allowing for more than two tries per free agent and the results are not modified in the essence.

Thus, we define the number of criminal attempts per month m as $A(m) = [\mathcal{A}(m)]$, where $[\dots]$ represents the integer part, and

$$\mathcal{A}(m) = 1 + \frac{N - N_j}{1 + \langle H \rangle}, \tag{1}$$

where N_j is the number of agents in jail, and $\langle H \rangle$ is the average honesty of the population. Then, the number of attempts decreases with the increasing number of inmates in prison and with an increasing average honesty. The simulation results obtained using other expressions for the number of attempts within the same philosophy but including, for example, random values for the number of tries, produce qualitatively similar results to the ones of this paper, presented in Section 4.

3.3 Criminals and victims

At each attempt, a *potential* criminal k is drawn at random among the population of free (not imprisoned) individuals. We consider more likely that the possible criminals have low honesty indexes and low wages. To simulate this, we define two probabilities:

$$p_H(k) = e^{-H(k)/\langle H \rangle}, \tag{2}$$

$$p_W(k) = e^{-W(k)/\langle W \rangle}, \tag{3}$$

then, once an agent k is chosen as potential criminal the crime will be committed with probability $p_H(k) \cdot p_W(k)$.

If the criminal k satisfies the above conditions, we simply select the *victim* v at random among the individuals not in jail and with non zero wealths.

In order to include a wide spectrum of booties, we will consider that the felon robs a random amount S and, of course, that amount cannot be larger than the victim's wealth.

This amount S is considered to be proportional to the victim's wage:

$$S = r_S \, c_S \, W_v, \tag{4}$$

where $r_S \in [0, 1]$ is a random number drawn afresh at each attempt and $c_S > 0$ is a coefficient (here we use arbitrarily $c_S = 10$, implying that robbery may concern amounts up to 10 times the victim's wage). If the value given by (4) is larger than the victim's wealth K_v, we set $S = K_v$.

Correspondingly, the victim's wealth is decreased by the stolen amount, to become $K_v - S$, while the criminal's wealth is increased to $K_k + S$.

3.4 Punishment probability and conviction

We assume that the criminal may be caught and punished with a probability, $\pi(S)$, that depends on the stolen amount. A simple way of representing this hypothesis is to define:

$$\pi(S) = \frac{p_1}{1 + \frac{p_1 - p_0}{p_0} e^{-\frac{S}{\langle W \rangle}}}, \tag{5}$$

which is a monotonic function that starts at its minimal value p_0 (for $S = 0$), increases almost linearly with S and gets close to the asymptotic value p_1 for large booties. A plot of the probability of punishment as a function of the importance of the booty is presented in Figure 1. The

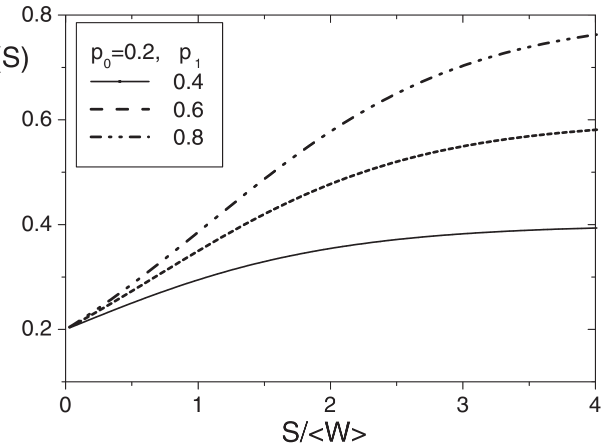

Fig. 1. Plot of the probability of punishment as a function of the importance of the booty, S, in units of the average wage $\langle W \rangle$.

probability increases, starting from p_0, and arrives to values near p_1 when the booty is of the order of several average wages.

A punished criminal is imprisoned for a number of months $1+[S/\langle W\rangle]$, i.e. proportional to the stolen amount. In addition, during this time he does not earn his monthly wage and cannot be selected as a potential offender nor as a victim.

If the crime is not punished only the victim suffers an economic loss, but when the crime is punished both victim and criminal suffer financial losses. This is to represent the fact that the capture and conviction of the offender do not guarantee that the crime is completely repaired. In our simulations the victim is returned an amount $f_R S$ (with $f_R < 1$), that is deduced from the criminal's wealth K_k. Thus, the victim does not recover the total stolen value. A monetary punishment is also inflicted to the offender besides incarceration (that represents also a monetary loss) through a fine or duty $D = f_D S$ which is also deduced from the criminal's wealth. Since we do not allow for negative wealth values, if $f_D S > K_k - f_R S$ the fine is decreased to $D = K_k - f_R S$, so exhausting the criminal's wealth. We put $f_R + f_D > 1$, so that the criminal's wealth after conviction is smaller than before the larceny. In our simulations, $f_R = 0.75$ and $f_D = 0.45$. Victims' and criminals' duties are assumed to contribute to maintain the law enforcement system.

3.5 Honesty dynamics

As we discussed in the presentation of the model, punishment has a dissuasive effect on the population. Following the empirical results of [19] we assume that the honesty will change as a function of the number of punished (or unpunished) crimes, but not with the importance of the penalty. More precisely, whenever a criminal is convicted, his honesty level remains unchanged while the honesty index of the rest of the population is increased by a fixed amount δH. On the contrary, crimes that go unpunished produce a decrease of the honesty levels, of $2\delta H$ for the criminal, but only of δH for the others. That is, unpunished criminals become even less honest than the rest of the population.

In the present simulations we do not allow H_i to become negative. Moreover, the lower bound of the distribution is absorbing. Thus, individuals reaching $H_i = H_{min}$ have their honesty index freezed, and henceforth are considered as intrinsic criminals. A dynamics with a non-absorbing lower honesty bound gives similar results to those presented here. A variant that we did not implement yet is to modify the honesty level of the criminal proportionally to the importance of the loot.

3.6 Monthly earnings and costs

At the end of each month m, after completion of the $A(m)$ criminal attempts, the individuals' assets $K_i(m)$ are updated: the total wealth of each agent, $K_i(m)$, is increased with his salary and decreased with his monthly expenses. Notice that the criminals' and the victims' wealths have been further modified during the month, according to the results of the criminal attempts and punishments.

We assume that individuals need an amount $W_{min} + f\,(W_i - W_{min})$ with $f < 1$ to cover their monthly expenses. Thus, individuals with higher wages spend a proportional part of their income in addition to the minimum wage W_{min}. This is a simplifying assumption which may be questionable since the richer the individuals, the smaller the fraction of income they need for living but, on the other hand, rich people spend more in luxury goods. Assuming a more involved model for the expenses would modify the monthly wealth distribution, making it more unequal, but we do not expect the qualitative results of our simulations to be much modified.

In order to quantify the expenditure of conviction and imprisonment, we assume that the monthly cost of maintaining a criminal in jail is equal to the minimal wage. This is clearly a too simple hypothesis, that does not take into account the fixed costs of maintaining the public enforcement against crime. It is just a means of assessing some social cost proportional to the criminal activity. The cumulated taxes should be decreased by an amount W_{min} per month for each convict in jail.

At the end of each month, the time that inmates have to remain in prison (excluding the criminals convicted during that month) is decreased by one; those having completed their sentence are released.

We would like to emphasize that the model simplifies a series of points, for example, neither particular consideration of relapses nor a particular analysis of the consequences of the prison system (rehabilitation or deterioration of convicted offenders) are studied in detail. Those points are under consideration for future development of the model.

4 Simulations results

Starting with the initial conditions, there are $A(m)$ criminal attempts each month m as described in Section 3.2. As a consequence of the criminal activity during the month, the wealth distributions of criminals and victims, as well as the honesty index distribution of the population, are shifted and distorted. For a low probability of punishment the system may even converge to a population where all the initially susceptible individuals end up as intrinsically criminal.

We simulated systems of $N = 1\,000$ agents for a period of 240 months, with triangular distributions of honesty and wages described in Section 3.1.

4.1 Time evolution of criminality

The system's time evolution is very sensitive to the punishment probability. Generally, the crime rate is low during the first months because the number of intrinsic criminals is small. After a transient of some months (the number depends on the initial conditions and on the values of p_0 and p_1) the crime rate attains a stable value with small fluctuations.

Figure 2 shows a typical evolution of the numbers of attempts, crimes, convictions and inmates. Figure 2(a) corresponds to a relatively low value $p_0 = 0.2$ (the probability of punishing small offenses) and an intermediate value of $p_1 = 0.5$ (the probability of punishing large offenses) in (5). Beyond about 75 months the number of attempts increases through an avalanche, goes trough a sharp peak and decreases to a rather stable value, of the order of $N/2$, while the number of inmates grows to include around half of the population. After the peak the level of crime remains more or less stable being of the order of $0.2N$. That means that a little less than half of the attempts are transformed in real crimes, while the number of convictions is relatively low, certainly less than half of the crimes are punished. It is evident that this level of punishment rate is not enough to keep criminality below any acceptable level.

On the other hand, Figure 2(b) (see the change in the ordinate scale) corresponds to a slightly higher value of $p_1 = 0.6$, showing the dramatic impact of a relatively low increase of the punishment probability of big offenses. All the quantities (attempts, crimes, etc.) present a drastic decrease with respect to the values on Figure 2(a).

Correspondingly, the earned wealth per individual (Fig. 3a) increases almost linearly with time when crime is limited. In contrast, in the highly criminal society ($p_1 = 0.5$) the wealth increases at much lower rate, mainly because most criminals are convicted and do not receive their wages, but also because, since the number of punished crimes is also high, the amount retained in the form of taxes is discounted from the total wealth of the society. In other words, the high cost of imprisonment, proportional to the total number of inmates, drains part of the wealth and contributes to decrease the wealth per capita. This is illustrated in Figure 3b, where the total assets per agent is decomposed into the assets hold by the intrinsic criminals and the part hold by the susceptible population (with honesty index $H_i > 0$). When the system reaches the high criminality regime the latter drops to zero because there are no more susceptible individuals (all agents go to the state with $H_i = 0$). Clearly, when the level of punishment is not high enough to guarantee an effective control of criminality, the cost of the repressive system is very high. It is remarkable that a very small increase of the upper value p_1 in (5) is enough for a complete change in the scenario.

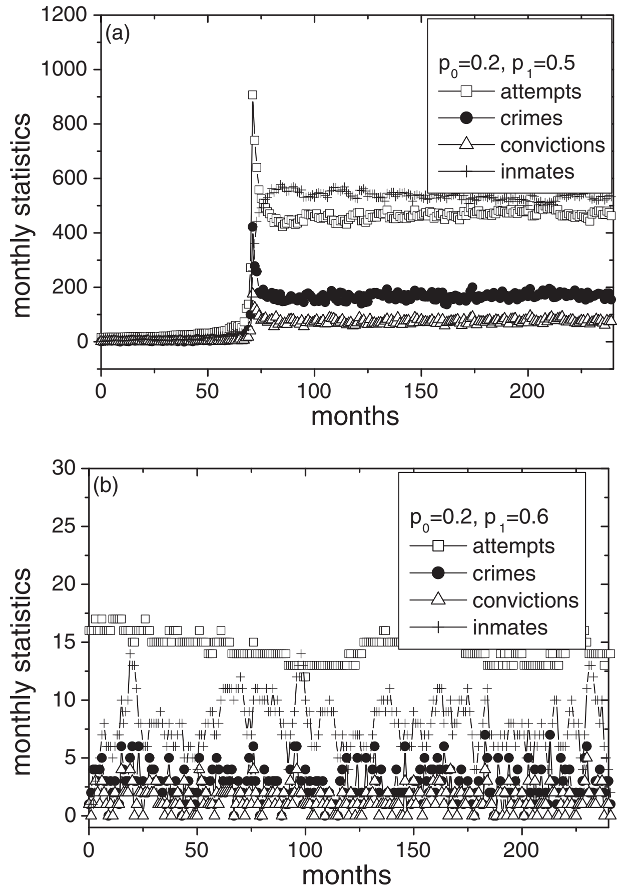

Fig. 2. Statistics of criminality per month for $p_0 = 0.2$ and $\delta H = 1$. Attempts: number of criminal attempts in the month. Crimes: number of crimes committed in the month. Convictions: number of criminals punished in the month. Inmates: number of criminals in jail. The difference between (a) and (b) panels is the maximum punishment probability p_1, $p_1 = 0.5$ in panel (a) and $p_1 = 0.6$ in panel (b). A small change in the probability of punishment induces an enormous change in the criminality (see the difference in scale in the ordinate axis).

Figure 3c presents the evolution of the Gini index of the population, defined by:

$$G = \frac{\sum_{j=1}^{N-1} \sum_{i=j+1}^{N} |K_i - K_j|}{(N-1) \sum_i^N K_i}. \tag{6}$$

The Gini index spans in the interval $[0, 1]$ and measures the inequality in the wealth distribution of the population.

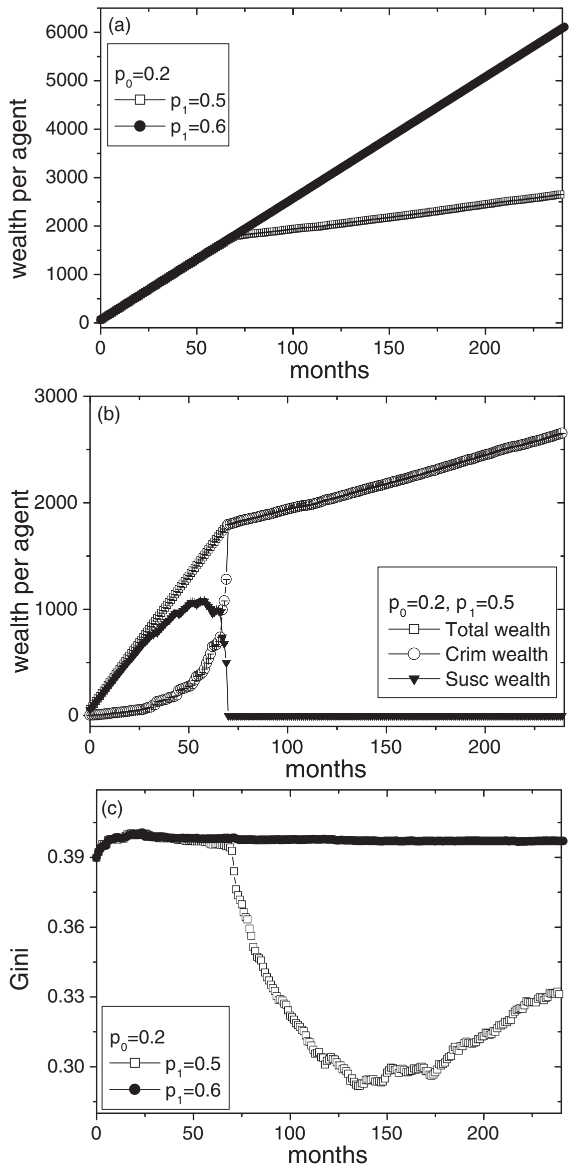

Fig. 3. Capital and Gini monthly statistics for $p_0 = 0.2$ with the same setting as in Figure 2: (a) cumulated wealth per agent as a function of time; (b) participation in the total wealth of intrinsic criminals (blue line) and susceptible ones (green line) for the case of high criminality ($p_1 = 0.5$); (c) Gini coefficient for the same two values of p_1 as in panel (a).

Its minimal value, $G = 0$, corresponds to a perfectly equalitarian society. The Gini index of the initial endowment, distributed according with a triangular probability density, is $G(0) = 0.4$. Due to the dynamics, when crime level is moderate ($p_1 = 0.6$) G is seen to slightly increase and then keep a constant value near 0.4. However, in the high criminality regime ($p_1 = 0.5$) when the high crime rate sets in, it first plummets down because successful criminals – mostly individuals with small incomes according to the probability of crime (5) – increase their wealths at the victims' expenses. As a result, the wealth becomes more evenly distributed. However, on the long run, the Gini starts to increase again. This is so because if everybody is a lawbreaker (lowest honesty index) criminals and victims are the same, stealing from each other. This result is also illustrated by the histograms of wealth distribution (Fig. 5), the relatively low value of the Gini coefficient after 240 months is due to a high concentration of poor agents, but the economic status of the population is worse than in the case with higher punishment probability. If we extend the simulations to a higher number of months the Gini coefficient continues to increase going above the value 0.4, meaning that in the long run, just a few agents escaping conviction are able to hold large assets thanks to crime, increasing the social inequality.

In fact, when p_0 is smaller than a critical value of the order of 0.5, on increasing p_1, the system presents an abrupt transition between a high crime-low honesty to a low crime-high honesty society. In the high criminality side, cumulated earnings are small, taxes are high and the Gini index is large. Conversely, on the low criminality side, i.e. for sufficiently high p_1, the cumulated wealth increases monthly according to the earned wages, and the Gini index reflects the distribution of the latter. We will show in Figure 5, below, typical histograms of wealth distribution.

4.2 Changing the punishment probability

In the previous section we have discussed the time evolution of criminality. Let's now consider the state of the society as a function of the probability of punishment. In order to make comparable experiments we have studied societies with the same initial conditions under different levels of punishment. Notice that these levels are constant during the simulated 240 months. The results presented in this section are *averages* over 240 months of evolution.

We consider different values of p_0 smaller than a critical value of the order of 0.5, and we study the variation of several social indicators as a function of p_1. We observe that the system ends up with either a high crime-low honesty population (for p_1 lower than a critical value) or a low crime-high honesty one. This transition, apparent on all the quantities, as may be seen in Figure 4, corresponds to a swing of the system between a regime of high criminality to one of moderate criminality.

In Figure 4a, the number of crimes and convictions per agent are represented for four representative values of p_0: 0.1, 0.2, 0.3 and 0.4. There is a sharp transition in the criminality when p_1 increases, at a critical value that strongly depends on the value of p_0. While for $p_0 = 0.4$ the transition happens for $p_1 \simeq 0.48$, for $p_0 = 0.1$ the critical value grows up to $p_1 = 0.6$. This is an indication that the permissiveness in coping with small crimes has a deleterious effect, since the probability of punishment needed to deter important crimes increases.

A simple argument allows to understand the abrupt transition found in the simulations. The average honesty level of the population increases roughly (we neglect the influence of criminals' different dynamics) by about δH if crimes are punished, and decreases by the same amount if not punished. Thus, we expect: $\bar{H}(m+1) \approx \bar{H}(m) - (1 - \pi)\delta H + \pi\delta H$ per crime, where π is the probability of the

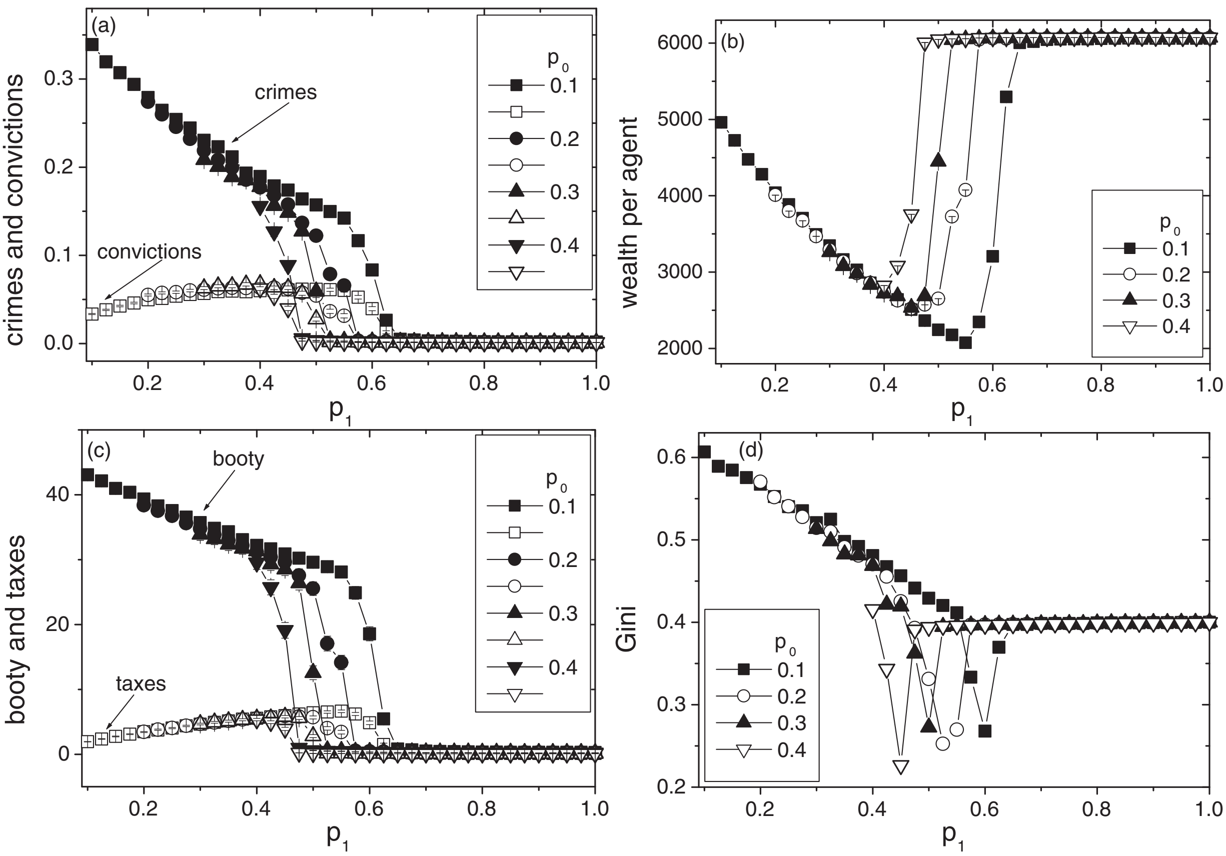

Fig. 4. Panel (a): average number of crimes and convicted criminals per capita over the simulated period. Panel (b): last month wealth per capita of the population. Panel (c): average loot (full symbols) and taxes (empty symbols). Panel (d): last month Gini index.

crime being punished. Clearly, there should be a change from an increasing honesty dynamics to a decreasing one for $\pi = 1/2$. If crimes were punished with the same probability whatever the value of the loot ($p_0 = p_1$), the change in the honesty dynamics would arise when this probability is equal to 1/2. If small crimes have a small probability to be punished, $p_0 < 1/2$, then p_1 must increase to keep the same dynamics on the average. Notice that the transition is found at $\pi \approx 1/2$ due to our symmetric treatment of the honesty dynamics, where the same amount δH is used both to increase and to decrease the honesty level of the population. If the honesty index increase due to conviction had a different value than the decrease due to absence of punishment, the transition would arise at a different value of π.

Figure 4b shows the total wealth of the society. The effect of wrongdoing is evident. From a strictly economic point of view the worse situation arises closely below the critical point: a high level of criminal actions together with a relatively high frequency of punishment (although not enough to control criminality) bring as a consequence a strong decrease in the total wealth (because the cumulated effect of not received wages, booties and taxes strongly reduces the total assets of the population). On the other hand, once the delinquency is under control the total wealth of the society arrives to a maximum level.

The opposite effect is observed in the plot of the booties and taxes (Fig. 4c). They are very large in the high criminality region (low values of p_1) and decrease strongly when p_1 is above its critical value.

Finally in Figure 4d we represent the Gini coefficient. If we observe this figure together with the wealth evolution of the society we can conclude that low criminality implies higher economic growth and less inequality. It is worth to note that the Gini coefficient goes through a minimum near the transition. However, this reverse peak coincides with a minimum in the total assets of the population. So this apparent decrease of inequality corresponds to a general impoverishment and not to a redistribution of wealth.

To better illustrate this point (the Gini coefficient is an average indicator) we present in Figure 5 the wealth distribution histograms, which provide a complementary indicator. The two panels (a) and (b) of Figure 5 correspond to the histograms for the two values of p_0 and p_1 used in Figures 2 and 3. It is clear that, for $p_1 = 0.5$, almost all the agents have a normalized wealth well below the initial distribution (Fig. 5c) clearly indicating a general impoverishment of the population. On the other hand, for $p_1 = 0.6$ the wealth distribution is near the initial one, corresponding to a constant increase of the wealth of the population. Finally and only for comparison the lower

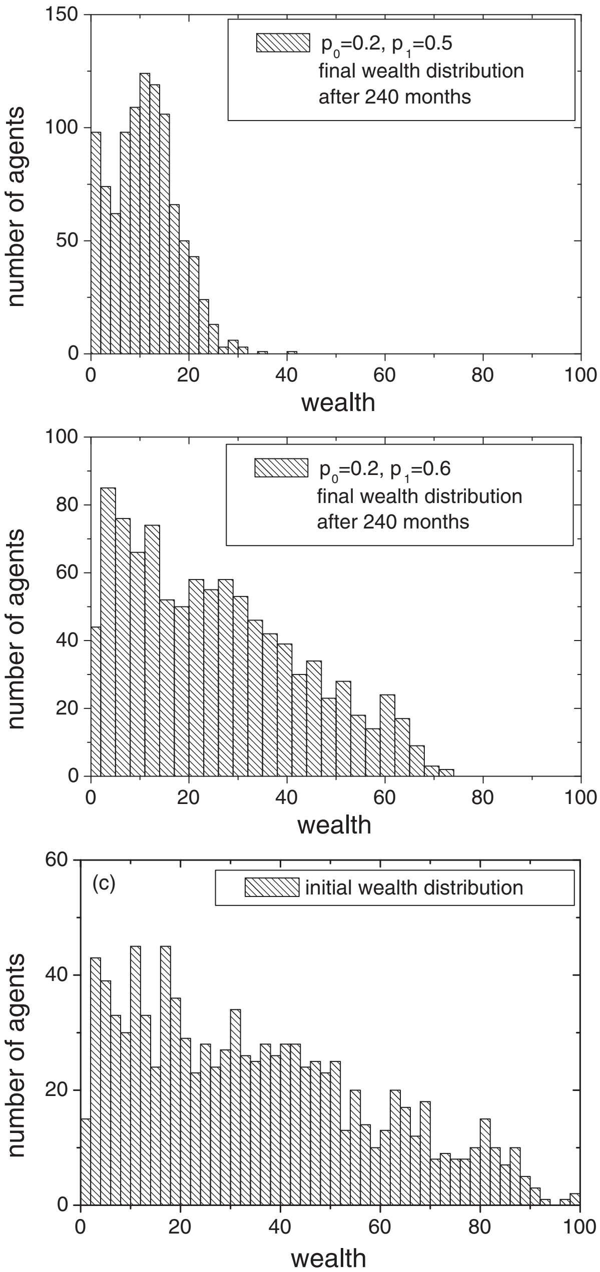

Fig. 5. Histograms of the population wealth normalized over 240 months. Upper figures: at the end of the 240 months with $p_1 = 0.5$ (a) and $p_1 = 0.6$ (b). Lower figure (c): initial state.

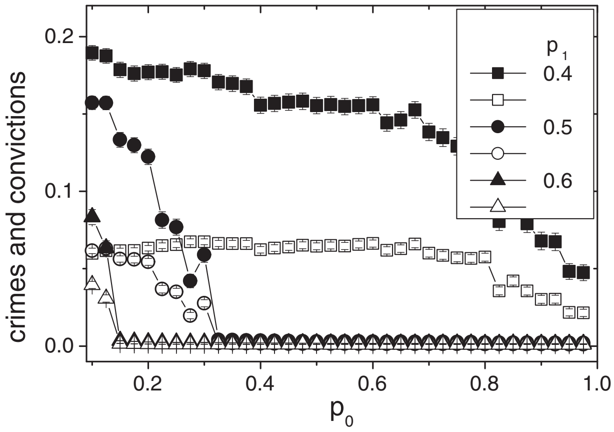

Fig. 6. Average number of crimes and convicted criminals per capita over the simulated period as a function of p_0 for different values of p_1.

figure represents the wealth distribution for the initial state that, according to our initialization, is equivalent to the wages distribution.

For completeness and for a detailed analysis of the dependence of the number of crimes on the probability of punishment one additional question should be answered: how the criminality varies as a function of the probability p_0 of punishment of small crimes? In order to illustrate this point, in Figure 6 we have represented the average number of crimes after 240 months as a function of p_0 for different values of p_1. One observes that for values of $p_1 \geq 0.5$ a sharp transition is apparent and the transition happens for low values of p_0. However, for $p_1 = 0.4$ a regime of high criminality is found even for values of p_0 close to 1. Combining these results with those presented in Figure 4a, we can conclude that, within the hypothesis of the model, severity with small crimes is important to reduce criminality with a relative lower cost. But, in order to be successful, this policy should be combined with an even higher probability of punishment of substantial crimes. If big crimes remain unpunished or, as it is the case for white collar crimes, small larcenies are penalized heavier than important crimes, a situation of high criminality may develop and settle down.

At last, we remind that the results presented in this section are a consequence of an evolution over a reasonable lapse of time (240 months) and that during this evolution all settings were kept constant. We expect that modifying either p_0 or p_1 or both as a function of time (as it may happen in real societies in order to correct an abnormal increase of criminality) would produce different results because the initial conditions before changing the probabilities would be different (here we assumed a low initial number of low honesty agents). In fact if one starts in a state of high criminality, a very high probability of punishment (much higher than the critical values presented here) would be needed in order to reduce the criminality back to acceptable levels. Once more, preventive actions should be less expensive and easier to apply than trying to recover from a very deteriorated security situation.

5 Discussion and conclusion

A central hypothesis of our model is that the honesty level of the population is correlated with the mere existence (or absence) of punishment, but not with its importance (which is proportional to the size of the loot). Thus, punishing small crimes is as effective as punishing large ones for increasing the population's honesty globally: the honesty level increases when crimes are punished and, on the contrary, impunity decreases it. Since small larcenies have

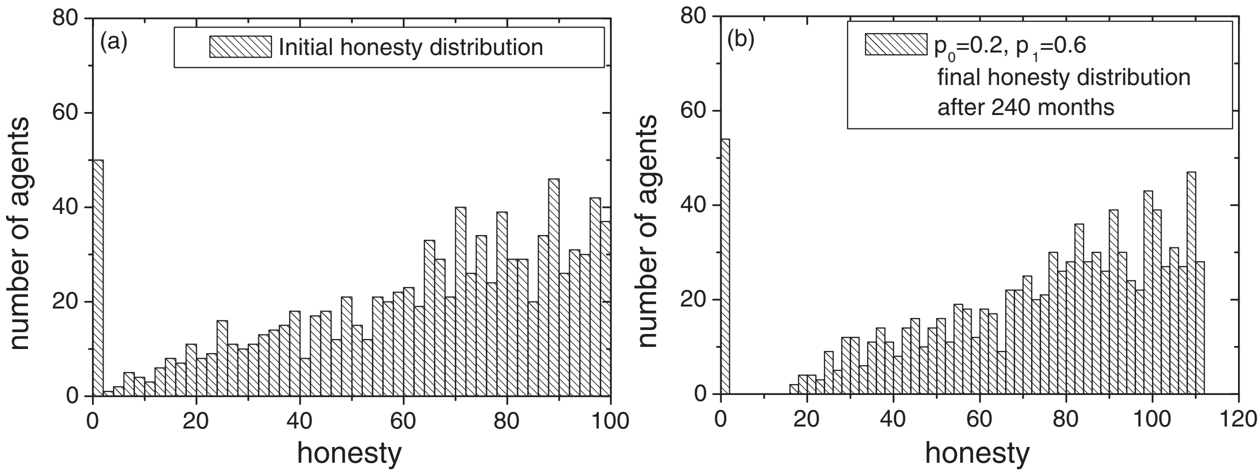

Fig. 7. Histograms of the population's honesty index. (a) initial distribution, where the small peak at zero honesty corresponds to the initial 5% concentration of intrinsic criminals. (b) final distribution for $p_0 = 0.2$ and $p_1 = 0.6$: more than half of the population exhibits a honesty index ≥ 100 while the number of intrinsic criminals remains the initial one. For $p_0 = 0.2$ and $p_1 = 0.5$ the honesty index of all the agents drops down to zero (and for this reason the plot is not included): a small change in the probability of punishment clearly induces a big change in the average honesty of the population.

a lower probability of being punished than large loots, the public effort on crime deterrence depends on the importance of the crime through the probability of punishment.

In Figure 7a we represent the initial honesty distribution, and in Figure 7b the final distribution for the case $p_0 = 0.2$ and $p_1 = 0.6$. We did not represent the histogram for $p_1 = 0.5$ because in that case the honesty index of the entire population drops to zero, as discussed above: $p_1 = 0.5$ is not large enough and the criminality level of the population is the highest.

Beyond those results, our model shows an interesting abrupt drop of the criminality level beyond a critical value of p_1, that depends on p_0, whenever $p_0 < 0.5$. When small larcenies have a high probability of being punished, the value of p_1 needed to reduce crime is smaller. Therefore, if we assume a proportionality between the amount of the booties and the expenses of the law enforcement system, we can conclude that being too permissive with small crimes may lead to high expenditures to control important crimes (if p_0 is small we need to increase p_1 to arrive to a low level of criminality). On the other hand the existence of a sharp transition is a good indicator that even a deteriorated situation with high criminality may be recovered with relatively small changes in the probability of punishment.

We remark that this behavior is very general, independent of the detailed parameters of the simulation, as we have shown using a general argument in Section 4.2. For example changes in parameters like the variation of honesty, δH, or the size of the booty, S, induce changes in the time to attain a stable state or in the absolute value of the final wealth, but without modifications in the general trends described.

On the other hand, the drop in criminality brings very positive consequences: increase of the global earnings because taxes decrease, and stabilization of the inequality at the level corresponding to the differences in wages. We think that this is a strong evidence that high criminality has a very negative effect on economic growth. Also, the fact of the Gini coefficient being lower in the state of low criminality implies that crime does not induce a "Robin Hood" redistributive effect. Also, this result is similar to that of reference [14], in the sense that low criminality is associated with low inequality. But an interesting twist in the present results is that inequality is a result of criminality; while in reference [14] they assume the reverse causality: crime as a consequence of inequality. Connecting both results, and considering empirical evidences that high inequality is at the origin of criminal activity, certainly inequality induces criminality, however, an increase of criminality does not impel the system to a lower inequality state, but the opposite.

So, to conclude, we have presented a simple model of crime and punishment that stands on the assumption that punishment has a deterrent effect on criminality. Our main result is that tolerance with respect to small felonies (small value of p_0) has a global negative consequence because it requires bigger efforts to cope with important crimes in order to keep a given level of honesty. We also observe an avalanche effect since a small change in the probability of punishment may reduce or increase the average criminality in a very significative way. Also, the economic consequences of criminality are remarkable, both in the observed total wealth of the population, as well as in the measure of wealth inequality – one should remind that in the model the wealth comes only from the salaries: we are essentially considering the distribution of wealth in the working class.

A less crude model should also include the effect of a particular treatment (or not) of recidivism. This is a point of discussion in countries like France, where some legislators ask for a minimum sentence for relapse. Also, one should be more careful in treating more "sophisticated" criminality, like organized crime, or criminals that choose the victim according to the expected loot. It would be interesting to analyze the effect of imprisonment: either to recover or to increase the inmates criminal tendencies. We are presently working on these points, which are of great importance.

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