Emerging Collective Behavior in a Simple Artificial Financial Market

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ABSTRACT

We consider a simple model of a society of economic agents, where each can invest a discrete quantity. Interactions among agents happen in a neighborhood and depend on the motivation level (insider information, economy prospects, etc.). The profit of the group fluctuates stochastically and is used to update individual motivations. We analyze the behavior, as a function of time, of the global persistence, given the initial quantity of money invested. Our simulations show that this quantity – a measure of the probability that the amount of money of the entire group remains at least equal to the initial amount – has a power law updating behavior. We have also performed simulations with heterogeneous agents, including deceiver and conservative agents. We show that, although there is no regular pattern regarding the average wealth, robust power laws for persistence exist, indicating that this can be used to model the emerging collective behavior. Besides, the updating of motivation and the presence of conservatives and deceivers is remarkable and has an influence on the persistence.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence
Multiagent systems, Coherence and coordination

General Terms

Economics

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Keywords

Multiagent Systems, Artificial Stock Market, Persistence, Game Theory

1. INTRODUCTION

In the fields of complex systems, physics, statistical mechanics, as well as of multiagent systems there is an extensive list of publications on complex systems in social sciences whose macroscopic behavior arises from interactions governed by micro-rules (see [5, 9] for other references). This has been thoroughly discussed in an article by Durlauf [4] especially in the context of socioeconomic behavior. Specifically, we are interested on agent-based simulation of artificial financial markets, an approach which is more descriptive than normative, given that it allows the study of the influence of individuals to elucidate macro phenomena. Our motivation stems from statistical mechanics, where a known set of simple rules of interaction between particles (microscopic physics) give rise to complex macroscopic phenomena (as a transition from liquid to gas). It is not our goal here to discuss whether or not the effects of the Prospect Theory (people tend to make decisions to maximize the value function, not the expected utility) [6] do cancel out in the market as a whole, leaving prices unaffected). Readers interested in this question are directed to [10]. Our focus is on the descriptive side of the issue, and in particular, on emerging patterns that cannot possibly be derived by analytical methods.

Agent-based simulation of artificial financial (or stock) markets (AFM) normally aims at analysing how asset prices are affected by investors. However, they can also have different fine-level objectives such as simulate different types of investors, which is also our aim here.

In our AFM, investment is made by agents depending on the actions of their nearest neighbors through a binary variable we call "motivation". This parameter is updated as a function of the profit of the group. The return per agent is a function of the average investment. Thus our basic model also relates to cooperative game-theory and we use the terms AFM and game indistinctively. Later, we analyze non-cooperative situations and introduce some personalities for the agents, analyzing how do specific idiosyncratic types perform. In particular we are interested in the influence of deceivers, a version of the egoist agents which appeared in the context of the Iterated Prisoners' Dilemma in [3].

Our analysis is centered on the concept of persistence [8], which measures the chance of keeping a positive balance relative to the initial amount invested. Persistence is a non–price component which we believe complements the already existing market performance quantities and price–dynamics analysis. Persistence focus on agent behavior as a market variable. Here it is analyzed and a scaling study is performed.

In the next two sections we discuss related works, and stochastic processes together with persistence in financial markets respectively. Section 4 presents our model, and investment updating rules for a society of L economic agents are defined. After that, we present details of the stochastic agent-based simulations performed and then, in the forthcoming section, our results are discussed. We finish the article with a brief summary of our results and conclusions.

2. RELATED WORK

A comprehensive list of papers on the subject would be too extensive due to its multidisciplinary character - one may find articles coming from the physics of complexity to multiagent systems. Thus we restrain ourselves here to those articles more closely related to the focus of the present paper, namely agent-based artifical financial markets.

Epstein and Axtell [5] applied an agent-based approach to economics back in 1996, through an artificial ants society where ants trade sugar and spice. Later, Axtell [2] emphasized the need for agents in economic sciences. B. Arthur and colleagues [1] used the Santa Fe Artificial Stock Market [7] to analyze price fluctuation when heterogenous investors — who can adapt their behavior — are present in a market.

Takahashi and Terano [10] investigate how asset prices are affected by investors as well as investment systems which are based on behavioral finance. Their artificial market contains two kinds of investors: fundamentalists (make prediction based on fundamental values), and non-fundamentalists (trend predictors). Moreover, their model is grounded in theories such as the Prospect Theory and over-confident investors. Their results are a plea to the use of agent-based approaches since the traditional finance theories (top down, analytical) may not be effective when it comes to real markets and real investors. In particular, they found that the survival of fundamentalists is related to their ratio in the market, and that when non-fundamentalists estimate the losses excessively (in accordance to the Prospect Theory), then trade price of assets deviates from the fundamental value.

Other models from behavioral finance cannot be discussed here due to lack of space (again, see [10] for brief discussion and references). They are sometimes referred to as "irrational" decision-making since they deal with the role emotions play. This issue is key when we want to model heterogeneous investors who have idiosyncrasies.

3. STOCHASTIC PROCESSES AND PERSISTENCE IN FINANCIAL MARKETS

Economic analyses have recently encompassed interactions among individuals, as opposed to individual–specific determinants of behavior, in the set of causal factors governing market decisions. This has led to a change in the mathematical apparatus from deterministic to stochastic approaches [4]. An interesting and well known quantity in the theory of stochastic process is the probability of random variable to return to its initial condition, also called first return probability. This return may or may not occur depending on the process involved. Besides, within the same class of processes, the return probability might depend on the particularities of the each process. Consider for example the case of a random walks in regular lattices, a paradigm in the field of Stochastic Processes. In its simplest form, a random walk is a process where, starting from a given position s', a walker can move in any of the four directions N,S,E,W by taking unit steps with probability 1/4 (this square grid is called $\mathbb{Z}^2$ lattice in mathematical parlance). The random variable is the position s of the walker at instant n (where n also represents the number of steps taken). Defining $P_n(s)$ as the probability for a walker to be in $s \in \mathbb{Z}^2$ after n steps, it is given by

$$P_n(s) = \sum_{s' \in \mathbb{Z}^2} p(s' \to s) P_{n-1}(s'),$$

where $p(s' \to s) = 1/4$ represents the probability of jumping from site s' to neighbor s. This situation, where the probability of being somewhere depends only on the previous positions (there is no memory attached to the system) defines what is called a Markov process. The main question is: does a walker have a finite probability of returning to its start position? Polya's Theorem states that in one and two dimensions the return always occurs. In one dimension this can be readily understood: if there is no bias in the jumping direction the walker will eventually recur to its original site (an example of an unbiased random walk is the tossing of a fair coin). In two dimensions the probability of returning is one, even if we cannot picture it as easily as in 1-d. However, in dimensions greater or equal to three, there is a positive probability that the walker never returns.

These ideas can be extended to many different contexts, as for example in population dynamics in Biology: given that a population has dwindle due to some plague, how much time will it take for them to recover their original population, if ever? An analogy with financial markets is also straightforward: if an investor starts out with a certain amount of money invested in assets, how long does it take for him to get at least the same value back on his assets as the one he initially paid for?

As in population dynamics, agents in a financial market are not alone and their individual actions cannot usually alter significantly the dynamics of markets. However, their collective behavior, as for example the selling stampede caused by some expectation of political turmoil may drive markets to crash. Markets are clearly governed by a series of other factors which may affect commodities prices, but it is unquestionable that market agents do play a significant role when acting as a group. Such complex behavior which arises from collective actions has been extensively studied in the context of nonequilibrium processes in statistical mechanics.

In a variety of situations where collective behavior can be seen as arising from the action of the elements which make up the system, statistical mechanics can be used to bear upon the problem. First passage (or recurrence) problems as discussed in the context of random walkers are examples of such complex systems if one extends the ideas to a system

Motivation Level	Investment Level
$\sigma_i = 1$ and $\sigma_{i+1} = 1$	$S_i = 2$
$\sigma_i = 0$ and $\sigma_{i+1} = 1$	$S_i = 1$
$\sigma_i = 1$ and $\sigma_{i+1} = 0$	$S_i = 1$
$\sigma_i = 0$ and $\sigma_{i+1} = 0$	$S_i = 0$

Table 1: Investment rules mapping motivation levels to investment, for each pair of agents

of walkers whose step probabilities depend on the behavior of other players.

Within this scenario, one of our goals is to characterize the statistical properties of economic games. For this purpose, we have performed a series of stochastic agent–based simulations for a game where the investment is made by agents depending on the actions of its nearest neighbors through a binary variable we call "motivation" to invest. This parameter is updated as a function of the profit of the group, as explained next. The global persistence is analyzed and a scaling study is performed.

4. THE MODEL

In our basic model we consider an economic institution and L investors or economic agents. The former acts as the interest provider, as it will be detailed below. As for the latter, each one of the L agents can invest a particular quantity $S_i \in \{0, 1, 2\}$. The group starts the game with a quantity w_0 of money for each agent. The agents invest cooperatively, in the sense that the average of the profit of the group influences the investment "motivation" level for each agent. This motivation level is modelled by a binary variable $\sigma_i \in \{0, 1\}$, where $\sigma_i = 1$ means that the agent is motivated and $\sigma_i = 0$ means that it is not. This abstraction aims at capturing issues such as insider information, economic prospects, etc.

The algorithm that describes the evolution of dynamics of the group is locally constrained, meaning that each agent has relationship with just the two closest neighbors. Thus, each time an interaction occurs, it is between agents labelled i and $i + 1$ ($i - 1$ also interact with i on its turn), with corresponding motivation levels σ_i and σ_{i+1}, and investment levels S_i and S_{i+1}. These interactions follow the rules presented in Table 1. Please note that we chose a one–dimensional system for the sake of clarity and convenience, there being no restraint on the dimensionality of the system (as in a small-world scenario).

As one may see from this table, if an agent is unmotivated, it influences its neighbor, causing it to invest less than it actually could.

To update the motivation level of the agents, we represent the average investment of all agents in the t−th iteration as:

$$Q(t) = \frac{1}{L} \sum_{i=1}^{L} S_i(t) \tag{1}$$

To keep the model simple, at this stage we assume that the overall profit is modulated by a random variable r uniformly distributed in $r \in [0,\ 1]$, and the return per agent is given by:

$$R_i(t) = Q(t) \left[\frac{1}{2} + r \right] - S_i(t) \tag{2}$$

According to this formula, profits ($1/2 < r < 1$) and losses ($0 < r < 1/2$) are allowed only within a range which depends on the mean investment $Q(t)$. Practically we can see the interest provider as increasing or decreasing the overall wealth by a factor of 1/2. However, individual agents can be better or worse off. Besides, each agent has at each time an accumulated wealth given by:

$$T_i(t + 1) = T_i(t) + R_i(t) \tag{3}$$

where $T_i(1) = w_0$, $i = 1, ..., L$.

We update the motivation level factor σ_i at each time t as in Table 2.

$\sigma_i(t + 1) \to 1$	if $(R_i(t) > 0)$
$\sigma_i(t + 1) \to 0$	otherwise

Table 2: Update rules for the motivation level

This update is based on a simple principle: the agents' wealth relies on the wealth of the group; if the group is wealthy, the agent is wealthy as well. Therefore, in principle there is an incentive to cooperate and behave according to the rules defined for this society (e.g. look at the motivation levels of neighbors and invest accordingly). However, since agents are autonomous and there is indeed room to cheat, we end up with two kinds of markets: one in which everyone is cooperative, and another where different types of individual behaviors are simulated. The former is shown in Section 6.1 while the latter appears in Section 6.2. In the next section we will give the details of the stochastic agent–based simulations to calculate the global persistence.

5. STOCHASTIC AGENT–BASED SIMULATIONS

In the time–dependent stochastic agent–based (MC) simulations performed in this work, the quantity

$$I(t) = \frac{1}{L} \sum_{i=1}^{L} T_i(t) \tag{4}$$

(average accumulated wealth) was obtained from averages over a certain number of runs (samples), N_s. Therefore, the average of $I(t)$ over N_s is given by:

$$\overline{I}(t) = \frac{1}{LN_s} \sum_{i=1}^{L} \sum_{j=1}^{N_s} T_{ij}(t), \tag{5}$$

We have performed time-dependent stochastic agent–based simulations in order to calculate the global persistence probability $P(t)$, *i.e.*, the probability that the $C(t) = I(t) - w_0$ does not change its sign up to time t. Let the accumulated distribution $p(t) = \sum_{t'=1}^{t} \frac{n(t')}{N_s}$ be the probability of having a sign change for the first time exactly at instant t. Then, the probability $P(t)$ is

$$P(t) = 1 - p(t) = 1 - \sum_{t'=1}^{t} \frac{n(t')}{N_s}, \tag{6}$$

where $n(t')$ is the number of samples for which the $C(t)$ changes its signal for the first time at instant t' and N_s is the total number of samples. The exponent θ_g may be

Algorithm 1 Estimative of global persistence in the context of the stochastic agent–based simulations

INPUT: $N_s \leftarrow$ number of sample; $N_{\max} \leftarrow$ max. number of stochastic agent–based steps
for $i = 1, ..., N_s$ **do**
  Choose a random configuration to start the simulations
  $N_{MC} := 1$ {initializing MC steps}
  while $\left((N_{MC} \leq N_{\max}) \text{ .or. } \left(\sum_{i=1}^{L} T(i) \geq L \cdot w_0 \right) \right)$ **do**
    $N_{MC} = N_{MC} + 1$
    compute investment of all agents according to the motivation rules in Table 1
    update motivation of agents according to the rules in Table 2
    if $\left(\sum_{i=1}^{L} T_i(N_{MC}) < L \cdot w_0 \right)$ **then**
      $p(N_{MC}) = p(N_{MC}) + 1$
    end if
  end while
end for
WRITE $N_{MC}, \left(1 - \frac{1}{N_S} \sum_{i=1}^{N_{MC}-1} p(i) \right)$; to $N_{MC} = 1, ..., N_{\max}$
END

obtained directly from the power law scale relation [8]

$$P(t) \sim t^{-\theta_g}, \quad (7)$$

from which we obtain $\ln P(t) = c - \theta_g \ln t$, where c is constant and each run requires a randomly chosen configuration of agents motivations. Algorithm 1 shows a high level description of the simulation procedure.

6. RESULTS

6.1 Basic simulations

Our aim is to perform stochastic agent–based simulations to estimate the global persistence exponent θ of the model. In general, we use $N_{\max} = 1000$ time steps, and set $N_s = 1000$ or $N_s = 2000$ runs regarding wealth and persistence respectively[1].

Numerical experiments were performed to construct a plot of the $\overline{I(t)}$ as function of time t. We simulate two different cases: in the first situation, hereafter called I, we do not change the motivation level during the simulations, while in case II the update of motivations is done according to Equation 1 and Table 2. Upper and lower boxes in Figure 1 show the average of wealth within time, for the cases I and II respectively. In both, the initial quantity of money invested by each agent, w_0, is equal to 5 and the number of investors (L) varies.

In case I (keeping the motivation level) a conservative investor ($\sigma_i = 0$) and an agressive one ($\sigma_i = 1$) remain so during the entire simulation. The curves depict a stochastic behavior similar to a random walk, as one may see in Figure 1. In case II (also Figure 1), where the motivation level of each agent is updated within the simulation according to the society wealth, this wealth, on average, remains close to the value $w_0 = 5$. As in the previous case, here we also observe a stochastic behavior for the time series of wealth

[1]A higher number of runs is required for persistence since it is an experimental distribution which is obtained via an histogram

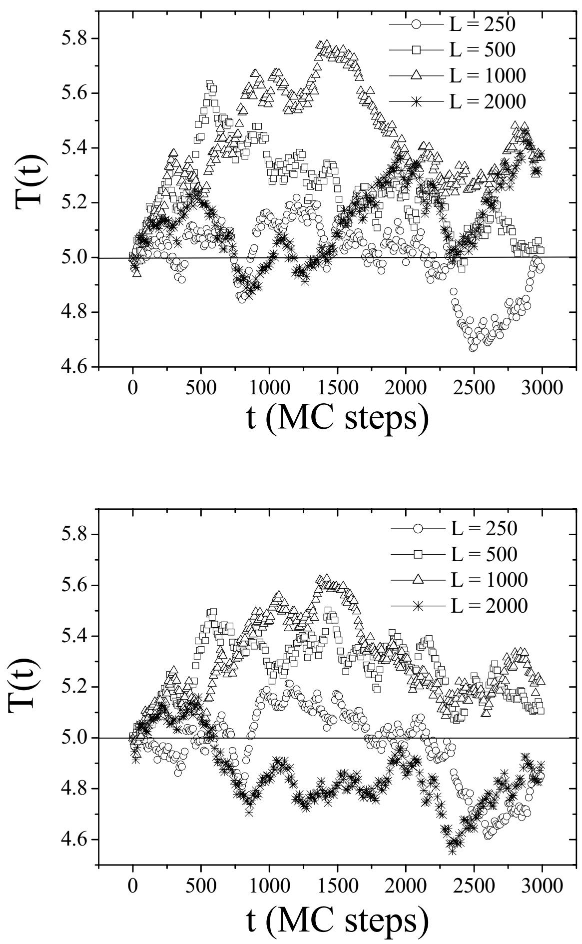

Figure 1: Average of the wealth as a function of time, for different number of investors L. Upper box: case I (no update of motivation level). Lower box: case II (update).

and there is hardly anything one can say about qualitative differences between these cases. Therefore, it is necessary to find another way to quantify the emerging collective behavior, which be capable of showing the differences between different profiles of the simulation parameters and/or mix of agents in the population. For our purposes we found that the global persistence could be this quantity, as we show next.

We measure the global persistence as function of time taking over $N_s = 2000$ samples and $N_{MC} = 1000$ stochastic agent–based steps, using Algorithm 1. In the plots for persistence, we used a log–log scale to allow for a better visualization of the scaling law.

In Figure 2 the lower curves of the global persistence were obtained from the time series with motivation levels unchanged. We observe a robust power law behavior: the probability that a particular time series for $T(t)$ (randomly selected) have a value larger than w_0 at time $t' > t$ decreases with time. Notice that this conclusion cannot be easily drawn from the plots of wealth (Figure 1).

Returning to Figure 2, the upper curves (case II, update of motivation), we do not observe quite the same robust law; there is a slight bending of the curves as time increases. The persistence becomes higher than in case I, suggesting that the system as a whole has a lower probability of having financial losses. The behavior is already markedly different (the persistence is 10 times higher) already at time $t = 100$ MC steps. The society has a lower chance to end up with a deficit, as compared to case I. However, the overall wealth does not increase. This just reflects the trade-off between low profit/low risk (case II) and high risk/high profit (case I). We would like to emphasize the fact that this kind of conclusion cannot be drawn from the plots for wealth. One needs a more accurate measure of the society's behavior, and this is given by the persistence.

Another way to show these conclusions is to numerically express the persistency exponent θ_g. This is estimated using time intervals of the same size in the log scale. Thus for each $t \in [10, t_{max}]$, a local measure of the exponent is taken, with t approaching t_{max}. The effective exponent is given by:

$$\theta_{eff}(t) = \frac{1}{\ln \Delta} \ln(\frac{P(t)}{P(t/\Delta)}) \quad (8)$$

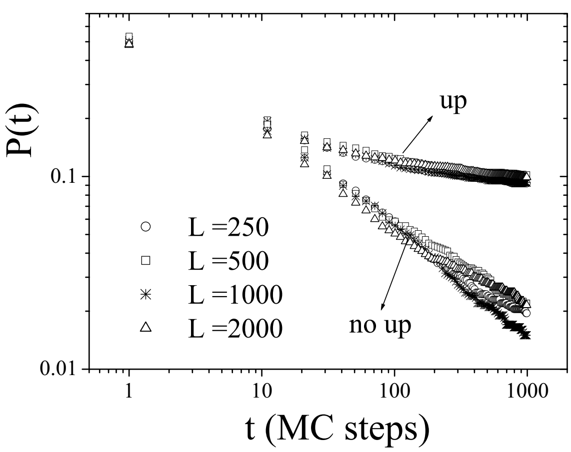

Figure 2: Global persistence P as a function of time in log-log scale. Lower curves: case I (motivation variable kept unchanged). Upper curves: case II (updating of motivation was performed).

Estimated values for the effective exponent of the persistence as function of the *inverse* of the time (for the sake of visualization) is depicted in Figure 3, for both case I and case II. In this figure we can observe that in case II, where the update of the motivation level is performed, the effective exponent has a tendency to increasingly smaller values ($\theta_{eff}(t \to t_{max}) \sim 0.1$) *in absolute terms* (the sign just shows the inclination), whereas in case I we notice $\theta_{eff}(t \to t_{max}) \sim 0.6$. This shows again that updating leads to risk-aversion.

6.2 Conservatives and Deceivers as Market Regulators

An interesting way to regulate the market is to introduce portions of agents that have idiosyncratic behaviors. We run

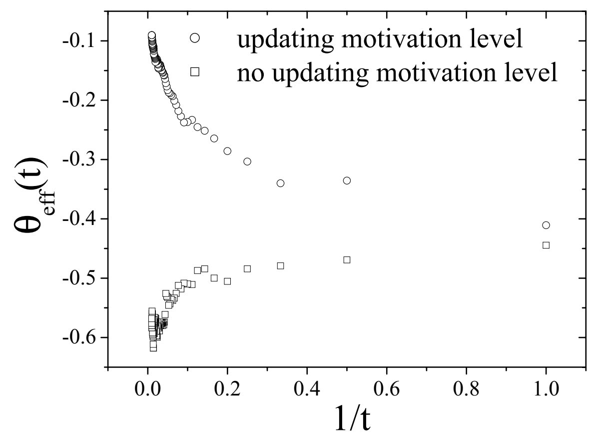

Figure 3: Effective persistence exponent

simulations with conservatives and deceivers. In some sense, both "lie" and/or do not update the individual motivation state. Our aim here is to study how persistent the market will be in these situations.

6.2.1 Conservatives

always have motivation level $\sigma = 0$ (they do not update) but do invest according to the rules given in Table 1. It is intuitive that, the higher the ratio of agents willing to invest *less* money, the more persistent the market will become. However this does not mean that the market is more profitable. On the other hand, a market in which a high number of agents invest the maximum quantity possible in each step can cause big losses due to the probabilistic nature of the return (Eq. 2).

Now we consider a new parameter in our simulations, z, which is the ratio of agents who remain at motivation level $\sigma_i = 0$ during the entire simulation. Experiments were performed changing z from 0 up to 1.

An analysis of Figure 4, upper box, shows that once more wealth alone is not a good source of information on the performance of the society. Thus we turn once again to persistence for a deeper look into the dynamics of the market. The upper plot tells us that the profit decreases with increasing z, which is obvious given that more agents are not motivated and influence others to invest less. For the first 300 MC steps in all cases the group has no profit but the deficit decreases with increasing z. The regions of high profit become less attractive. In particular, for $z = 1$, there is a stagnation of the market (neither profit nor deficit) because nobody invests and of course the probability to return to w_0 (P) is one. As to what regards the persistence, for $z = 0$ the behavior is the same as in Figure 2, namely $P(t) = 0.1$. As we increase z – for instance at $z = 0.2$ – we notice that the society turns less persistent. For $z = 0.4$, it seems that the effect of the conservatives is felt before in comparison to the previous cases so that the stagnation tendency occurs before and tends to a more persistent one. We plan to investigate in the future whether there is a transition between a case with and without update of the motivation level, specifically for the case with conservatives.

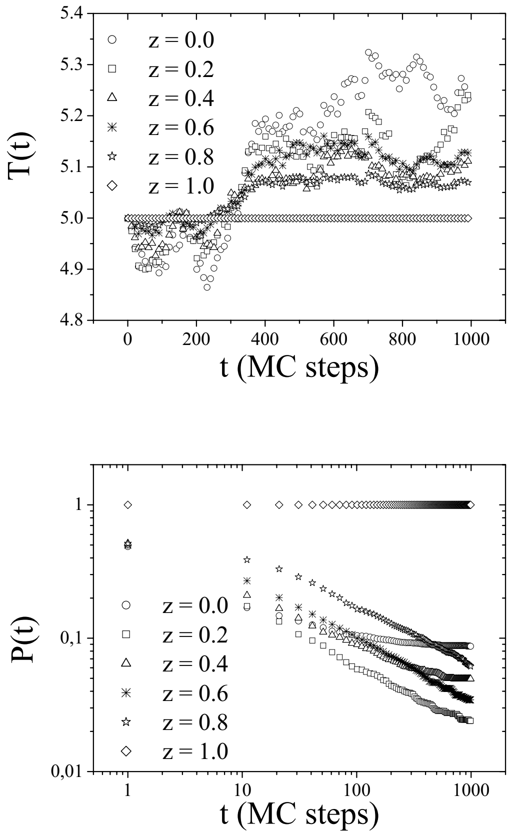

Figure 4: Effect of ratio of conservatives (z) on the mean wealth (upper plot) and on the persistence (lower plot) as a function of time.

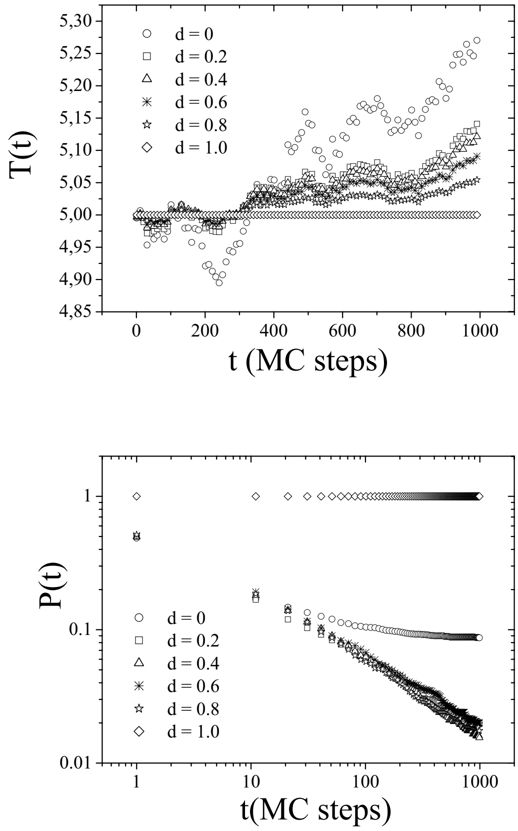

Figure 5: Effect of ratio of deceivers ($0 < d < 1$) on the mean wealth (upper box) and on persistence (lower box) as a function of time.

6.2.2 *Deceivers*

do update their motivation levels (as opposed to conservatives) but they do not invest a cent! Therefore they influence others to invest driven by the expectation that the more people invest, the higher the overall wealth will be and therefore they will benefit without investing anything.

In the curves shown in Figure 5 we vary the ratio of deceivers. When everyone is a deceiver, nobody invests and we have a situation similar to that when everyone is conservative, namely stagnation of the market. Also, for the first 300 MC steps, the market has mostly deficit, although here only the ratio $d = 0$ is really bad for the whole society. For other ratios the losses are not so high as in the case of conservatives. After 300 MC steps, there is a trend to increasing profit, which is higher the less deceivers there are. This only reinforces the common intuition that a small number of liars are better off but when everyone behaves like this, then the overall performance of the society decreases drastically, as can be seen from the plots for persistence in Figure 5: the probability to remain at w_0 is one in case $d = 1$.

7. SUMMARY AND CONCLUSIONS

Traditional economic analysis has recently been extended to include, in the set of causal factors determining individual decision making, interactions among individuals, as opposed to individual–specific determinants of behavior. From the perspective of the classical theory, this approach introduces two new features: on the one hand group–level influences on individuals are not directly mediated by markets. Second, and perhaps more significant is the fact that this approach introduces new and explicit sociological perspective on agent's individual behavior [4]: an agent might feel motivated by its peers to act as they do, based on feelings of belonging to a group ("to go with the pack").

In this work we presented a simple model of a society of L economic agents, where each can invest a discrete quantity in a cooperative or non-cooperative way, based on his nearest–neighbors' motivation level. The motivation is introduced to capture such features as insider information, economy prospects or personal or even moral stands on economic issues. The profit of a group fluctuates stochastically and is used to update the motivation level of individual agents after each stochastic agent–based step. We also introduce idiosyncratic personalities (conservative or deceiver agents) to see how the dynamics of the society's investment is altered.

Our results show that the analysis of the accumulated wealth over time is not always a good measure of the performance of the group as one would expect. A more informative analysis is given by the behavior of the first return probability as a function of time (which measures the probability that the total amount of money of an entire group remains higher than or equal to the initial amount up to time t), and wether or not it has a power law behavior. In general, the simulations show that the global persistence has a robust power behavior. Thus it can be used as an appropriate measure of the time the group remains wealthy.

In particular, the following issues should be stressed:

- the emerging collective behavior has a power law behavior in the persistence, and this does not vary significantly with the number of agents. In other words, it is robust;

- the update of motivation has a positive influence on the persistence: although it increases with update (by a factor of 10), the group becomes more risk-averse. Note that in our model the interest paid to agents depends on a random variable (r) and therefore the update prevents big losses because agents get a feedback via what happened with their neighbors. A market which is more persistent implies that agents are unwilling to take risks and hence their profits are smaller (but so are their losses);

- conservative and deceivers alter significantly the persistence of markets. Conservative investors make the market initially less persistent, but as their number increases, the persistence increases again. This behavior resembles what is called in physics a "phase–transition" and could indicate the existence of quite different dynamical behavior depending on the ratio of conservatives in the market. The presence of deceivers make the market highly profitable for them as long as their ratio remains small. If too many deceive, the market stagnates and profits decrease (deceivers invest no money).

It would be interesting to study a situation where agents can also change their type, going from conservative to aggressive and deceiver. This should reflect the fact that markets influence people, changing their behavior, as for example the news that bond prices are going up. This kind of information might lead one to feel tempted to make a bigger profit on a short stint and, from this perspective, to better characterize how agents gain or lose one has to go from the society (global persistence) to the individual (local persistence) level. This we plan on investigating in a future work.

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