Dynamics of a public investment game: from nearest-neighbors lattices to small-world networks

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1 Introduction

In the fields of complex systems and multiagent systems there is an extensive list of publications on nontrivial phenomena which arise due to the interplay between microscopic (individual) rules and macroscopic (group) behavior. In the context of socioeconomic behavior, this has been thoroughly discussed by Durlauf [2]. Within this scenario we study here a variation of a simple "public investment game" [1]. In its original version, one wishes to model public spending on public goods. Players can invest their money in a common pool, and profits are equally distributed among all participants irrespective of their contributions. Clearly it would be "fair" for people with similar amounts of money to invest similar quantities. However individuals are different: each player, being "blind" as to what regards others' contributions, would, if it were also "rational" , default and invest nothing - for purely rational players the dominant solution is to default.

To give the model a more realistic flavor we let agents interact and invest according to the actions of their immediate neighbors, as controlled by a binary variable we call *motivation*, and whose update depends on a random variable [3]. This aims at simulating natural causes which may affect the way players assess the investment. The return per agent is considered a function of the average investment, in close relation to cooperative game-theory.

We first consider periodic boundary conditions (players in a ring). To explore more complex networks which allow agents to be influenced by others far away from them, we also consider small-world networks [4]. Such networks are built from a neighborhood of size $k = 4$. With probability p one edge can be reconnected to a vertex chosen randomly.

In this paper we explore the dynamics of a fraction of agents with a deficit given that each player starts the game with the same quantity of money. We also measure the probability of a particular agent not losing all its money up to time t as function of parameter p of the small-world. We study the non-trivial dynamics that emerges out of this system by looking for the density of motivated agents in the model. In section

2 we perform these studies in small-world networks with arbitrary coordination k. Section 3 presents some numerical experiments. Section 4 brings some conclusions and open questions.

2 The model in a small-world network

We consider a game of L investors or economic agents which, starting the game with a quantity w_0 of money, can invest a particular quantity S_i. Agents invest cooperatively, *i.e.* the average profit of the group influences the investment motivation level of each agent, modelled by a binary variable $\sigma_i \in \{0, 1\}$ ($\sigma_i = 1$ means an agent is motivated while $\sigma_i = 0$ means it is not). This abstraction aims at capturing issues such as insider information and economic prospects as perceived by agents.

An agent $i = 1, ..., L$, in a small-world network built from a regular lattice with arbitrary coordination k has a set of neighbors we denote ξ_i. We define

$$S_i(t) = \sigma_i(t) + k(\ \rho_i(t)\ ). \tag{1}$$

where the function $k(\ \rho_i(t)\ )$ depends on the density of neighbors of an agent i at instant t $\rho_i(t) = (1/\left|\xi_i\right|) \sum_{j\in\xi_i} \sigma_j(t)$, as follows

$$k(\ \rho_i(t)\ ) = \begin{cases} v_1(t)\ if\ \rho_i(t) > 1/2 \\ v_2(t)\ if\ \rho_i(t) = 1/2 \\ v_3(t)\ if\ \rho_i(t) < 1/2 \end{cases}$$

where $v_1(t)$, $v_2(t)$ and $v_3(t)$ are arbitrary functions and $|\xi_i|$ is the cardinality of set ξ_i. An interesting choice for the v_i's is

$$v_1(t) = v_2(t) = v_3(t) = \rho_i(t)\left|\xi_i\right| = \sum\nolimits_{j\in\xi_i} \sigma_j(t) \tag{2}$$

since it allows for a wide range of investments, namely $0 \leq S_i \leq \sum_{j=1}^{L} \sigma_j(t)$. However, for the sake of clarity, we restrict our investment to four possibilities $S_i \in \{0, 1, 2, 3\}$, *i.e.*

$$v_l(t) = 3 - l \tag{3}$$

where $l = 1, 2, 3$ (see table 1).

To update the motivation level of the agents, we represent the average investment of agents in the $t-$th iteration as:

$$S(t) = \frac{1}{L}\sum_{k=1}^{L} S_k(t) \tag{4}$$

where periodic boundary conditions are imposed and S_k is obtained from (1).

σ_i (motivation)	ρ_i (density of neighborhood)	S_i (investment)
0	$< 1/2$	0
0	$= 1/2$	1
0	$> 1/2$	2
1	$< 1/2$	1
1	$= 1/2$	2
1	$> 1/2$	3

Table 1. Investment rules relating motivation levels to investment

We assume that the overall profit is modulated by a random variable r (noise) uniformly distributed in $r \in \ [-1,\ 1]$, and the return per agent is given by:

$$g_k(t) = (a + br)\, S(t) - S_k(t) \tag{5}$$

According to this formula, when $b = 0$ we have the deterministic case. On the other hand, if $a = 1$ and $b = 1/2$, profits $(0 < r < 1)$ and losses $(-1 < r < 0)$ are allowed only within a range which depends on the mean investment $S(t)$. Individually agents can be better off or not. Besides, at each time each agent has an accumulated wealth given by:

$$W_k(t+1) = W_k(t) + g_k(t) \tag{6}$$

where $W_k(1) = w_0$, $k = 1, ..., L$. We update the motivation at each time step by the profit rate $g_k(t)$:

$$\sigma_k(t+1) = \begin{cases} \frac{1}{2}\left(1 + \frac{g_k(t)}{|g_k(t)|}\right) & \text{if} g_k(t) \neq 0 \\ \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

This update is based on a simple principle: an agent's wealth relies on the wealth of the group. However, since agents are autonomous and there is room for cheating, we end up with two kinds of situations: one in which everyone is cooperative, and another where different types of individual behaviors are simulated. In the next section we present some results from numerical experiments.

3 Numerical experiments and results

We perform numerical simulations starting with half of the agents motivated, *i.e.* $\rho_0 = 1/2$, randomly chosen in small worlds networks built from lattices with coordination $k = 4$ (i initially connected to nodes $i-2$, $i-1$, $i+1$ and $i+2$) and periodic boundary conditions. We first measured the time evolution of the average density of motivated agents, considering many runs over random different initial conditions (different small worlds and configurations of motivated agents with fixed density $\rho_0 = 1/2$). However, particularly for study of time evolution density only

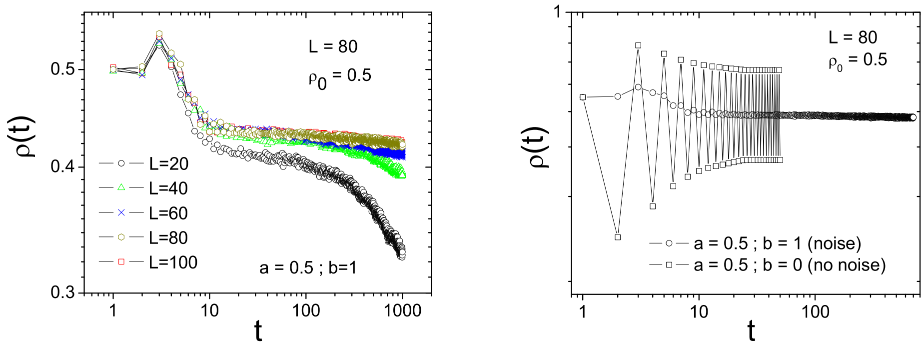


Fig. 1. (a) Time evolution of the density of motivated agents for $a = 1/2$, $b = 1$, $\rho_0 = 1/2$ for different lattice sizes ($L = 20$, 40, 60, 80,100) with coordination $k = 4$. (b) A comparison between a particular case with noise ($b = 1$) and the no noise ($b = 0$) one.

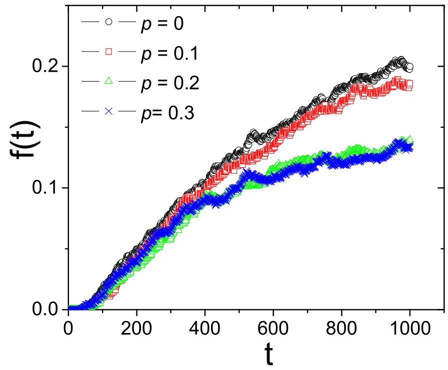
the case $p = 0$ was considered and the average is only over different configurations of motivated ones.

We explore the effects of finite size in the density of motivated agents. Our results (see fig 1 (a)) for the particular case with noise ($a = 1/2$ and $b = 1$) shows a deviation from $\rho(t)$ vs t for small values of L. We can also observe in this graph a tendency of the density towards a constant after ~ 20 MC-steps, where were used $N_s = 1000$ runs for computing of average and $t_{\max} = 1000$ MC-steps. For $L = 80$ we also simulated the case without noise $a = 1/2$ and $b = 0$. An oscillatory behavior for the density of motivated agents is found, as can be seen in the figure 1 (b). After the 27th MC-step the density oscillates between two fixed values of density ($\rho_1 = 0.29947$ and $\rho_2 = 0.62375$). The noisy and noiseless case are depicted together .

We also assessed the bankruptcy of agents. For this we studied four small worlds $p = 0$, 0.1, 0.2, 0.3. We measure the average fraction of bankrupt agents that is $f(t) = (1/L)\#\{W_k(t) < 0$ since $W_k(0) = w_0$, $k = 1, ..., L\}$. For our simulations we considered $w_0 = 10$. A plot of $f(t)$ is shown in figure 2 (a). To quantify the influence of p in the bankruptcy we used the concept of first return probability $Q(t)$, which measures the probability of an agent remaining wealthy $W_k(t') > 0$ for all $t' < t$. Our results show that this probability decay as a power law and $Q(t)$ vs t is less steep with growing p.

4 Conclusions and summaries

We explored the emergent dynamics in a modified public investment game in small-world networks, where benefits are determined by two parameters: a deterministic (a) and a random one (b). Investment depends on the motivation level of an agent



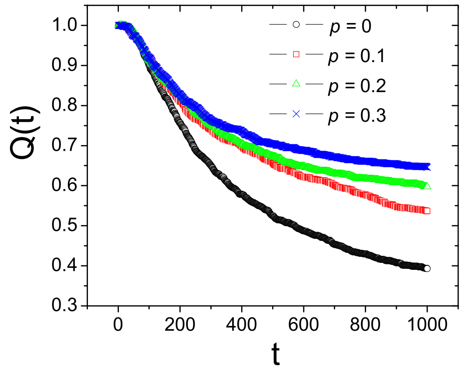


Fig. 2. (a) Time evolution of the fraction of agents in a situation of bankruptcy. (b) Probability of a particular agent randomly chosen does not reaching bankruptcy.

and of its neighbors. We performed some simulations of $f(t)$ (fraction of brankrupt agents) and $Q(t)$ (probability that an agent does not go bankrupt up to time t) for different small worlds (parameter p) generate from regular lattices with coordination $k = 4$. Our results show that both $f(t)$ and $Q(t)$ depend on$p$. For the regular case ($p = 0$) we studied a noisy ($a = 1/2$ and $b = 1$) and a noiseless ($a = 1/2$ and $b = 0$) situation. A finite-size dependence is observed for the case with noise and an oscillatory behavior in the noiseless one. Actually a phase diagram can be obtained in a simplified situation $S_i = \sigma_i + \sigma_{i-1}$ for $p = 0$ and $k = 2$, as shown in [3]. In the future we expect to extend the results to obtain the phase diagram for the general case, for lattices with bigger coordination numbers and small-world networks. The probability for the entire group to remain with an amount of money above the initial quantity up to time t, taking into account different values of p should also be analysed.

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