Lista II.5 - FIS01208 - Semestre 2010/1

1. Determine the probability distribution for various values of the momentum for the normal state of a particle in an infinite deep square potential well. (Exercício do Landau-Lifschitz, vol. 3. Por estado normal entende-se o costumeiro estado fundamental).

2. Determine the probability distribution of the various values of the momentum for an oscillator. (Exercício do Landau-Lifschitz, vol. 3).

3. A particle moves in the homogeneous gravitational field over the earth's surface, the latter being assumed to elastically reflect the particle e.g. a steel ball dancing over an horizontal plate of glass). This basic problem of classical mechanics is to be treated according to quantum theory. Solve the Schrödinger equation to obtain is eigenfunctions and its eigenvectors. [Just in case you can't solve the problem without any help: see Landau and Lifschitz and/or the article *A quantum bouncing ball, The American Journal of Physics, 67, 776-782 (1999)*]

4. A particle in a harmonic oscillator potential

$$V(x) = \frac{1}{2}m\omega^2 x^2$$

has an initial wave function

$$\Psi(x, t = 0) = \frac{1}{\sqrt{2}}\left[\phi_0(x) + \phi_1(x)\right]$$

where ϕ_0 and ϕ_1 are the $n = 0$ and $n = 1$ normalized eigenstates for the harmonic oscillator.
a) Write down $\Psi(x,t)$ and $|\Psi(x,t)|^2$. For this part, you may leave the expression in terms of $\phi_0(x)$ and $\phi_1(x)$.
b) Find the expectation value of x as a function of time. Notice that it oscillates with time. What is the amplitude of the oscillation in terms of m, ω, and fundamental constants? What is its angular frequency?
c) Find the expectation value of p as a function of time. Use your result from part b), and check if Ehrenfest's Theorem holds for this potential.

5. An experimenter asks for funds to observe visually through a microscope the quantum behavior of a small oscillator. According to his proposal, the oscillator consists of an object 10^{-4} cm in diameter and estimated mass of 10^{-12} g. It vibrates on the end of a thin fiber with a maximum amplitude of 10^{-3} cm and frequency 1000 Hz. You are referee for the proposal
a) What is the approximate quantum number for the system in the state described?
b) What would be its energy in eV if it were in its lowest-energy state? Compare with the average thermal energy (25 meV) of air molecules at room temperature.
c) What would be its classical amplitude of vibration if it were in its lowest-energy state? Compare this with the wavelength of visible light (500 nm) by which it is presumably observed.
d) Would you, as referee of this proposal, recommend award of a grant to carry out this research?

6. Find the energy levels of a particle of mass m moving in a one-dimensional potential:

$$V(x) = \begin{cases} \infty & x \leq 0 \\ \frac{1}{2}m\omega^2 x^2 & x > 0 \end{cases}$$

7. Calculate the probability of finding a particle in the classically forbidden region of a harmonic oscillator for the sates n=0,1,2,4. Are these results compatible with their classical counterparts? Given:

$$\int_1^\infty \exp(-y^2)dy = 0.1394 \qquad \int_{\sqrt{3}}^\infty y^2 \exp(-y^2)dy = 0.0495$$
$$\int_{\sqrt{5}}^\infty (4y^2 - 2)^2 \exp(-y^2)dy = 0.6740 \quad \int_{\sqrt{9}}^\infty (16y^4 - 48y^2 + 12)^2 \exp(-y^2)dy = 26.86$$

8. Find the energy levels and the wave functions of two harmonic oscillators of masses m_1 and m_2, having identical frequencies ω, and coupled by the interaction $\frac{1}{2}k(\hat{x}_1 - \hat{x}_2)^2$. [Hint: This is a two-particle problem. As in Classical Mechanics, it is more convenient to describe the dynamics of a two-particle system in terms of the center of mass (CM) and relative motions].

9. Consider a particle of mass m and charge q moving under the influence of a one-dimensional harmonic oscillator potential. Assume it is placed in a constant electric field ε. Derive the energy expression and the wave function of the nth excited state.