Lista II.3 - FIS01208 - Semestre 2010/1

1. Neutrons from a reactor are made to pass through a column of (polycrystalline) beryllium. This material is chosen because it does not absorb neutrons appreciably. It is found that the neutrons which emerge at the other end are "cold", they have kinetic energies corresponding to temperatures below 50 K. The "warmer" neutrons, with kinetic energies corresponding to room temperature, are found to be strongly scattered out of the beam by the beryllium. Can you give an explanation of these phenomena? (Lattice constants: a=0.2275 nm, b=0.358 nm).

2. One of the favorite arguments of those who want to refute the uncertainty relation goes as follows. A monoenergetic beam of electrons, of momentum p, is incident normally on the screen S_1 from the left. This screen has a circular hole, of diameter a. At a distance d from the screen S_1 we have another screen S_2, which likewise has a circular hole of diameter a. We assume that the two holes are lined up in the direction of the incident beam. Some of the electrons which pass through the first hole might be deflected, but some of them will go to pass through the second hole. Consider an electron which has passed through the second hole. The uncertainty in its lateral position is of order $\Delta x \simeq a$. The magnitude of its momentum is p, the same as in the incident beam, because the electrons do not lose or gain energy in this experiment. Since we know that the electron has passed through both holes the uncertainty in the direction of the momentum must be less than, or equal to, $\Delta\theta \simeq a/d$. It follows that the uncertainty in the lateral component of the momentum of the electron is of order $\Delta p \cong (a/d)p$. We thus have

$$\Delta x \Delta p \cong \frac{a}{d} ap$$

for the product of the uncertainties in lateral position and lateral momentum. By making a small and d large we can make this product as small as we please, and hence violate the uncertainty relation, which is one of the cornerstones of quantum mechanics.
Can you demolish this argument? (Exercise from vol 4, Berkeley).

3. Electrons impinge on a double slit and form an interference pattern on a far-away screen with spatial period s. The slits have equal width that is much smaller than the electrons de Broglie wavelength. The contrast C of the interference pattern is defined as $C = (I_{max} - I_{min})/(I_{max} + I_{min})$, where I_{max} and I_min are the maximum and minimum intensity on the screen.
a) Assume that we have a way of changing the phase of the wavefunction at each of the slits without changing its amplitude. If we change the phase of the wavefunction at slit 1 by φ_1, and the phase of the wave function at slit 2 by φ_2, what happens to the electron interference pattern on the screen? (Position and contrast.) Explain your answer with a formula or a sentence.
b) Now assume that we do not change the phases of the wavefunctions at the slits, but instead make slit 1 half as wide as slit 2. What happens to the interference pattern on the screen? (Position and contrast.) Explain your answer with a formula or a sentence.
c) What happens to the interference pattern if we replace the electrons impinging on the double-slit by muons of the same energy? If the interference pattern changes, specify the change quantitatively. The mass of the muon is 207 times larger than that of the electron, the charge is the same. Both the electrons and the muons are assumed to be non-relativistic.

4. Show that the addition of an imaginary part to the potential in the quantal wave equation describes the presence of sources or sinks of probability. (Work out the appropriate continuity equation.)
Solve the wave equation for a potential of the form $V = -Vo(1 + i\zeta)$, where Vo and ζ are positive real constants. If $\zeta \ll 1$, show that there are stationary state solutions that represent plane waves with exponentially attenuated amplitude, describing absorption of the waves. Calculate the absorption coefficient.

5. Prove the following theorem:
A function $\psi(x)$ and its Fourier Transform $\phi(k)$ have the same normalization, i.e.

$$\int_{\infty}^{\infty} |\psi(x)|^2 \, dx = \int_{\infty}^{\infty} |\phi(k)|^2 \, dk$$

6. a) Show that the probability current can be written as

$$j(x,t) = \frac{1}{2m} \left\{ \psi^*(x,t)\hat{p}\psi(x,t) + [\psi^*(x,t)\hat{p}\psi(x,t)]^* \right\}$$

b) Show that a complex potential $(V(x))^* \neq V(x)$ contradicts the continuity equation.

7. We have defined the expectation value of a function g(p) of momentum in terms of the probability density in wavevector (or momentum) space as

$$\langle g(p)\rangle = \int_{-\infty}^{\infty} dk\ g(\hbar k)|\tilde{\phi}(k)|^2 = \int_{-\infty}^{\infty} p|\phi(p)|^2\ dp.$$

For $g(p) = p$, this is simply $\langle p\rangle = \int_{-\infty}^{\infty} \hbar k|\tilde{\phi}(k)|^2\ dk = \int_{-\infty}^{\infty} p|\phi(p)|^2\ dp$.
Show that we can instead calculate the expectation value of momentum directly from the spatial wavefunction as

$$\langle p\rangle = \int_{-\infty}^{\infty} \psi^*(x)\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)\psi(x)\ dx$$

and in general

$$\langle p^n\rangle = \int_{-\infty}^{\infty} \psi^*(x)\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)^n \psi(x)\ dx$$

This means that momentum p is represented in the spatial domain as an operator $\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}$
Show that similarly the expectation value of the position operator can be written as

$$\langle x\rangle = \int_{-\infty}^{\infty} \phi^*(p)\left(i\hbar\frac{\partial}{\partial p}\right)\phi(p)\ dp$$

and in general

$$\langle x^n\rangle = \int_{-\infty}^{\infty} \phi^*(p)\left(i\hbar\frac{\partial}{\partial p}\right)^n \phi(p)\ dp$$

What is the representation of the particle's domain in position x representation?