Lista II.2 - FIS01208 - Semestre 2010/1

1. A 45kW broadcasting antenna emits radiation emits radio wave at a frequency of 4 MHz. a) How many photons are emitted per second? b) Is the quantum nature of the eletromagnetic radiation important in analyzing the radiation emitted from this antenna?

2. Consider a mass-spring system where a 4 kg mass is attached to a massless spring of constant k= 196 N/m; the system is set to oscillate on a frictionless, horizontal table. The mass is pulled 25 cm away from the equilibrium position and then released.
a) Use classical mechanics to find the total energy and frequency of oscillations of the system.
b) Treating the oscillator with quantum theory, find the energy spacing between two consecutive energy levels and the total number of quanta involved. Are the quantum effects important in this system?

3. Estimate the energy of the electrons that we need to use in an electron microscope to resolve a separation of 0.27 nm.

4. In a double-slit experiment with a source of monoenergetic electrons, detectors are placed along a vertical screen parallel to the y-axis to monitor the diffraction pattern of the electrons emitted from the two slits. When only one slit is open, the amplitude of the electrons detected on the screen is $\psi_1(y,t) = A_1 e^{-i(ky-wt)}/\sqrt{1+y^2}$, and when only the other is open the amplitude is $\psi_2(y,t) = A_2 e^{-i(ky+\pi y-wt)}/\sqrt{1+y^2}$, where A_1 and A_2 are normalization constants that need to be found. Calculate the intensity detected on the screen when
a) both slits are open and a light source is used to determine which of the slits the electron went through
b) both slits are open and no light source is used
Plot the intensity registered on the screen as a function of y for cases (a) and (b)
It is given: $\int_{-\infty}^{\infty} dy/(1+y^2) = \pi$.

5. a) Find the Fourier transform for

$$\phi(k) = \begin{cases} A(a-|k|) & \text{if } |k| \le a \\ 0 & \text{for } |k| > a \end{cases}$$

where a is a positive parameter and A is a normalization factor to be found.
(b) Calculate the uncertainties Δx and Δp and check whether they satisfy the uncertainty principle.

6. a) Calculate the final size of the wave packet representing a free particle after travelling a distance of 100 m for the following four cases where the particle is:
i) a 25 eV electron whose wave packet has an initial width of 10^{-6} m
ii) a 25 eV electron whose wave packet has an initial width of 10^{-8} m
iii) a 100 MeV electron whose wave packet has an initial width of 1 mm
iv) a 100 g object of size 1 cm moving at a speed of 50 m/s
b) Estimate the times required for the wave packets of the electron in (i) and the object in (iv) to spread to 10 mm and 10 cm, respectively. Discuss the results obtained.

7. A neutron is confined in space to 10^{-14} m. Calculate the time its packet will take to spread to a) four times its original size; b) a size equal to the Earth's diameter; c) a size equal to the distance between the Earth and the Moon.

8. Consider the bound state of two quarks having the same mass m and interacting via a potential energy $V(r) = kr$ where k is a constant.
a) Using the Bohr model, find the speed, the radius and the energy of the system in the case of circular orbits.
b) Determine also the angular frequency of the radiation generated by a transition of the system from energy state n to energy state m.
c) Obtain numerical values for the speed, the radius and the energy for the case of the ground state, $n = 1$, by taking a quark mass of $mc^2 = 2$ GeV and $k = 0.5$ GeV fm$^-1$.

9. A free particle moving in one dimension is in the state

$$\psi(x) = \int_{-\infty}^{\infty} dk \exp\left(-\frac{(ak)^2}{2} + ikx\right)(i \sin(ak))$$

a) What values of the momentum will not be found?
b) In which momentum state is the particle most likely to be found?

10. Consider a particle of mass m inside a box of size a with infinite walls

$$V(x) = \begin{cases} 0 & \text{for } 0 \leq x \leq a \\ \infty & \text{elsewhere.} \end{cases}$$

The wavefunction is specified at $t = 0$ to be

$$\psi(x, t = 0) = C\left[3\sin(2kx) - 2\sin(3kx)\right]$$

where $k = \pi/a$.
a) Determine the normalization coefficient C.
b) Expand the wavefunction at the initial time $\psi(x, t = 0)$ in terms of the eigenfunctions of the infinite box, i.e. determine the expansion coefficients c_n.
c) Write down $\psi(x, t)$, at an arbitrary later time t.
d) If a measurement of the particle's energy at time t is performed, what will be the possible outcomes, and with what probability will those values be measured? What is the average energy $\langle E\rangle$ of the particle in the box? Is $\langle E\rangle$ changed by the measurement?
e) Calculate the probability current $j(x, t = 0)$ at the initial time. Does it depend on the complex phase that you have chosen for C, and why (not)? Will the value of j remain unchanged at later times? Express the fact that the particle does not leave the box as a mathematical condition on j.

11. In quantum mechanics the quantum of lattice vibration is called a **phonon**. Consider a longitudinal polar-optic phonon in GaAs that has energy $\hbar\omega = 36.3$ meV, where $\hbar$ is Planck's constant and ω is the angular frequency of the lattice vibration. Assuming a particle mass $m = 72 \times \mathrm{m}_p$, where m_p is the mass of a proton, estimate the amplitude of oscillation of this phonon relative to the nearest-neighbor spacing. The lattice constant of GaAs is $\mathrm{L} = 5.65\times\ 10^{-10}$ m.