Lista II.1 - FIS01208 - Semestre 2010/1

1. To what velocity would an electron (neutron) have to be slowed down, if its wavelength is to be 1 meter? Are matter waves of macroscopic dimensions a real possibility?

2. For the observation of quantum mechanical Bose-Einstein condensation, the interparticle distance in a gas of noninteracting atoms must be comparable to the de Broglie wavelength, or less. How high a particle density is needed to achieve these conditions if the atoms have mass number A = 100 and are at a temperature of 100 nanokelvin?

3. Assume $\phi(k_x) = \sqrt{2\pi}$ for $\bar{k}_x - \delta \leq k_x \leq \bar{k}_x + \delta$, and $\phi(k_x) = 0$ for all other values of k_x.
Calculate $\psi(x, 0)$, plot $|\psi(x, 0)|^2$ for several values of δ, and show that $\Delta x \Delta k_x \cong 1$ holds if Δx is taken as the width at half maximum.

4. Assume $\psi(x, 0) = e^{-\lambda|x|}$ for $-\infty < x < \infty$. Calculate $\phi(k_x)$ and show that the widths of $\phi(k_x)$ and $\psi(x, 0)$, reasonably defined, satisfy the reciprocal relation $\Delta x \Delta k_x \cong 1$.

5. By choosing reasonable numerical values for the mass and velocity, convince yourself that $\Delta x \Delta p_x \geq \hbar/2$ does not in practice impose any limitations on the precision with which the position and momentum of a macroscopic body can be determined.

6. Consider a wave packet satisfying the relation $\Delta x \Delta p_x \approx \hbar$. Show that if the packet is not to spread appreciably while it passes through a fixed position, the condition $\Delta p_x << p_x$, must hold (see Merzbacher).

7. Can the atoms in liquid helium at 4 K (interatomic distance about 0.1 nanometer = 1Å) be adequately represented by nonspreading wave packets, so that their motion can be described classically?

8. Make an estimate of the lower bound for the distance Δx, within which an object of mass m can be localized for as long as the universe has existed ($\approx 10^{10}$ years). Compute and compare the values of this bound for an electron, a proton, a one-gram object, and the entire universe. (Hint: see how the width of a gaussian wavepacket evolves with time).

9. A high-resolution neutron interferometer narrows the energy spread of thermal neutrons of 20 meV kinetic energy to a wavelength dispersion level of $\Delta\lambda/\lambda = 10^{-9}$. Estimate the length of the wave packets in the direction of motion. Over what length of time will the wave packets spread appreciably?

10. A one-dimensional initial wave packet with a mean wave number $\bar{k}_x$, and a Gaussian amplitude is given by

$$\psi(x, 0) = C \exp\left[-\frac{x^2}{4\Gamma^2} + i\bar{k}_x x\right].$$

Calculate the corresponding k_x-distribution and $\psi(x, t)$, assuming free particle motion. Plot $|\psi(x, t)|^2$ as a function of x for several values of t, choosing Γ small enough to show that the wave packet spreads in time, while it advances according to the classical laws. Apply the results to calculate the effect of spreading in some typical microscopic and macroscopic experiments.