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FROM FORMULAE TO CONCEPTUAL EXPERIMENTS: INTERACTIVE MODELLING IN THE PHYSICAL SCIENCES AND IN MATHEMATICS

V. D. TEODORO

Faculty of Science and Technology, New University of Lisbon, 2825 Campus de Caparica, Portugal

E-mail: vdt@mail.fct.unl.pt

Physical sciences are the sciences of constructing models (simplified descriptions or explanations) about the physical world. Models based on functions, differential equations and difference equations can describe many patterns of the physical world. In the traditional learning environments, these types of models are difficult to be mastered by many students. These difficulties can be rooted in the fact that most students do not have tools with which they can explore formal objects as «objects-to-think-with», as «objects-to-experiment-with». Experimentation with conceptual objects is a new kind of experiment—a conceptual experiment—only possible with computer tools, based on graphical user interfaces.

In this paper, I will present some aspects of a computer tool (*Modellus*, developed at the New University of Lisbon, and available worldwide through a US publisher), and will discuss conditions for its successful use, based on research on learning and innovation.

> «To create a world and watch it evolve is a remarkable experience. It can teach one what it means to have a model of reality, which is to say what it is to think. It can show both how good and how bad such models can be. And by becoming a game played for its own sake it can be a beginning of purely theoretical thinking about forms.» (Ogborn, 1989)

1 THE PLACE OF MODELS IN SCIENCE AND MATHEMATICS EDUCATION

Before Galileo and Newton, most of the ideas in the physical sciences were expressed only in words. For example, the famous statement by Aristotle that «speed of falling is proportional to weight» was expressed in his book *Physics* without any equation. Some authors attribute to Galileo the idea of science as we see it today, since Galileo was the first to assume that «Mathematics is the language of Nature».

If we look up the word «model» in a dictionary (e.g., the *Oxford Concise Dictionary*, Oxford University Press, 1996), we find that it is «a simplified (often mathematical) description of a system etc., to assist calculations and predictions.» According to Webb and Hassell (1988), there are five families of models:

- dynamic system models;
- spatial distribution models;
- qualitative models of logical reasoning;
- probabilistic event models;
- data analysis models.

Models are created to describe natural systems, capturing only the essence of major objects and processes. A model is just a simplified representation of a system and does not aim to represent all the features of that system. Many models in the physical sciences are dynamic system models, i.e., models that establish some sort of mathematical relation between physical quantities and time, considered as an independent variable. In this paper, I am specially interested in dynamic system models and in computer tools to explore this kind of model.

A simple example of a dynamic system model is the model of an object, considered as a particle (an object with mass but no dimensions), moving with constant velocity (since velocity is a vector quantity, both the direction and the magnitude are constant). The distance travelled by the particle, *s*, can be expressed as a linear function of time, *t*,

$$s = v\,t \quad .$$

Another way of expressing this model is by representing the *x*-coordinate in a particular reference frame, where the direction of *Ox* is coincident with the trajectory of the particle, as a linear function of time,

$$x = v_x t + x_0 \quad .$$

In this equation, v_x represents the scalar component of the velocity in the *Ox* direction and x_0 represents the value of *x* when time is zero.

We can also represent this model using the concept of rate of change. Since the rate of change of the *x*-coordinate is constant, we can write that as

$$\frac{dx}{dt} = v_x \quad .$$

These three models are equivalent and they are used in different stages of learning. Typically, the first one is used with junior high school students, the second with senior high school students and the third with college students. Both the first and the second models are functions and the third is a first-order differential equation.

I shall note that none of these models are «explanations» of the motion of the object. Explaining a motion is describing the forces that are responsible for the motion. This means that explaining a motion with constant velocity is just saying that the sum of all forces acting on the particle is null. This *explanation* is really a *description* of the interaction between the system (the particle) and its environment.

We can still consider another equivalent model for the motion of a particle with constant velocity. Using the equal sign as a means to represent that the left side of the equation is substituted by the right side, we can write that as

$$x = x + v_x dt$$

where *dt* is a «small» time interval. This model, a difference equation, is of particular interest in certain conditions, especially when we are interested in discussing how computers can use rates of change to make simple or complex computations. Some authors (e.g., Ogborn, 1984) and more recently projects such as the IoP project Physics 16-19[1], give special attention to difference equations as a simple way of introducing students to Calculus concepts, such as rate of change, differential equations, integration, etc.

Models are mental constructions distilled from theory and data about the physical world. For example, we can use data (time and position) of a motion with constant velocity to adjust a function such as $x = v_x t + x_0$. From a few assumptions and definitions, Newton was able to use the then new precisely defined concept of force (the instantaneous rate of change of the momentum) to explain uniform rectilinear motion. It must be emphasized that, as Bertrand Russel pointed out (1948), scientists use mathematical models to describe/explain phenomena not because they know the physical world very well but because they know very little: they can only discover mathematical properties of physical entities, such as particles, fields, heat, planets, galaxies, etc.

Learning models of motion is a process of creating mental representations, not from scratch as Galileo and Newton did, but from socially shared knowledge, in books and in learning communities. This process, as most of school learning processes, involves both *learning by reception* and *learning by doing*. Learning by reception since students cannot rediscover scientific principles and ideas (that took a long time and effort to formulate), and learning by doing because as the constructivist movement in education has shown us in the last two decades, learning is an idiosyncratic process where new ideas interact with previous knowledge.

Computer modelling tools, i.e., computer software that allows the user to create and explore computer-based models without writing a program in a high-level computer programming language, are essential tools both for learning by reception (students can explore models built by the teacher or someone else) and for learning by doing (students can build and explore their own models.)

The process of modelling as part of powerful learning environments, as defined by de Corte (1989), can lead to better understanding. For example, modelling can (Webb & Hassell, 1988):

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[1] A project from the Institute of Physics, aiming to revitalise physics post 16 in the UK (http://www.iop.org/IOP/AP).

1. raise the level of cognitive process, by encouraging students to think at a higher level, generalising concepts and relationships;
2. encourage pupils to define their ideas more precisely;
3. provide pupils with opportunities to test their own cognitive models and detect and correct inconsistencies.

2 *MODELLUS*: MAKING «CONCEPTUAL EXPERIMENTS» WITH A COMPUTER

Modellus (Teodoro, Vieira, & Clérigo, 1997) is a computer tool that allows students and teachers to perform conceptual experiments using mathematical models expressed as functions, derivatives, rates of change, differential equations and difference equations. The theory under *Modellus* has been developed in recent years, with contributions from many authors (especially, Mellar, Bliss, Boohan, Ogborn, & Tompsett, 1994; Perkins, Schwartz, West, & Wiske, 1995).

From a computational point of view, *Modellus* can be seen as a computer microworld for both students and teachers alike, based on a non-programming metaphor: in the «Model window» (see Figure 1) the user can write mathematical models, almost always the same way as he would on paper. There is no new language to learn, either visual or written, such as in Logo or in the modelling software STELLA (High Performance Systems, 1988).

Two essential features of Modellus are *multiple representations* and *direct manipulation*. Multiple representations means that the user can create, see and interact with analytical, analogical and graphical representations of mathematical objects. Direct manipulation means that the user can work directly with all kinds of objects that appear on the computer screen, without the mediation of any symbolic representation such as a programming language. Objects in *Modellus* can be classified as a new class of objects: concrete-abstract objects (Hebenstreit, 1987). Concrete in the sense that they can be manipulated directly with a computer and abstract in the sense that they are representations of mathematical constructs.

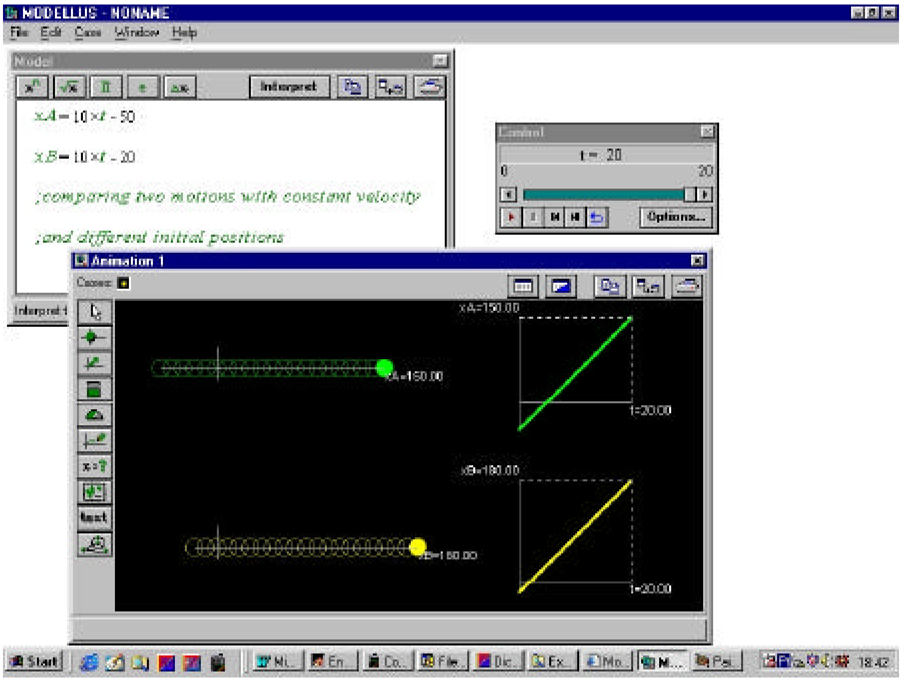

Figure 1 A simple exploration of rectilinear motion with constant velocity done with *Modellus.*

From an educational point of view, *Modellus* incorporates both expressive and exploratory modes of learning activities (Bliss & Ogborn, 1989). In an expressive learning activity, students can build their own models and create ways of representing them. In an exploratory mode, students can use models and representations made by others, analysing how different things relate to one another. Teachers and curriculum developers can also take advantage of the educational design of *Modellus*, since the software can be used as an authoring language for creating visual representations. A teacher or curriculum developer can specify what is presented to the student in a certain learning situation (for example, s/he can prepare a model of the Earth-Mars-Sun system to show the trajectory of Mars as seen from the Earth or as seen from outside the solar system, not showing to the student the complexity of the equations that govern the model (see Figure 2).

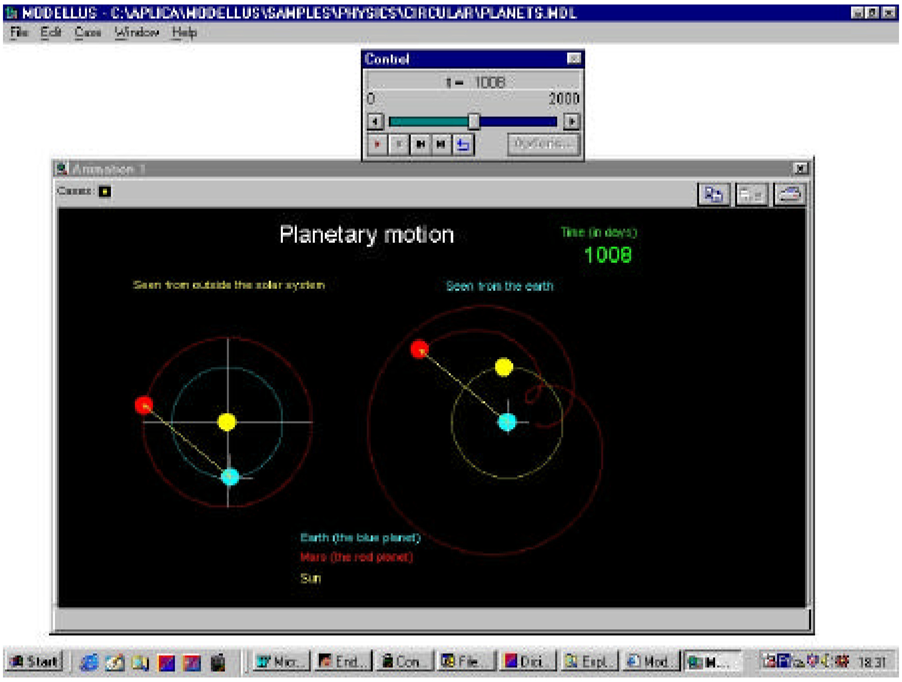

Figure 2 The trajectory of Mars as seen from a point outside the solar system (left) and as seen from the Earth (right). The interface is simplified (the user cannot access the full spectrum of *Modellus* tools).

The pedagogical design of *Modellus* assumes that the computer is a cognitive tool, but not a replacement of the higher order human skills. Contrary to the design options of other educational systems, such as those usually adopted in artificial intelligence, it is assumed that *Modellus* is a tool to «impart wheels to the mind» but the intelligence, emotion, culture, poetry, and art, reside in the user, not in the software.

Technology, science, and mathematics have been the fundamental forces that shaped our century. The importance of these human activities is not reflected in human culture. Several studies, all over the world, have shown that students and adults have persistent misconceptions or alternative conceptions about scientific matters, even the most basic ideas (Pfundt & Duit, 1991). Some of these misconceptions can be easily associated with the way science and mathematics are taught in schools: teaching tends to overemphasize verbal learning, where objects, particularly mathematical objects, are taught as complete abstract entities. Computer tools like *Modellus* can help change the emphasis in a direction where mathematical objects are *objects-to-think-with*, objects to perform experiments with, and objects to study interactively. Learning can then become concrete—mathematical objects behave like «real» objects and simultaneously maintain their abstractness: they represent the essential features of a phenomenon, not the whole phenomenon.

However, it is important to take some relevant limitations into consideration when using exploratory environments such as computer-based modelling tools or computer simulations. For example, teachers should be aware that learning with exploratory environments does not take place spontaneously: regulation and control are fundamental. Probably, the best way to regulate and control learning is the use of written materials, with guided inquiry approaches, where students read, discuss with peers, confront conceptions and descriptions, and write (the process of writing is a «disaccelerator» of information, specially visual information, and an accelerator of knowledge construction). In addition, we must be aware of the fact that for certain students and teachers, a computer interface can be an obstacle for the use of meaningful learning environments.

3 SOME CONSIDERATIONS ON THE WAY SCIENCE AND MATHEMATICS TEACHING CAN CHANGE

In the 60's, science education reformers concentrated their efforts on curricular development. PSSC Physics, BSSC Biology, CBA Chemistry and many other projects required the work of thousands of people. We are now witnessing a similar effort in the 90's but, in spite of developing curricula, educators, pushed by politicians, develop standards about what students should know and be able to do (e.g., American Association for the Advancement of Science, 1993).

What can be the fundamental aspects of an effort to change science and mathematics education towards a direction where these areas are taught as related and integrated subjects with the computer assuming a fundamental role?

First, it should be clear that it is not possible, due to the nature of computer use and to the availability of resources, to concentrate efforts simultaneously on all segments of the curriculum. We must concentrate upon one segment only: upper secondary school, where students are more prepared to use formal and abstract tools. Today, it is almost impossible to do science, engineering and mathematics without computers tools. The same should be valid when learning these subjects. The advantages of the use of computers, especially with exploratory tools like *Modellus*, are enormous: from giving mathematical objects a new epistemological status (thus becoming more like concrete objects, and allowing students to build formal reasoning skills more easily), to giving students the sense of using state-of-the-art scientific tools to learn and perform science and mathematics, by measuring, controlling, communicating, etc.

Nickerson (1995) proposes five principles for fostering understanding when teaching with computers in science and mathematics based on research on learning: 1) start where the student is; 2) promote active processing and discovering; 3) use appropriate representations and models; 4) use simulations; and 5) provide a supportive environment. The first, an ausubelian principle (Ausubel, Novak, & Hanesian, 1978), is a principle which a teacher or a curriculum developer must always keep in mind. The second, the third and the fourth have been considered in the previous sections. The fifth is a fundamental principle when we consider what to do if we want to disseminate new ideas, new tools and new approaches. Creating supportive environments have at least three different aspects: organizational, curricular, and teacher training and support.

Organizational aspects and school resource management play a determinant role in how and how often computers are used in schools (Cuban, 1986). A possible solution to resource management of computers in schools is the creation of computational laboratories, where students can work in groups of two or three. These laboratories could be used either by both science and mathematics teachers. The necessary investments in these laboratories, that should have access to the Internet, allowing students and teachers to share and discuss their work with a larger community and use the Internet as a global resource, may have the power to invert the trend of the increasing «mathematophobia» and «technophobia» and a decreasing enrolment in science and technological studies.

Simultaneously, a strong investment on curriculum development and on teacher training and support are of the utmost importance. The «standards approach» can only be of some use if there is a field where standards can grow. Moreover, this field can only be the result of an educated population of teachers, with good teaching materials and a supportive environment. It is fundamental to consider that curriculum development is a process undertaken with teachers, not for teachers. The view that reduces teachers to mere users of materials made by experts has been widely criticised and cannot be considered a valuable approach anymore.

Finally, a last and important comment about the relationship between software and books, the most important curriculum materials. If we want teachers to use software and computers to teach, schoolbooks and software tools must constitute an integrated and complementary tool (Teodoro, 1992). It is not possible to use a piece of software extensively if it is not frequently quoted and used within the schoolbook, the tool that gives a unified view of what is done in class.

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