# NEUTRINO PAIR SYNCHROTRON EMISSION FROM MAGNETARS 

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Typical emission rates of neutrino pairs in the GeV range, produced by the process $e^{-}\left(e^{+}\right)+B \rightarrow e^{-}\left(e^{+}\right)+B+\nu \bar{\nu}$ in the magnetosphere of highly magnetized neutron stars, are calculated as well as their contribution to the extragalactic background. Neutron stars having magnetic fields as high as $10^{16} \mathrm{G}$ can be potential sources for detectors like AMANDA, ANTARES or ICE-CUBE.

## 1 Introduction

The magnetic field is one of the most important parameters affecting the evolution of pulsars, since it determines the energy loss rate, the luminosity in different spectral regions, modifies the beta equilibrium in the inner crust and hence the equation of state (Chakrabarty et al. 1997; Suh \& Mathews 2001).

Over the past years there has been increasing recognition that neutron stars are born with a considerable diversity of magnetic field strengths and of initial periods (Regimbau \& de Freitas Pacheco 2000, 2001). The existence of neutron stars with magnetic fields in excess of $10^{14} \mathrm{G}$, dubbed magnetars, was suggested by Duncan \& Thompson (1992).

Observations of Soft Gamma-Ray Repeaters and Anomalous X-Ray Pulsars (Kouveliotou et al. 1994, 1998, 1999; Cline et al. 2000; Vasisht \& Gotthelf 1997; Mereghetti 1999) give some support to the assumption that these objects are associated to highly magnetized neutron stars, if the braking mechanism is the standard magnetic dipole radiation. The crust of newborn magnetars may be under strong magnetic stresses, which induces a tectonic activity. In this case, the crust may be deformed or suffer a cracking. Field annihilation in faults at the surface heats the medium, producing a sudden increase in the local photon emission, that could explain the recurrence of Soft Gamma-Ray Repeaters (de Freitas Pacheco 1998). Moreover, the Parkes multibeam survey
(Manchester et al. 2001) detected several new radio pulsars with surface fields near $10^{14} \mathrm{G}$, deduced from spin-down rates. Simulations based on population synthesis methods suggest that the statistical properties of these high field objects are compatible with those derived for neutron stars born in the high side tail of the magnetic field distribution (Regimbau \& de Freitas Pacheco 2001).

Presently, the idea that rotating magnetized neutron stars generate huge potential differences between different parts of their surfaces is widely accepted. The associated electric fields are able to accelerate charged particles to energies up to $10^{12-14} \mathrm{eV}$. In the magnetosphere, these particles can flow only along the field lines and they escape through the "light cylinder", beyond which the field lines become open. This outflow of charged particles from the corotating magnetosphere causes a "gap" near the surface where $\mathbf{E} \cdot \mathbf{B} \neq \mathbf{0}$ (Ruderman \& Sutherland 1975, hereafter RS75), which becomes unstable when curvature photons produce an avalanche of $e^{-} e^{+}$pairs.

The acceleration and radiative losses of these pairs inside the gap have extensively been studied in the past decades, since it is believed that they may play an important role in the pulsar emission mechanism. However, besides synchrotron and curvature radiation, other mechanisms could be important in the presence of a strong magnetic field. In particular, neutrino pair production by the process $e^{-}\left(e^{+}\right)+B \rightarrow e^{-}\left(e^{+}\right)+\nu \bar{\nu}+B$ may occur (Yakovlev \& Tschaepe 1981; Merenkov 1985; Kaminker et al. 1992; Kaminker \& Yakovlev 1993), since the magnetic field guarantees the conservation of momentum. This process may be an important source of GeV neutrinos as well as an additional energy loss mechanism, which will impose limits to the maximum energy that can be attained by electrons in the accelerating region. In the present work, we estimate the typical energies of neutrinos produced by such a mechanism, the neutrino luminosity of magnetars and their contribution to the background.

## 2 The Polar Cap Electrodynamics

In the polar cap, the electric field along the magnetic field results from deviations of the local charge density from the corotational charge density of Goldreich \& Julian (1969), corrected by relativistic effects (see, for instance, Muslimov \& Tsigan 1992). The original gap model developed by RS75 was demonstrated to suffer from the binding energy problem, since the cohesive energy of ${ }^{56} \mathrm{Fe}$ may avoid the gap formation (Usov \& Melrose 1995; Xu et al. 1999). This problem was recently revisited by Gil \& Mitra (2001), who found
that a gap can be formed if the magnetic field is strong enough $\left(\geq 10^{13} \mathrm{G}\right)$ and if there is a significant distortion with respect to a dipole configuration. They have considered two different mechanisms by which seed photons could form $e^{-} e^{+}$pairs: curvature radiation and resonant inverse Compton scattering. The former requires field curvature radius $\leq 10^{5} \mathrm{~cm}$ at the surface, whereas the later requires values $\geq 5 \times 10^{4} \mathrm{~cm}$ (Gil \& Melikidze 2002). However, these gap models predict X-luminosities from heating of polar caps by flowing back particles in excess of the observed values (Helfand et al. 1980). This problem can be overcame if the flux of downward particles is only a fraction of the upward flux. This condition can be achieved in space-charge limited flow scenarios (Arons 1981; Harding \& Muslimov 1998, 2001). In these models, the accelerating field arises not from deviations of the Goldreich-Julian charge, but from a small imbalance between the corotation and the actual charge. The field grows with the height above the polar cap until the $e^{-} e^{+}$pairs screen the field at a critical distance $z_{*}$, above which it decays exponentially (Harding \& Muslimov 2001).

In our calculations, we adopted the approximate formulae derived by Harding \& Muslimov (2001) for the accelerating electric field and for the critical screening distance $z_{*}$. In this case, the energy of an electron at a distance z above the surface is

$$
\begin{equation*}
E \approx 0.02 \frac{B_{14}}{P^{3 / 2}} z^{2} \mathrm{MeV} \tag{1}
\end{equation*}
$$

where $B_{14}$ is the surface magnetic field in units of $10^{14} \mathrm{G}, \mathrm{P}$ is the rotation period in seconds, z is the distance above the surface in cm , and we have adopted a canonical radius of 10 km for the neutron star. The critical distance above the surface at which the fields decays is approximately given by

$$
\begin{equation*}
z_{*}=2.5 \times 10^{4} \frac{P^{11 / 14}}{B_{14}^{4 / 7}} R_{10} \mathrm{~cm} \tag{2}
\end{equation*}
$$

Both Eqs. are valid if $z_{*} \ll r_{p}$, where $r_{p}$ is the radius of the polar cap.
If inside the accelerating region the charge density is given approximately by the Goldreich-Julian corotating charge, corrected by relativistic effects ( $\zeta$ and $\alpha$ terms, see below), then the number of relativistic electrons with energies between $E$ and $E+d E$ is

$$
\begin{equation*}
\frac{d N(E)}{d E} d E=\frac{\zeta}{\alpha} \frac{\left(\boldsymbol{\Omega} \cdot \mathbf{B}_{\mathbf{s}}\right)}{2 \pi e c} \pi r_{p}^{2} \frac{d z}{d E} d E \tag{3}
\end{equation*}
$$

where $r_{p}=\left(\frac{2}{3}\right)^{3 / 4} R\left(\frac{\Omega R}{c}\right)^{1 / 2}$ is the radius of the polar cap considering a dipole structure, $\zeta=1-\left(\frac{r_{g}}{R}\right)\left(\frac{I}{M R^{2}}\right)$ and $\alpha=\sqrt{1-\frac{r_{g}}{R}}$. In these last expressions, $r_{g}$
is the gravitational radius and I, M, R are respectively the moment of inertia, mass and radius of the neutron star.

From these equations one obtains

$$
\begin{equation*}
\frac{d N(E)}{d E}=5.7 \times 10^{21} B_{14}^{1 / 2} R_{10}^{3} P^{-5 / 4} E^{-1 / 2} \quad \mathrm{MeV}^{-1} \tag{4}
\end{equation*}
$$

where the energy is in MeV and the numerical value corresponds to a "canonical" neutron star of mass equal to $1.4 M_{\odot}$ and radius equal to 10 km . This energy spectrum is valid essentially for $z<z_{*}$.

## 3 The Neutrino Production

If $W_{\nu \bar{\nu}}(E, \hbar \omega)$ is the probability per unit of time and per energy interval for a relativistic electron of energy $E$ to emit a neutrino pair of (total) energy $\hbar \omega$, then the neutrino production rate $Q_{\nu \bar{\nu}}(\hbar \omega)$ per energy interval in the non-quantized limit is

$$
\begin{equation*}
Q_{\nu \bar{\nu}}(\hbar \omega)=\int W_{\nu \bar{\nu}}(E, \hbar \omega) \frac{d N(E)}{d E} d E \tag{5}
\end{equation*}
$$

The mean energy of the emitted neutrino pair is

$$
\begin{equation*}
\hbar \omega \approx E \frac{\chi}{(1+\chi)} \tag{6}
\end{equation*}
$$

where $\chi=\hbar \omega_{L} \frac{E^{2}}{\left(m c^{2}\right)^{3}}$ and $\omega_{L}=\frac{e B}{m c}$ is the electron Larmor's frequency. For magnetic fields $B \geq 10^{14} \mathrm{G}$, the condition $\chi \gg 1$ is satisfied if the electron energy is higher than 5 MeV . In this case, the energy of the emitted neutrino pair is comparable to the electron energy and the neutrino probability emission per unit of time and energy can be written as (Baier et al. 1998)

$$
\begin{equation*}
W_{\nu \bar{\nu}}(E, \hbar \omega)=\frac{G_{F}^{2}\left(m c^{2}\right)^{6}}{216 \pi^{3}(\hbar c)^{7}} \frac{c \chi^{2}}{E} g^{2}(\ln \chi-1.96) \delta(E-\hbar \omega) \tag{7}
\end{equation*}
$$

where $G_{F}$ is the Fermi constant and $g^{2}=g_{A}^{2}+g_{V}^{2}$. The effective axial and vector coupling constants are defined in terms of the Weinberg angle $\theta_{W}$ as: $g_{A}=\frac{1}{2}$ and $g_{V}=\frac{1}{2}+2 \sin ^{2} \theta_{W}$ for the electron neutrino, while $g_{A}=-\frac{1}{2}$ and $g_{V}=-\frac{1}{2}+2 \sin ^{2} \theta_{W}$ for other neutrino types. This approach is constrained by the condition $\chi \ll\left(\frac{M_{W}}{m}\right)^{2}$, where $M_{W}$ is the mass of the gauge boson $W$. Such a condition imposes an upper limit for the electron energy (or equivalently to the neutrino pair energy). In order that Eq. (7) be valid we should have $E \ll 60 B_{14}^{-1 / 2} \mathrm{GeV}$.

Substituting Eqs. (4) and (7) into (5) and computing numerically the constants, one obtains for the neutrino production rate
$Q_{\nu \bar{\nu}}(\hbar \omega)=3.9 \times 10^{17} B_{14}^{5 / 2} R_{10}^{3} P^{-5 / 4}(\hbar \omega)^{5 / 2}\left[\lg \left(8.68 B_{14}(\hbar \omega)^{2}\right)-1.96\right] s^{-1} \mathrm{MeV}^{-1}$
where the neutrino pair energy $\hbar \omega$ is given in MeV . From this equation, the expected neutrino pair luminosity in the range $0.01-20 \mathrm{GeV}$ is

$$
\begin{equation*}
L_{\nu \bar{\nu}} \approx 1.3 \times 10^{32} B_{14}^{5 / 2} R_{10}^{3} P^{-5 / 4} \mathrm{erg} \mathrm{~s}^{-1} \tag{9}
\end{equation*}
$$

## 4 Contribution to the Background

If magnetars are simply neutron stars born in the high side of the magnetic field distribution, then objects with fields $B \geq 10^{14} \mathrm{G}$ may represent about $19 \%$ of the total population. These neutron stars evolve quite rapidly to long periods and, as a consequence, their radio-luminosity decreases very fast, difficulting their detection. Moreover, as the period increases, the emission beam narrows, reducing even more the detection probability. This seems to be the case of the radio-pulsar J2144-3933 (Young, Manchester \& Johnston 1999), which has a period of 8.5 s and a very narrow pulse profile. The statistical study performed by Regimbau \& de Freitas Pacheco (2001) suggests that the predictions from population synthesis, concerning the number of high field neutron stars in the Galaxy, are not in contradiction with present data.

If the galactic population of magnetars estimated from simulations is typical for other galaxies, it is interesting to estimate the expected neutrino flux resulting from the contribution of a fair volume of the universe.

Let $F_{\nu_{0}}$ be the neutrino pair flux with energy $\hbar \omega_{0}$ in the observer's frame, integrated over all sources, namely,

$$
\begin{equation*}
F_{\nu_{0}}=\int_{0}^{z_{m}} f_{\nu_{0}} d R(z) \tag{10}
\end{equation*}
$$

where the neutrino flux of a single source located between redshifts $z$ and $z+d z$ is

$$
\begin{equation*}
f_{\nu_{0}}=\frac{1}{4 \pi d_{L}^{2}} \frac{d Q_{\nu}}{d \hbar \omega}(1+z) \tag{11}
\end{equation*}
$$

$d_{L}=(1+z) r$ is the distance luminosity, $\frac{d Q_{\nu}}{d \hbar \omega}$ is the spectral distribution of the total number of neutrinos emitted by the source and $\hbar \omega=(1+z) \hbar \omega_{0}$ is the neutrino pair energy in source frame. The event rate as observed in our
frame is

$$
\begin{equation*}
d R(z)=\lambda_{p} \frac{R_{c}(z)}{(1+z)} \frac{d V}{d z} d z \tag{12}
\end{equation*}
$$

where $R_{c}(z)$ is the cosmic star formation rate corrected for time dilation, $\lambda_{p}$ $=0.0048 M_{\odot}^{-1}$ is the mass fraction of newborn neutron stars and the element of comoving volume is

$$
\begin{equation*}
d V=4 \pi r^{2} \frac{c}{H_{0}} \frac{d z}{L\left(\Omega_{i}, z\right)} \tag{13}
\end{equation*}
$$

where $H_{0}$ is the Hubble parameter, $L\left(\Omega_{i}, z\right)=\sqrt{\Omega_{v}+\Omega_{m}(1+z)^{3}}, \Omega_{v}$ and $\Omega_{m}$ are respectively the density parameters of vacuum and matter (baryonic and non-baryonic).

The spectral distribution in Eq. (10) can be estimated through different steps: the magnetar period evolves as $P=P_{0}\left(1+t / \tau_{*}\right)^{1 / 2}$, where $P_{0}$ is the initial period and the characteristic magnetic braking timescale is defined as $\tau_{*}=\frac{3}{4 \pi^{2}} \frac{I c^{3} P_{0}^{2}}{B^{2} R^{6}}$. Integrating with respect to time Eq.(8) and taking into account that the neutrino pair energy scales as $P^{-3 / 2}$ (see Eq. 1), one obtains

$$
\begin{equation*}
\frac{d Q_{\nu \bar{\nu}}}{d \hbar \omega}=3.1 \times 10^{29} P_{0}^{5} R_{10}^{-9 / 2} \tau_{*}^{-1 / 4}(\hbar \omega)^{5 / 2}\left[\lg \left(a(\hbar \omega)^{2}\right)-1.96\right] \mathrm{MeV}^{-1} \tag{14}
\end{equation*}
$$

where $a=845 \frac{P_{0}}{\tau_{*}^{1 / 2} R_{10}^{3}}$ and we have substituted the magnetic field by the relation defining the magnetic braking timescale. In the equation above $\tau_{*}$ is given in years. In the following step, we computed the mean values of $P_{0}$ and $\tau_{*}$ in Eq. (13), according to their power dependence. The updated distribution functions derived originally by Regimbau \& de Freitas Pacheco $(2000,2001)$ correspond to a Gaussian distribution of initial periods of mean equal to 0.24 s and dispersion equal to 0.08 s , while the braking timescale (in years) can be represented by a log-normal distribution with mean equal to 9.5 and dispersion $\sigma_{l g \tau_{*}}=3.5$. Performing the relevant averages one obtains:

$$
\begin{equation*}
\frac{d Q_{\nu \bar{\nu}}}{d \hbar \omega}=2.5 \times 10^{36} R_{10}^{-9 / 2}(\hbar \omega)^{5 / 2}\left[\lg \left(4.4 \times 10^{6} \hbar \omega^{2}\right)-1.96\right] G e V^{-1} \tag{15}
\end{equation*}
$$

where now the neutrino pair energy is in GeV .
In the present calculations, we assumed a flat universe characterized by $\Omega_{v}=0.7$ and $\Omega_{m}=0.3$, in agreement with recent angular power spectra data of CMB by WMAP, BOOMERANG,MAXIMA and distances from SNIa supernovae. The Hubble parameter was taken equal to $H_{0}=65 \mathrm{~km} / \mathrm{s} / \mathrm{Mpc}$. The cosmic star formation rate up to redshifts $z \sim 5$ was taken from models
by Porciani \& Madau (2001). Then, from Eqs. (9)-(13), one obtains for the integrated flux

$$
\begin{equation*}
F_{\nu_{0}}=6.2 \times 10^{-20}\left(\hbar \omega_{0}\right)^{5 / 2}\left[\lg \left(4 \times 10^{7} \hbar \omega_{0}^{2}\right)-1.96\right] \mathrm{cm}^{-2} . \mathrm{s}^{-1} . \mathrm{sr}^{-1} . \mathrm{GeV}^{-1} \tag{16}
\end{equation*}
$$

Integrating in energy, one obtains an expected background flux $F_{\nu_{0}}(1-$ $20 \mathrm{GeV}) \approx 1.3 \times 10^{-14} \mathrm{~cm}^{-2} \mathrm{sr}^{-1} \mathrm{~s}^{-1}$.

## 5 Concluding Remarks

The integrated flux calculated above is many orders of magnitude less than the present sensitivity of $k m^{2}$ neutrino detectors like ANTARES, AMANDA or ICE-CUBE. However, we cannot discard the possibility that individual galactic magnetars could be detected.

The number of muons produced by muon-neutrinos crossing the detector is

$$
\begin{equation*}
N_{\mu}=A_{e f f} T \int P_{\mu}\left(E_{\mu}\right) J\left(E_{\mu}\right) d E_{\mu} \tag{17}
\end{equation*}
$$

where $A_{\text {eff }}$ is the effective area of the detector, $T$ is the integration time, $J\left(E_{\mu}\right)$ is the muon-neutrino flux of energy $E_{\mu}$ and $P_{\mu}\left(E_{\mu}\right)$ is the probability for a muon to be produced and detected inside the detector. The expected number background events in the range $1-20 \mathrm{GeV}$ is

$$
\begin{equation*}
N_{\mu, b a c}=2.7 \times 10^{4}\left(A_{e f f} / 1 \mathrm{~km}^{2}\right)(T / 1 y r) \tag{18}
\end{equation*}
$$

If the detector has a resolution $\theta \sim 2^{\circ}$, then the background is reduced by a fraction $\Delta \Omega_{*} \approx \frac{\pi \theta}{4} \sim 10^{-3}$, corresponding to the solid angle aperture.

Using Eq. (8), the expected number of muons produced by a source at a distance $D$ is

$$
\begin{equation*}
N_{\mu, s}=0.002 \frac{B_{14}^{5 / 2} R_{10}^{3} P^{-5 / 4}}{D_{k p c}^{2}}\left(A_{e f f} / 1 \mathrm{~km}^{2}\right)(T / 1 y r) \tag{19}
\end{equation*}
$$

If we require a comparable number of events (background and signal), then a typical magnetar at a distance of 1 kpc and with a period $P \sim 10 \mathrm{~s}$ must have a magnetic field of $10^{16} \mathrm{G}$ in order to be detected. A more detailed discussion concerning the detection probability of these objects will be presented in a forthcoming paper.

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