MAGNETIZED NEUTRON STARS : AN OVERVIEW

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In the presence of strong magnetic field reported to have been observed on the surface of some neutron stars and on what are called magnetars, a host of physical phenomena, from the birth of a neutron star to the free streaming neutrino cooling phase, will be modified. In this review we discuss the effect of a magnetic field on the equation of state of high density nuclear matter by including the anomalous magnetic moment of the nucleons into consideration. We would then go over to discuss the neutrino interaction process in strong as well as in weak magnetic fields. Neutrino processes are important in studying the propagation of neutrinos and energy loss. Their study is a prerequisite for the understanding of the actual dynamics of supernova explosion and on the stabilization of radial pulsation modes through the effect on bulk viscosity. The anisotropy introduced in the neutrino emission and through the modification of the shape of the neutrino sphere may explain the observed pulsar kicks.

1 Introduction

Large magnetic fields $\sim 10^{14}$ Gauss have been reported to exist on the surface of pulsars. Recent observations [1] of γ-ray repeaters and spinning X-ray pulsars (magnetars) hints to the existence of fields in excess of 10^{14} Gauss. It then follows from the Scalar Virial theorem that the fields in the core could even reach a value as large as 10^{18} Gauss. There is, however, an upper limit on the magnetic field discussed by Chandershekhar beyond which the magnetic energy exceeds the gravitational energy and the star is no longer stable. In the presence of such magnetic fields, neutron star properties in all its phases, from its evolution from protoneutron star to cold neutrino emitting phase, would be modified. This arises because in the presence of a magnetic field, the motion of the charged particles is quantized in a plane perpendicular to the magnetic field and the charged particles occupy discrete Landau levels. This has the effect of not only modifying the energy eigenvalues but also the particle wave functions. The quantum state of a particle in a magnetic field is specified by its momentum components p_z, p_y, spin s and Landau quantum number ν. The anomalous magnetic moments of protons and neutrons further modify the energy eigenvalues. The time scales involved during all phases of neutron stars, from their birth to neutrino burst through thermal neutrino emission from the trapped neutrino sphere to the freely streaming neutrino cooling phase, are large compared to the interaction time scales of strong, electromagnetic and weak interactions and the matter is in β equilibrium. The magnetic field would modify the equilibrium and all neutrino interaction processes including scattering, absorption and production.

The strategy then is to solve first the Dirac equation for all particles in the presence of a magnetic field including their anomalous magnetic moments, to obtain the energy eigenvalues, to construct the Grand Partition Function taking into account strong interactions in some model dependent way and to obtain the equation of state (EoS) of nuclear matter. The next step is to calculate the scattering cross-sections for all neutrino processes by using the exact wave functions and by modifying the phase space integrals for arbitrary values of degeneracy, density, temperature and magnetic field. The various phenomenon that we will address are :

1. Composition of matter in neutron stars, proton fraction, effective nucleon mass, and others.
2. Cooling of neutron stars in the free streaming regime.
3. Neutrino transport in neutron and collapsing stars which is an essential prerequisite for an understanding of supernova explosion, structure of protoneutron star and observed pulsar kicks.
4. Damping of radial oscillations and secular instability through the calculation of bulk viscosity.

2 Nuclear Matter Composition

For determining the composition of dense, hot, magnetized matter, we employ a relativistic mean field theoretical approach in which the baryons (protons and neutrons) interact via the exchange of $\sigma-\omega-\rho$ mesons in a constant uniform magnetic field. Following Ref.[2], in a uniform magnetic field B along the z axis corresponding to the choice of the gauge field $A^\mu = (0, 0, xB, 0)$, the relativistic mean field Lagrangian can be written as

$$\mathcal{L} = \sum \bar{\psi_B}\left[i\gamma_\mu D^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu - g_{\rho B}\,\tau_{3B}\,\gamma_\mu\,\rho^\mu + \kappa_B\sigma_{\mu\nu}F^{\mu\nu}\right]\psi_B$$
$$+\frac{1}{2}\left[(\partial^\mu\sigma)^2 - (m_\sigma^2\sigma^2)\right] - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu}$$
$$+\frac{1}{2}m_\rho^2\rho_\mu\rho^\mu - U(\sigma) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_l \bar{\psi}_l(i\gamma_\mu D^\mu - m_l)\psi_l \quad (1)$$

in the usual notation, with $D^\mu = \partial^\mu + ieA^\mu$ and κ_B as the anomalous magnetic moment given by

$$\kappa_p = \frac{e}{2m_p}\left[\frac{g_p}{2} - 1\right] \quad ; \kappa_n = \frac{e}{2m_n}\frac{g_n}{2} \tag{2}$$

where $g_p = 5.58$ and $g_n = -3.82$ are the Lande's g-factor for protons and neutrons, respectively. Replacing the meson fields in the relativistic mean field approximation by their density dependent average values $< \sigma_0 >$,$< \omega_0 >$ and $< \rho_0 >$, the equations of motion satisfied by the nucleons in the magnetic field become

$$[-i\alpha_x \frac{\partial}{\partial_x} + \alpha_y(p_y - eBx) + \alpha_z p_z + \beta(m_i - g_\sigma < \sigma_0 >) - i\kappa_i \alpha_x \alpha_y B] f_{i,s}$$
$$= (E^i - U_0^i) f_{i,s}(x) \tag{3}$$

where

$$U_0^{p,n} = g_{\omega N} < \omega_0 > \pm \frac{g_{\rho N}}{2} < \rho_0 > \ . \tag{4}$$

These equations are first solved for the case when the momentum along the magnetic field direction is zero and then boosting along that direction till the momentum becomes p_z. For the neutrons and protons we thus get

$$\mathcal{E}_s^n = \sqrt{m_n^{*2} + \overrightarrow{p}^2 + \kappa_n^2 B^2 + 2\kappa_n B s \sqrt{p_x^2 + p_y^2 + m_n^{*2}}}$$
$$= E_s^n - U_0^n \tag{5}$$

and

$$\mathcal{E}_{\nu,s}^p = \sqrt{m_p^{*2} + p_z^2 + eB(2\nu + 1 + s) + \kappa_p^2 B^2 + 2\kappa_p B s \sqrt{m_p^{*2} + eB(2\nu + 1 + s)}}$$
$$= E_{\nu,s}^p - U_0^p \tag{6}$$

where $2\nu = (2n + 1 + s)$, with ν and n being integer known as Landau and principal quantum numbers, respectively. $s = \pm 1$ indicates whether the spin is along or opposite to the direction of the magnetic field and $m_B^* = m_B - g_\sigma \sigma$ is the effective baryon mass. The energy spectrum for the electrons is given by

$$E_{\nu,s}^e = \sqrt{m_e^2 + p_z^2 + (2\nu + 1 + s)eB}\,. \tag{7}$$

The mean field values $< \sigma_0 >$, $< \omega_0 >$ and $< \rho_0 >$ are determined by minimizing the energy at fixed baryon density $n_B = n_p + n_n$ or by maximizing the pressure at fixed baryon chemical potential μ_B. We thus get

$$< \sigma_0 >= \frac{g_\sigma}{m_\sigma^2}(n_p^s + n_n^s) \; ; < \omega_0 >= \frac{g_\omega}{m_\omega^2}(n_n + n_p) \; ; < \rho_0 >= \frac{1}{2}\frac{g_\rho}{m_\rho^2}(n_p - n_n) \tag{8}$$

where n_i and n_i^s (i=n,p) are, respectively, the number and scalar number densities for protons and neutrons. In the presence of the magnetic field, the phase space volume is replaced by

$$\int \frac{d^3p}{(2\pi)^3} \longrightarrow \frac{eB}{(2\pi)^2} \sum_{\nu=0}^{\nu_{max}} (2-\delta_{\nu,0}) \int dp_z\,. \tag{9}$$

The expressions for the number and scalar number densities for neutrons and protons are, respectively, given by

$$n_n = \frac{1}{(2\pi)^3} \sum_s \int \frac{d^3p}{1+e^{\beta(\mathcal{E}_s^n - \mu_n^\star)}} \tag{10}$$

$$n_n^s = \frac{m_n^\star}{(2\pi)^3} \sum_s \int \frac{d^3p}{\mathcal{E}_s^n (1+e^{\beta(\mathcal{E}_s^n - \mu_n^\star)})} \tag{11}$$

$$n_p = \frac{eB}{2\pi^2} \sum_s \sum_\nu \int_0^\infty \frac{dp_z}{1+e^{\beta(\mathcal{E}_{\nu,s}^p - \mu_p^\star)}} \tag{12}$$

$$n_p^s = \frac{eB}{2\pi^2} m_p^\star \sum_s \sum_\nu \int \frac{dp_z}{\mathcal{E}_{\nu,s}^p (1+e^{\beta(\mathcal{E}_{\nu,s}^p - \mu_p^\star)})} \tag{13}$$

and the net electron and neutrino number densities are

$$\bar{n}_e = \frac{eB}{2\pi^2} \sum (2-\delta_{\nu,0}) \int dp_z \left[\frac{1}{(1+e^(\beta(\mathcal{E}_\nu^e - \mu_e)))} - \mu_e \leftrightarrow (-\mu_e) \right] \tag{14}$$

$$\bar{n}_{\nu_e} = \frac{1}{(2\pi)^3} \int d^3p \left[\frac{1}{(1+e^{\beta(\mathcal{E}^{\nu_e} - \mu_{\nu_e})})} - \mu_{\nu_e} \leftrightarrow (-\mu_{\nu_e}) \right]. \tag{15}$$

The thermodynamic potentials for the neutron, proton, electron and neutrino are

$$\Omega_n = -\frac{1}{\beta(2\pi)^2} \sum_s \int d^3p \ln[1+e^{-\beta(\mathcal{E}_s^n - \mu_n^\star)}] \tag{16}$$

$$\Omega_p = -\frac{1}{\beta(2\pi)^2} \sum_s \sum_\nu \int dp_z \ln[1+e^{-\beta(\mathcal{E}_{\nu,s}^p - \mu_p^\star)}] \tag{17}$$

$$\Omega_e = -\frac{1}{\beta(2\pi)^2} \sum_s \sum_\nu \int dp_z \ln[1+e^{-\beta(\mathcal{E}_{\nu,s}^e - \mu_e)}] \tag{18}$$

$$\Omega_{\nu_e} = -\frac{1}{\beta(2\pi)^2} \int d^3p \ln[1+e^{-\beta(\mathcal{E}^{\nu_e} - \mu_{\nu_e})}] \tag{19}$$

and the thermodynamic potential for the system is

$$\Omega = -\frac{1}{2}m_\omega^2\omega_0^2 - \frac{1}{2}m_\rho^2\rho_0^2 + \frac{1}{2}m_\sigma^2\sigma^2 + U(\sigma) + \frac{B^2}{8\pi} + \sum_i \Omega_i \tag{20}$$

where $i = n, p, e, \nu_e$. The various chemical potentials are determined by the conditions of charge neutrality and chemical equilibrium. In later stages of core collapse and during the early stages of protoneutron star, neutrinos are trapped and the chemical potentials satisfy the relation $\mu_n = \mu_p + \mu_e - \mu_{\nu_e}$. These situations are characterized by a trapped lepton fraction $Y_L = Y_e + Y_{\nu_e}$ where $Y_e = (n_e - n_{e_+})/n_B$ is the net electron fraction and $Y_{\nu_e} = (n_{\nu_e} - n_{\bar{\nu}_e})/n_B$ is the net neutrino fraction.

The evolution of a protoneutron star begins from a neutrino-trapped situation with $Y_L \sim 0.4$ to one in which the net neutrino fraction vanishes and chemical equilibrium without neutrinos is established. In this case, the chemical equilibrium is modified by setting $\mu_{\nu_e} = 0$. In all cases, the condition of charge neutrality requires

$$n_p = \bar{n}_e\,. \tag{21}$$

In the nucleon sector, the constants $g_{\sigma N}$, $g_{\omega N}$, $g_{\rho N}$, b and c are determined by fitting the nuclear matter equilibrium density $n_0 = 0.16fm^{-3}$, the binding energy per nucleon ($\sim 16MeV$), the symmetry energy ($\sim 30 - 35MeV$), the compression modulus ($200MeV \leq k_0 \leq 300MeV$) and the nucleon Dirac effective mass $M^* = (0.6 - 0.7) \times 939MeV$ at n_0. Numerical values of the coupling constants so chosen are :

$$\frac{g_{\sigma N}}{m_\sigma} = 3.434fm^{-1}\,;\ \frac{g_{\omega N}}{m_\omega} = 2.694fm^{-1}\,;\ \frac{g_{\rho N}}{m_\rho} = 2.1fm^{-1}\,;$$

$$b = 0.00295\,;\ c = -0.00107\,.$$

3 Weak Rates and Neutrino Emissivity

The dominant mode of energy loss in neutron stars is through neutrino emission. The important neutrino emission processes leading to neutron star cooling are the so called URCA processes

$$n \longrightarrow p + e^- + \bar{\nu}_e \qquad ; \qquad p + e^- + \longrightarrow n + \nu_e\,. \tag{22}$$

At low temperatures, for degenerate nuclear matter, the direct URCA process can take place only near the fermi energies of participating particles and simultaneous conservation of energy and momentum require the inequality

$$p_F(e) + p_F(p) \geq p_F(n)$$

to be satisfied in the absence of the magnetic field. This leads to the well known threshold[3] for the proton fraction $Y_p = \frac{n_p}{n_B} \geq 11\%$ thus leading to strong suppression in nuclear matter. This condition is satisfied for $n_B \geq 1.5 n_0$ (where $n_0 = 0.16 fm^{-3}$, is the nuclear saturation density), in a relativistic mean field model of interacting n-p-e gas for $B = 0$. The standard model of the long term cooling is the modified URCA process:

$$(n,p) + n \longrightarrow (n,p) + p + e^- + \bar{\nu}_e \;\; ; \;\; (n,p) + p + e^- \longrightarrow (n,p) + n + \nu_e \quad (23)$$

which differ from the direct URCA reactions by the presence in the initial and final states of a bystander particle whose sole purpose is to make possible conservation of momentum for particles close to the Fermi surfaces. For weak magnetic fields, the matrix element for the process remains essentially unaffected and the modification comes mainly from the phase space factor. Treating the nucleons non-relativistically and electrons ultra-relativistically, the matrix element squared and summed over spins is given by

$$\sum |M|^2 = 8G_F^2 \cos^2\theta_c (4m_n^* m_p^*) E_e E_\nu [(1 + 3g_A^2) + (1 - g_A^2)\cos\theta_c] \quad (24)$$

where $g_A = 1.261$ is the axial-vector coupling constant. The emissivity expression is given by

$$\dot{\mathcal{E}}_\nu = \left[\prod_i \int \frac{1}{(2\pi)^3 2E_i} d^3 p_i \right] E_\nu \sum |M|^2 (2\pi)^4 \delta^4 (P_f - P_i) S \quad (25)$$

where the phase space integrals are to be evaluate over all particle states. The statistical distribution function is $S = f_n(1 - f_p)(1 - f_e)$, where the $f_i's$ are the Fermi-Dirac distributions. We can now evaluate the emissivity in the limit of extreme degeneracy, a situation appropriate in neutron star cores by using the standard techniques to perform the phase space integrals [2]

$$\dot{\mathcal{E}} = \frac{457\pi}{40320} G_F^2 \cos^2\theta_c (1 + 3g_A^2) m_n^* m_p^* eBT^6 \sum_{\nu=0}^{\nu_{max}} (2 - \delta_{\nu,0}) \frac{1}{\sqrt{\mu_e^2 - m_e^2 - 2\nu eB}} \quad (26)$$

where $\nu_{\max} = Int(\mu_e^2 - m_e^2/2eB)$. In the limit of a vanishing magnetic field, the sum can be replaced by an integral and we recover the usual expression, i.e. the one for the case $B = 0$

$$\dot{\mathcal{E}}_\nu(B = 0) = \frac{457\pi}{20160} G_F^2 \cos^2\theta_c (1 + 3g_A^2) m_n^* m_p^* \mu_e T^6 \, . \quad (27)$$

Modified URCA processes are considered to be the dominant ones for neutron star cooling. Similarly the energy loss expression with the appropriate electron phase space, for the modified URCA process $\dot{\mathcal{E}}_{URCA}$ is calculated to be

$$\begin{aligned}\dot{\mathcal{E}}_{URCA} &= \frac{11513}{60480}\frac{G_F^2\cos^2\theta_c}{2\pi}g_A^2 m_n^{*3}m_p^*\left(\frac{f}{m_\pi}\right)^4\alpha_{URCA}T^8\\ &\times\sum_{\nu=0}^{\nu_{max}}(2-\delta_{\nu,0})\left(\frac{1}{\sqrt{\mu_e^2-m_e^2-2\nu eB}}\right)\end{aligned} \tag{28}$$

where f is the $\pi - N$ coupling constant ($f^2 \simeq 1$) and α_{URCA} has been estimated to be equal to 1.54. The above equation in the $B \to 0$ limit goes over to the standard result[4]

$$\dot{\mathcal{E}}_{URCA}(B=0) = \frac{11513}{30240}\frac{G_F^2\cos^2\theta_c}{2\pi}g_A^2 m_n^{*3}m_p\left(\frac{f}{m\pi}\right)^4\alpha_{URCA}\ \mu_e T^8\,. \tag{29}$$

In the case of super strong magnetic fields such that $B > B_c^e$ (with $B_c^e = 4.41\times10^{13}$ Gauss) all electrons occupy the Landau ground state at $T=0$ which corresponds to a $\nu=0$ state with electron spins pointing in the direction opposite to the magnetic field. Charge neutrality now forces the degenerate non-relativistic protons also to occupy the lowest Landau level with proton spins pointing in the direction of the field. In this situation we can no longer consider the matrix elements to be unchanged and they should be evaluated using the exact solutions of the Dirac equation. Further, because nucleons have anomalous magnetic moment, matrix elements need to be evaluated for specific spin states separately. The electron in the $\nu=0$ state has energy $E_e = \sqrt{m_e^2+p_{ez}^2}$ and the positive energy spinor in the $\nu=0$ state is given by

$$U_{e,-1} = \frac{1}{\sqrt{E_e+m_e}}\begin{pmatrix}0\\ E_e+m_e\\ 0\\ -p_{ez}\end{pmatrix}. \tag{30}$$

Protons are treated non-relativistically and the energy in the $\nu=0$ state is $E_p \simeq \tilde{m}_p + \frac{P_z^2}{2\tilde{m}_p} + U_0^p$ and the non-relativistic spin up operator $U_{p,+1}$ given by

$$U_{p,+1} = \frac{1}{\sqrt{2\tilde{m}_p}}\begin{pmatrix}\chi_+\\ 0\end{pmatrix} \tag{31}$$

where $\tilde{m}_p = m_p^* - \kappa_p B$. For neutrons we have

$$\Psi_{n,s}(r) = \frac{1}{\sqrt{L_xL_yL_z}}e^{-ip_{n,s}\cdot r}U_{n,s}(E_{n,s}) \tag{32}$$

$$U_{n,s} = \frac{1}{\sqrt{2m_n^*}} \begin{pmatrix} \chi_s \\ 0 \end{pmatrix} , \tag{33}$$

and $E_{n,s} \simeq m_n^* + \overrightarrow{p_n}^2 - \kappa_n B s + U_0^n$ in the non-relativistic limit. The neutrino wave function is given by

$$\Psi_\nu(r) = \frac{1}{\sqrt{L_x L_y L_z}} e^{-ip_\nu \cdot r} U_{\nu,s}(E_\nu) . \tag{34}$$

Here $U_{\nu,s}$ is the usual free particle spinor, χ_s is the spin spinor and the wave function has been normalized in a volume $V = L_x L_y L_z$. Using the explicit form for the spinors given above we can now calculate the matrix element squared and summed over neutrino states to get

$$\sum |M_+^\dagger M_+| = |K_+|^2 4(4m_n^* \tilde{m_p})(1+g_A)^2 (E_e + p_{ez})(E_{\nu + p_{\nu z}}) \tag{35}$$

and

$$\sum |M_-^\dagger M_-| = |K_-|^2 16(4m_n^* \tilde{m_p}) g_A^2 (E_e + p_{ez})(E_\nu - p_{\nu z}) . \tag{36}$$

The neutrino emissivity is calculated by using the standard techniques for degenerate matter and we obtain

$$\begin{aligned} \dot{\mathcal{E}} = \; & \frac{457\pi}{40320} G_F^2 \cos^2\theta_c eB \frac{\tilde{m_p} m_n^*}{p_F(e)} T^6 \left[\frac{(1+g_A)^2}{2} \theta\left(p_F^2(n,+)\right) exp\left(\frac{-p_F^2(n,+)}{2eB}\right) \right. \\ & + \theta\left(p_F^2(n,+) - 4p_F^2(e)\right) exp\left(\frac{-(p_F^2(n,+) - 4p_F^2(e))}{2eB}\right) \\ & \left. + 2g_A^2 p_F(n,+) \longrightarrow p_F(n,-) \right] , \end{aligned} \tag{37}$$

where $p_f(n,+)$ and $p_F(n,-)$ are the neutron Fermi momenta for spins along and opposite to the magnetic field direction respectively and are given by

$$\frac{p_F^2(n,\pm)}{2m_n^*} = \mu_n - m_n^* - U_0^n \pm \frac{\kappa_n B}{2m_n^*} . \tag{38}$$

We thus see as advertised that in the presence of quantizing magnetic field the inequality $p_F(e) + p_F(p) \geq p_f(n)$ is no longer required to be satisfied for the process to proceed, regardless of the value of the proton fraction and we get a non-zero energy loss rate.

4 Bulk Viscosity of Magnetized Neutron Star Matter

The source of bulk viscosity of neutron star matter is the deviation from β equilibrium, and the ensuing non-equilibrium reactions, implied by the compression and rarefaction of the matter in the pulsating neutron star. These important reactions are the URCA and the modified URCA processes. Since the source of bulk viscosity is the deviation from β equilibrium, these reactions are driven by the non-zero values of $\Delta\mu = \mu_n - \mu_p - \mu_e$. We calculate the bulk viscosity of neutron star matter in the presence of a magnetic field for direct URCA processes in the linear regime, i.e. $\Delta\mu << kT$. The bulk viscosity ζ is defined by[5]

$$\zeta = 2\left(\frac{dW}{dt}\right)_{av}\frac{1}{v_0}\left(\frac{v_0}{\Delta v}\right)^2\frac{1}{\omega^2}\,. \tag{39}$$

Here v_0 is the specific volume of the star in equilibrium configuration, Δv is the amplitude of the periodic perturbation with period $\tau = \frac{2\pi}{\omega}$ and $v(t) = v_0 + \Delta v\sin(2\pi t/\tau)$. The quantity $(\frac{dW}{dt})_{av}$ is the mean dissipation rate of energy per unit mass and is given by the equation

$$\left(\frac{dW}{dt}\right)_{av} = -\frac{1}{\tau}\int P(t)\frac{dv}{dt}dt\,. \tag{40}$$

The pressure $P(t)$ can be expressed near its equilibrium value P_0, as

$$P(t)=P_0+\left(\frac{\partial P}{\partial v}\right)_0\Delta v+\left(\frac{\partial P}{\partial n_p}\right)_0\Delta n_p+\left(\frac{\partial P}{\partial n_e}\right)_0\Delta n_e+\left(\frac{\partial P}{\partial n_n}\right)_0\Delta n_n\,. \tag{41}$$

The change in the number of neutrons, protons and electrons per unit mass over a time interval $(0,t)$ due to URCA reactions (23) is given by

$$-\Delta n_n = \Delta n_p = \Delta n_e = \int_0^t \frac{dn_p}{dt}dt\,. \tag{42}$$

The net rate of production of protons, $\frac{dn_p}{dt}$, is given by the difference between the rates Γ_1 and Γ_2 of the URCA reactions. At equilibrium, the two rates are obviously equal and the chemical potentials satisfy the equality $\Delta\mu = \mu_n - \mu_p - \mu_e = 0$. A small volume perturbation brings about a small change in the chemical potentials and the above inequality is no longer satisfied; now $\Delta\mu$ is not zero and consequently the reaction rates are no longer equal. The net rate of production of protons will thus depend upon the value of $\Delta\mu$. In the linear approximation, $\frac{\Delta\mu}{kT} \ll 1$, the net rate can be written as

$$\frac{dn_p}{dt} = \Gamma_1 - \Gamma_2 = -\lambda\Delta\mu\,. \tag{43}$$

Using the thermodynamic relation $\frac{\partial P}{\partial n_i} = -\frac{\partial \mu_i}{\partial v}$ and employing the above result we obtain

$$\delta P = P(t) - P_0 = -\frac{\partial(\Delta\mu)}{\partial v} \int_0^t \lambda \Delta\mu(t) dt \,. \tag{44}$$

The change in the chemical potential $\Delta\mu(t)$ arises due to modifications in the specific volume Δv and in the concentration of various species, viz, neutrons, protons and electrons. Thus

$$\Delta\mu(t) = \Delta\mu(0) + \left(\frac{\partial \Delta\mu}{\partial v}\right)_0 \Delta v + \left(\frac{\partial \Delta\mu}{\partial n_n}\right)_0 \Delta n_n + \left(\frac{\partial \Delta\mu}{\partial n_p}\right)_0 \Delta n_p + \left(\frac{\partial \Delta\mu}{\partial n_e}\right)_0 \Delta n_e \tag{45}$$

and we arrive at the following equations for $\Delta\mu$:

$$\frac{d\Delta\mu}{dt} = \omega A \frac{\Delta v}{v_0} \cos(\omega t) - C\lambda\Delta\mu \tag{46}$$

where

$$A = v_0 \left(\frac{\partial \Delta\mu}{\partial v}\right)_0$$
$$C = v_0 \left(\frac{\partial \Delta\mu}{\partial n_p} + \frac{\partial \Delta\mu}{\partial n_e} - \frac{\partial \Delta\mu}{\partial n_n}\right)_0 . \tag{47}$$

Since for small perturbations, λ, A and C are constants, equation (46) can be solved analytically to give

$$\Delta\mu = \frac{\omega A}{\omega^2 + C^2\lambda^2} \frac{\Delta v}{v_0} \left[- C\lambda e^{-C\lambda t} + \omega \sin(\omega t) + C\lambda \cos(\omega t) \right] \tag{48}$$

and we obtain the following expressions for ζ

$$\zeta = \frac{A^2 \lambda}{\omega^2 + C^2\lambda^2} \left[1 - \frac{\omega C\lambda}{\pi} \frac{1 - e^{-C\lambda\tau}}{\omega^2 + C^2\lambda^2} \right] . \tag{49}$$

Given the number densities of these species in terms of their respective chemical potentials, one can determine the coefficients A and C; given the rates Γ_1 and Γ_2 for the two URCA processes, one can determine λ and hence ζ for any given baryon density and temperature. For weak magnetic fields, several Landau levels are populated and the matrix elements remain essentially unchanged and one needs to account for the correct phase space factor. For non-relativistic degenerate nucleons, the decay rate constant λ is given by [6]

$$\lambda = \frac{17}{480\pi} G_F^2 \cos^2_{\theta_c} T^4 (1+3g_A^2) eB\, m_p^*\, m_n^*\; \theta\Big(p_F(p) + p_F(e) - p_F(n) \Big)$$
$$\times \sum_{\nu=0}^{\nu_{max}} [2-\delta_{\nu,0}] \frac{1}{\sqrt{\mu_e^2 - m_e^2 - 2\nu eB}} \,. \tag{50}$$

For a strong magnetic field, electrons are forced into the lowest Landau level. Using the exact wave functions for protons and electrons in the lowest Landau level and carrying out the energy integrals for degenerate matter, the decay constant λ is given by[6]

$$\lambda = \frac{17}{960\pi} G_F^2 \cos^2_{\theta_c} eB \frac{m_n^* m_p^*}{p_F(p)} T^4 [4g_A^2 + (g_v + g_A)^2] \Bigg[exp\Big(-\frac{[p_F^2(n) - 4p_F^2(e)]}{2eB} \Big)$$
$$\times \theta\Big(p_F^2(n) - 4p_F^2(e) \Big) + exp\Big(-\frac{p_F^2(n)}{2eB} \Big) \theta\Big(p_F(n) \Big) \Bigg] .$$

It is clear from above that in the case of completely polarized electrons and protons, the direct URCA decay rate always gets a non-zero contribution from the second term in the last square bracket, irrespective of whether the triangular inequality $p_F(e) + p_F(p) \geq p_F(n)$ is satisfied or not.

5 Neutrino Opacity in Magnetized Hot and dense Nuclear Matter

We calculate the neutrino opacity for magnetized, interacting dense nuclear matter for the following limiting cases: a) nucleons and electrons, highly degenerate with or without trapped neutrinos, b) non-degenerate nucleons, degenerate electrons and no trapped neutrinos and finally, c) when all particles are non-degenerate. The important neutrino interaction processes which contribute to opacity are the neutrino absorption process

$$\nu_e + n \rightarrow p + e \tag{51}$$

and the scattering processes

$$\begin{aligned} \nu_e + N &\rightarrow \nu_e + N \\ \nu_e + e &\rightarrow \nu_e + e \end{aligned} \tag{52}$$

both of which get contributions from charged as well as neutral current weak interactions. For the general process

$$\nu(p_1) + A(p_2) \rightarrow B(p_3) + l(p_4)\,. \tag{53}$$

The cross-section per unit volume of matter or the inverse mean free path is given by

$$\frac{\sigma(E_1)}{V} = \lambda^{-1}(E_1) = \frac{1}{2E_1}\prod_{i=2,3,4} d\rho_i\; W_{fi}\; f_2(E_2)[1-f_3(E_3)]\,[1-f_4(E_4)] \tag{54}$$

where $d\rho_i = \frac{d^3p_i}{(2\pi)^3 2E_i}$ is the density of states of particles and the transition rate is given by $W_{fi} = (2\pi)^4\delta^4(P_f - P_i)\,|M|^2$.

Weak Magnetic Field: for a weak magnetic field, several Landau levels are populated, the matrix element remain essentially unchanged and one needs to account only for the correct phase space factor. We first consider the neutrino-nucleon processes. In the presence of weak magnetic fields, the matrix element squared and summed over initial and final spins in the approximation of treating nucleons non-relativistically and leptons relativistically is given by

$$\sum |M|^2 = 32G_F^2\cos^2_{\theta_c}\, m_p^\star m_n^\star\Bigg[(C_V^2 + 3C_A^2) + (C_V^2 - C_A^2)cos\theta\Bigg]E_eE_\nu \tag{55}$$

where $C_V = g_V$ =1, $C_A = g_A$ =1.23, for the absorption process; $C_V = -1$, $C_A = -1.23$, for neutrino scattering on neutrons and $C_V = -1 + 4sin^2\theta_w$ =0.08, C_A =1.23, for neutrino proton scattering. We now obtain the neutrino cross-sections in the limits of extreme degeneracy or for non-degenerate matter.

Degenerate Matter: the absorption cross-section for highly degenerate matter can be calculated by using (78) in (79) by the usual techniques and we get for small B^7

$$\frac{\sigma_A(E_\nu,B)}{V} = \frac{G_F^2\cos^2_{\theta_c}}{8\pi^3}(g_V^2 + 3g_A^2)m_p^* m_n^* T^2\frac{\left(\pi^2 + \frac{(E_\nu-\mu_\nu)^2}{T^2}\right)}{\left(1+e^{\frac{(\mu_\nu-E_\nu)}{T}}\right)}eB$$
$$\times\Bigg[\theta\Big(p_F(p)+p_F(e)p_F(n)-p_F(\nu)\Big)+\frac{[p_F(p)+p_F(e)-p_F(n)+p_F(\nu)]}{2E_\nu}$$
$$\times\Bigg(\theta\Big(p_F(\nu)-|p_F(p)+p_F(e)-p_F(n)|\Big)\Bigg)\Bigg]\sum_{\nu=0}^{\nu_{max}}(2-\delta_{\nu,0})\frac{1}{\sqrt{\mu_e^2-m_e^2-2\nu eB}}\,. \tag{56}$$

The case of freely streaming, un-trapped neutrinos is obtained from the above equation by putting μ_ν =0 and replacing μ_e by $(\mu_e + E_\nu)$. When the magnetic field is much weaker than the critical field for protons, only electrons are affected and the neutrino-nucleon scattering cross-section expression remain unchanged by the magnetic field. The numerical values are, however, modified due to changed chemical composition. The cross-sections are given by

$$\frac{\sigma_{\nu N}(E_\nu)}{V} = \frac{G_F^2 \cos^2_{\theta_c}}{16\pi^3}(C_V^2 + 3C_A^2){m_N^*}^2 T^2 \mu_e \frac{\left(\pi^2 + \frac{(E_\nu - \mu_\nu)^2}{T^2}\right)}{\left(1 + exp\frac{(E_\nu - \mu_\nu)}{T}\right)} . \tag{57}$$

If neutrinos are not trapped, we get in the elastic limit

$$\frac{\sigma_{\nu N}(E_\nu)}{V} = \frac{G_F^2 \cos^2_{\theta_c}}{16\pi^3}(C_V^2 + 3C_A^2){m_N^*}^2 T^2 E_\nu . \tag{58}$$

The neutrino-electron scattering cross section is

$$\frac{\sigma_{\nu e}}{V} \simeq \frac{2G_F^2 \cos^2_{\theta_c}}{3\pi^3}(C_V^2 + C_A^2)\frac{\mu_e^2 T E_\nu^2}{1 + e^{-\beta(E_\nu - \mu_\nu)}} , \tag{59}$$

which in the un-trapped regime goes over to

$$\frac{\sigma_{\nu e}}{V} \simeq \frac{2G_F^2 \cos^2_{\theta_c}}{3\pi^3}(C_V^2 + C_A^2)\mu_e^2 T E_\nu^2 . \tag{60}$$

Non-Degenerate Matter: we now assume the nucleons are non-relativistic and non-degenerate such that $\mu_i/T \ll -1$ and thus the Pauli-blocking factor $1 - f_N(E_i)$ can be replaced by 1; the electrons are still considered degenerate and relativistic. The various cross-sections are given by

$$m\frac{\sigma_A(E_\nu, B)}{V} = \simeq \frac{G_F^2 \cos^2_{\theta_c}}{2\pi}(C_V^2 + 3C_A^2)n_n(E_\nu + Q)\frac{1}{1 + e^{-\beta(E_\nu + Q - \mu_e)}}$$
$$\times\, eB \sum_{\nu=0}^{\nu_{max}} (2 - \delta_{\nu,0})\frac{1}{\sqrt{(E_\nu + Q)^2 - m_e^2 - 2\nu eB}} \tag{61}$$

where n_N is the nuclear density and $Q = m_n - m_p$.

$$\frac{\sigma_{\nu N}(E_\nu, B)}{V} \simeq \frac{G_F^2 \cos^2_{\theta_c}}{4\pi}(C_V^2 + 3C_A^2)n_N E_\nu^2 . \tag{62}$$

If the electrons are also considered non-degenerate, we get

$$\frac{\sigma_A}{V} \simeq \frac{G_F^2 \cos^2_{\theta_c}}{\pi}(g_V^2 + 3g_A^2)n_N E_\nu^2$$
$$\frac{\sigma_{\nu N}}{V} \simeq \frac{G_F^2 \cos^2_{\theta_c}}{4\pi}(C_V^2 + 3C_A^2)n_N E_\nu^2 \tag{63}$$

$$\frac{\sigma_{\nu e}}{N} \simeq \frac{4G_F^2 \cos^2_{\theta_c}}{\pi^3} T^4 E_\nu (C_V + C_A)^2 \,. \tag{64}$$

Quantizing Magnetic Field: for quantizing magnetic field, the square of the matrix elements can be evaluated in a straight forward way [7] and we get

$$\begin{aligned} |M|^2_A &= 8G_F^2 cos^2\theta_c m_2^* m_3^* (E_4 + p_{4z}) \left[(g_V + g_A)^2 (E_1 + p_{1z}) + 4g_A^2 (E_1 - p_{1z}) \right] \\ &\times exp \left[-\frac{1}{2eB} ((p_{1x} + p_{2x})^2 + (p_{3x} + p_{4y})^2) \right] \,; \end{aligned} \tag{65}$$

$$\begin{aligned} |M|^2_{\nu p} &= 16G_F^2 \cos^2_{\theta_c} C_A^2 m_2^{*2} (p_1 \cdot p_4 + 2p_{1z} p_{4z}) \\ &\times exp \left[-\frac{1}{2eB} ((p_{4x} + p_{1x})^2 + (p_{4y} - p_{1y})^2) \right] \,; \end{aligned} \tag{66}$$

$$\begin{aligned} |M|^2_{\nu e} &= 16G_F^2 \cos^2_{\theta_c} \Big[(C_V^2 + C_A^2)((E_1E_4 + p_{1z}p_{4z})(E_2E_3 + p_{2z}p_{3z}) \\ &\quad -(E_1 p_{4z} + E_4 p_{1z})(E_2 p_{3z} + E_3 p_{2z})) + 2C_V C_A ((E_1 E_4 + p_{1z} p_{4z}) \\ &\times (E_2 p_{3z} + E_3 p_{2z}) - E_1 p_{4z} + E_4 p_{1z})(E_2 E_3 + p_{2z} p_{3z})) \Big] \\ &\times exp \left(-\frac{1}{2eB} ((p_{4x} - p_{1x})^2 - (p_{4y} - p_{1y})^2) \right) \,. \end{aligned} \tag{67}$$

The absorption cross-section is now given by

$$\begin{aligned} \frac{\sigma_A(E_1, B)}{V} &= \frac{1}{2E_1 L_x} \int \frac{d^3 p_z}{(2\pi)^3 2E_2} \int_{-eBL_x/2}^{eBL_x/2} \int_{-\infty}^{\infty} \frac{dp_{3y} dp_{3z}}{(2\pi)^2 2E_3} \\ &\times \int_{-eBL_x/2}^{eBL_x/2} \int_{-\infty}^{\infty} \frac{dp_{4y} dp_{4z}}{(2\pi)^2 2E_4} (2\pi)^3 \delta(P_y) \delta(P_z) \delta(E) \left| M \right|^2 \\ &\times f_2(E_2)[1 - f_3(E_3)][1 - f_4(E_4)] \,. \end{aligned} \tag{68}$$

The corresponding scattering cross-sections are obtained by interchanging the particle 2 with 4. Performing the integrals, we have

$$\begin{aligned} \frac{\sigma_A(E_1, B)}{V} \simeq & \frac{eB}{2E_1} \frac{1}{(2\pi)^3} \frac{1}{8} \int dE_2 \frac{dp_{3z}}{E_3} \frac{dp_{4z}}{E_4} \delta(E) \left| M \right|^2 \\ & \times f_2(E_2)[1 - f_3(E_3)][1 - f_4(E_4)] \,. \end{aligned} \tag{69}$$

To make further progress, we consider the cases of extreme-degeneracy and non-degeneracy separately.

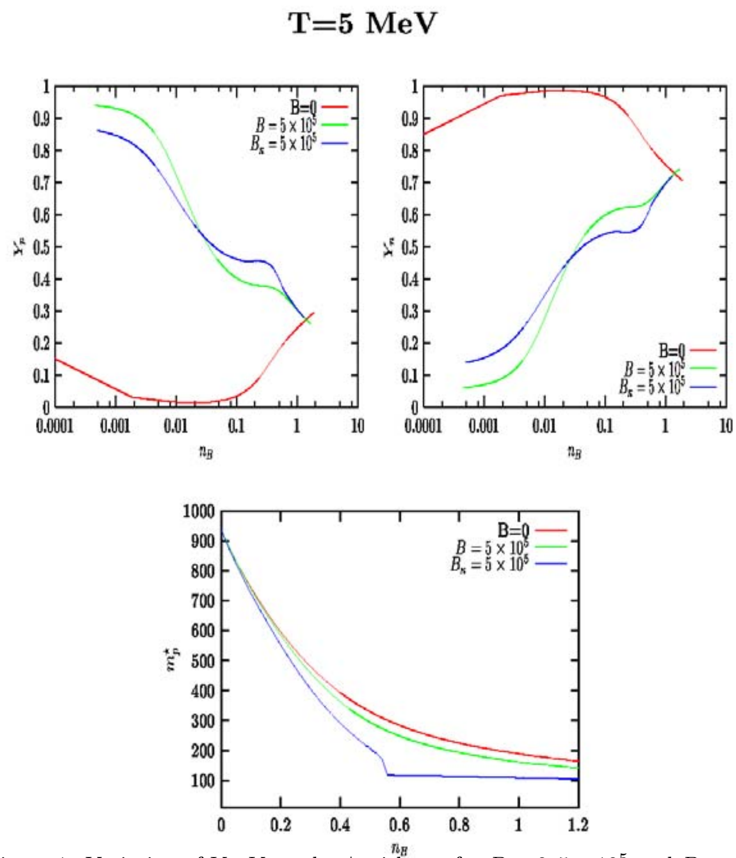

Figure 1. Variation of Y_p, Y_n and m_p^* with n_B for $B = 0, 5 \times 10^5$ and $B_\kappa = 5 \times 10^5$ where in B_κ the effect of anomalous magnetic moment has been included.

Degenerate Matter: for strongly degenerate matter, particles at the top of their respective fermi-seas alone contribute and in this approximation we get[7]

$$\frac{\sigma_A(E_\nu, B)}{V} \simeq \frac{G_F^2 cos^2\theta_c}{(2\pi)^3} eB \frac{m_n^* m_p^*}{p_F(\nu)E_\nu} \frac{T^2}{2} \frac{(\pi^2 + (\frac{E_\nu}{T})^2)}{(1+e^{-E_\nu/T})} \Bigg[(g_V + g_A)^2 (E_\nu + p_{\nu z}) + 4g_A^2 (E_\nu - p_{\nu z}) \Bigg] \Bigg[exp\Bigg(- \frac{[p_F(n) + p_F(\nu)]^2}{2eB} \Bigg) \theta\Bigg(p_F(n) + p_F(p) \Bigg) + exp\Bigg(- \frac{[p_F(n)+p_F(\nu)]^2 - 4p_F^2(e)}{2eB} \Bigg) \theta\Bigg([p_F(n) + p_F(\nu)]^2 - 4p_F^2(e) \Bigg) \Bigg]. \quad (70)$$

The case of freely streaming neutrinos is obtained by putting $p_F(\nu) = 0$ everywhere.

$$\frac{\sigma_{\nu p}(E_\nu, B)}{V} \simeq \frac{G_F^2 cos^2\theta_c}{(2\pi)^3} \frac{m_p^{*2}}{p_F^2(p)} g_A^2 T \frac{E_\nu^2}{2} \frac{eB}{2}(1 + 2cos^2\theta) \tag{71}$$

$$\frac{\sigma_{\nu e}(E_\nu, B)}{V} \simeq \frac{G_F^2 cos^2\theta_c}{8\pi} T^2 eB\mu_\nu \left[(C_V^2 + C_A^2)(1 + cos^2\theta) - 4C_V C_A cos\theta\right] .$$

Non-Degenerate Matter: similarly performing the integrals in the non degenerate limit, the various cross sections are

$$\frac{\sigma_A(E_\nu, B)}{V} \simeq \frac{G_F^2 cos^2\theta_c}{4\pi} eB \cos_\theta n_N \frac{1}{e^{-(E_\nu + Q - mu_e)\beta} + 1}$$
$$[((g_V + g_A)^2 + 4g_A^2) + ((g_V + g_A)^2 - 4g_A^2)] \tag{72}$$

$$\frac{\sigma_{\nu p}(E_\nu, B)}{V} \simeq \frac{2G_F^2 cos\theta_c{}^2 C_A^2}{2\pi} E_\nu^2 n_p \tag{73}$$

$$\frac{\sigma_{\nu e}(E_\nu, B)}{V} \simeq \frac{G_F^2}{\pi} n_e T E_\nu \left[(C_V^2 + C_A^2)(1 + cos^2\theta) - 4C_V C_A cos\theta\right] . \tag{74}$$

We see that in the neutrino free case ($Y_{\nu_e} = 0$), the neutrino absorption cross section, i.e. the direct URCA process which is highly suppressed for degenerate matter in the absence of the magnetic field, proceeds at all densities for the quantizing magnetic field.

6 Results and Discussion

We first calculate the composition of matter for arbitrary magnetic fields for both the neutrino free and neutrino trapped cases over a wide range of density and temperature. We find that the effect of magnetic field is to raise the proton fraction and is pronounced at low densities. The effect of including the anomalous magnetic moment also becomes significant at field strengths $\sim 10^5 B_e^c$ which can be seen from Fig.1 where we have plotted the proton fraction and the effective nucleon mass as a function of density. We then calculate the effect of the magnetic field on neutrino emissivity for the direct URCA process and find that this effect is not important at weak magnetic fields. However, for the interesting case of quantizing magnetic field capable of totally polarizing the electrons and protons, we see from Fig.2 that the threshold for direct URCA process is evaded and the emissivity is enhanced by up to two orders of magnitude and develops anisotropy.

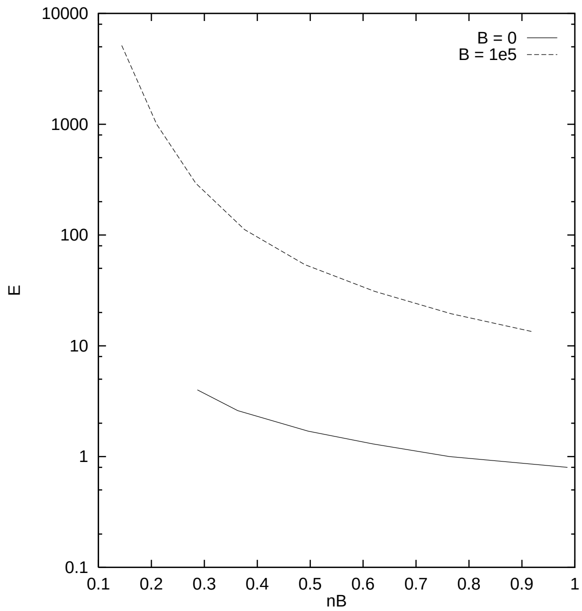

Figure 2. Neutrino emissivity as as function of Baryon density for $B = 0$ and $B = 10^5$, when the electrons and protons are completely polarized

γ-ray burst events, if they arise due to pulsations and the maximum rotation frequency, are influenced by the bulk viscosity of neutron star matter through: 1) damping of radial oscillations and 2) by influencing gravitational radiation reaction instability limiting maximum rotation rate. In Fig.3 we plot the bulk viscosity as a function of baryon density for different values of the magnetic field and find that the effect of the magnetic field is very pronounced at low densities where only the lowest Landau level contributes and the direct URCA process is no longer inhibited. We also find that the viscosity decreases very rapidly with density.

Neutrinos emitted during the cooling of protoneutron star have a momentum ~ 100 times the momentum of pulsars and therefore, one percent anisotropy in neutrino emission would give a kick velocity consistent with observation. This can occur in two different ways since the neutrino dispersion relations in magnetized medium are modified. Firstly, due to neutrino scattering reactions on polarized electrons and protons and secondly [8] due to matter induced MSW $\nu_e \leftrightarrow \nu_\tau$ neutrino oscillations in the presence of a mag-

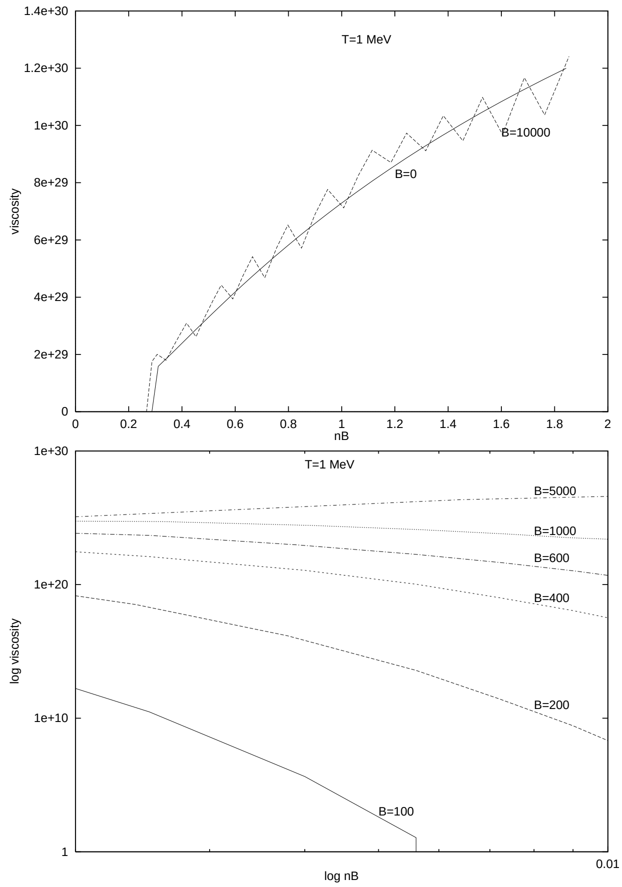

Figure 3. Bulk viscosity as a function of Baryon density at $T = 1$ MeV for different values of the magnetic field

netic field when the ν_τ sphere develops asymmetry and the resonant surface gets distorted and the neutrinos escaping from different depths emerge with different energies. In Figs.4 and 5 we have plotted the neutrino absorption and scattering mean free paths with density for different values of the temperature and magnetic field for both the free streaming and trapped regimes. In Fig.6 we show the variation of the absorption mean free path with density and observe substantial decrease in addition to developing anisotropy with magnetic field which is particularly pronounced at low densities.

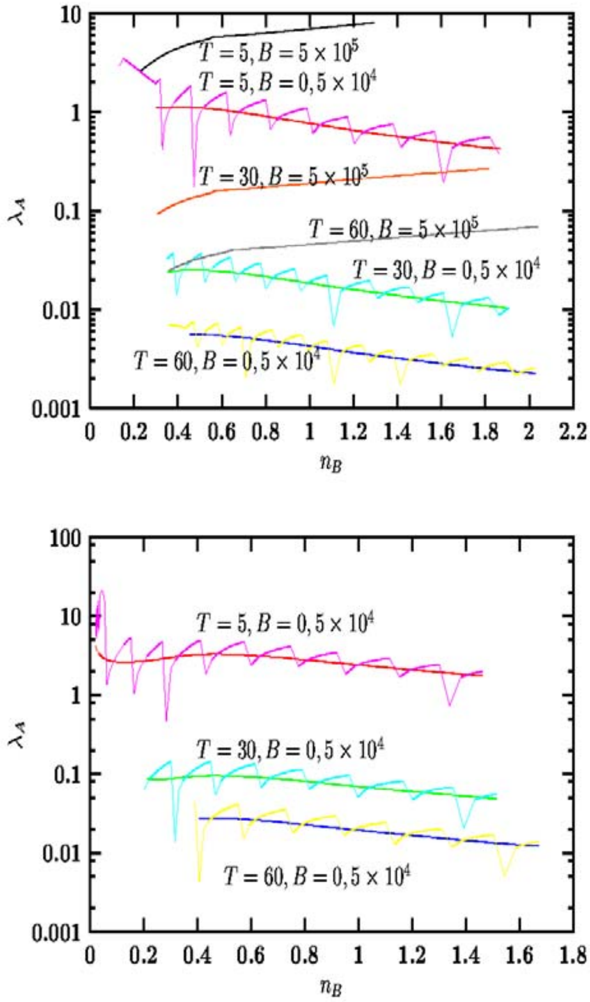

Figure 4. Variation of absorption mean free path with Baryon density. The first figure is for un-trapped degenerate matter and the second figure for trapped degenerate matter.

To conclude, we find that the effect of quantizing magnetic field and of the inclusion of anomalous magnetic moment of nucleons is to increase the proton

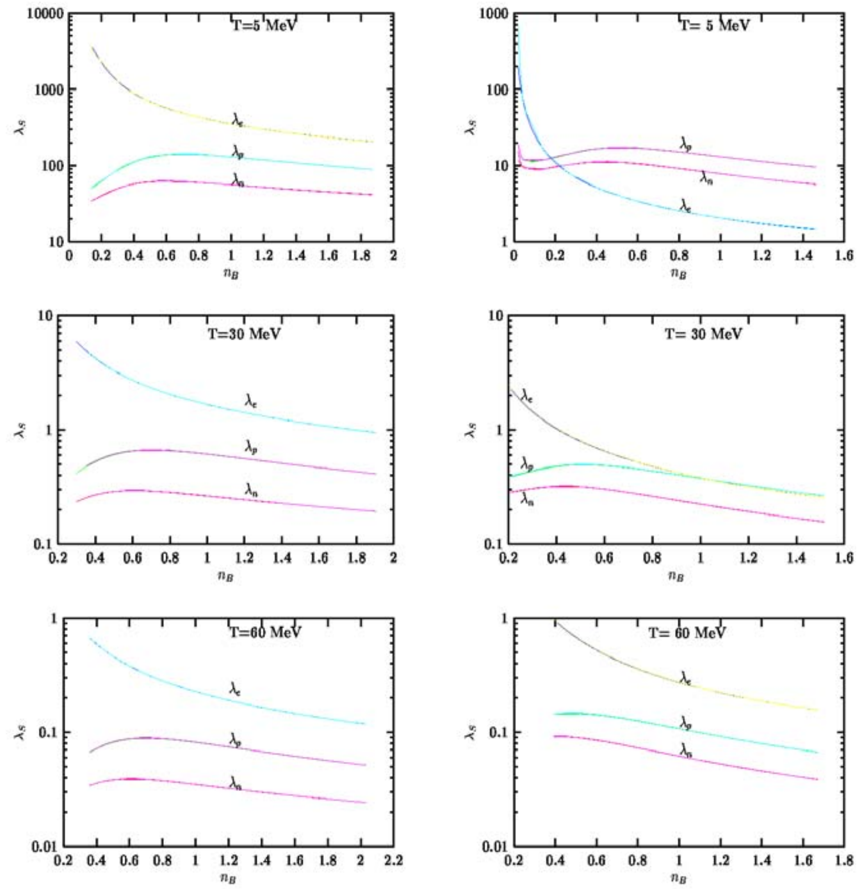

Figure 5. Variation of neutrino scattering mean free path with baryon density; the left panel is for un-trapped matter and the right panel is for trapped matter

fraction and lower the effective nucleon mass so as to evade the threshold for the direct URCA processes to proceed. This results in the enhancement of neutrino emission and bulk viscosity. This also has the effect of substantially decreasing the neutrino absorption mean free path in addition to developing

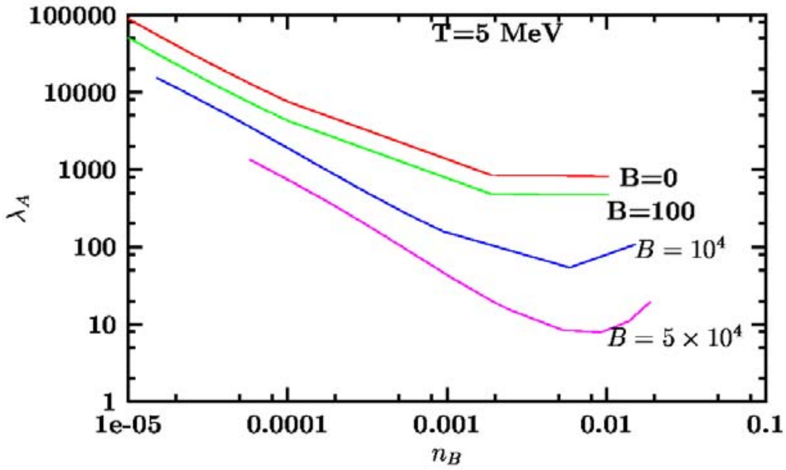

Figure 6. Variation of neutrino absorption mean free path with Baryon density for un-trapped non-degenerate matter.

anisotropy.

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