COLLAPSE AND FLATTENING OF NUCLEONIC BAGS IN ULTRA-STRONG MAGNETIC FIELD

S. MANDAL AND S. CHAKRABARTY
Department of Physics, University of Kalyani, Kalyani 741 235, India
soma@klyuniv.ernet.in, somenath@klyuniv.ernet.in

It is shown explicitly using the MIT bag model that in the presence of ultra-strong magnetic fields, a nucleon either flattens or collapses in the direction transverse to the external magnetic field in the classical or quantum mechanical pictures, respectively, which gives rise to some kind of mechanical instability. Alternatively, it is here argued that the bag model of confinement may not be applicable in this strange situation.

1 Introduction

One of the oldest subjects in physics, "the effect of strong magnetic fields on dense matter", has gotten a new life after the observational discoveries of a few strongly magnetized exotic stellar objects- known as magnetars [1,2,3,4,5]. These uncommon objects are believed to be strongly magnetized young neutron stars and their strong magnetic fields are supposed to be the possible sources of X-rays from anomalous X-ray pulsars (AXP) and low energy γ-radiation form the soft gamma-ray repeaters (SGR). It is believed that such objects may also act as the central engine for gamma ray bursts (GRB). The measured value of the magnetic field strength at the surface of these objects are $\sim 10^{14} - 10^{15}$G. Then it can be shown by the scalar virial theorem that the magnetic field strength at the core region may go up to 10^{18}G. These objects are also assumed to be too young compared to the decay/expulsion time scale of magnetic fields from the core region. Now, in the presence of such intense magnetic fields, most of the physical properties of dense stellar matter, for example, the equation of state and the quark-hadron phase transition must change significantly[6,7,8]. Moreover, some of the physical processes[9,10], in particular, weak and electromagnetic decays and reactions, neutrino opacities and others, at the core region of compact neutron stars, will also be affected in the presence of ultra-strong magnetic fields. The transport properties (e.g, shear and bulk viscosities, thermal and electrical conductivities) of dense neutron star matter also change both qualitatively and quantitatively in the presence of a strong magnetic field[11,12]. Furthermore, these intense magnetic fields could cause structural deformation of the exotic objects. In the classical general relativistic theory, it is shown, by use of the Maxwell stress tensor that such exotic objects get flattened[13,14,15] for the macroscopic field B_m, whereas in the quantum mechanical scenario they collapse in the direction transverse to the magnetic field[16,17]. In the case of an ultra-strong magnetic field, the structure of these objects could become either disk-like (in a classical picture) or cigar-like (in a quantum mechanical scenario) from their usual spherical shapes. In the extreme case, they may be converted into black disks or black strings. Therefore, in some sense, these strange stellar objects become mechanically unstable in the presence of ultra-strong magnetic fields. Long ago, Chandrasekhar and Fermi in their studies on the stability of magnetized white dwarfs explained the possibility of such strange behavior[18]. Their conclusions are also valid for strongly magnetized neutron stars, where the white dwarf parameters have to be replaced by typical neutron star parameters; the upper limit of the magnetic field strength for a stable typical neutron star is found to be 10^{18}G. In a recent work we have shown that if the magnetic field is extremely high to populate only the zeroth Landau level (with fully polarized spin states) of electrons, then stable neutron star/protoneutron star matter can not exist if the β-equilibrium condition[19,20] holds. It was also shown by Bander and Rubinstein[21], in the context of the stability of neutron and protons in a strong magnetic field, that in presence of an extremely strong magnetic field, protons become unstable by gaining effective mass, whereas neutrons, loosing effective mass, become stable. In their calculations, a delicate interplay between the anomalous magnetic moments of neutron and proton makes neutrons stable and protons unstable: then, decays of protons into neutrons via e^+ and neutrino emission can occur.

In this article, following the recent work of Martínez et al[16,17] and Kohri et al[22], we show that even nucleonic (proton or neutron) bags cannot be stable in the presence of ultra-strong magnetic fields: they either collapse or elongate in the transverse direction of the corresponding one to the ultra-strong external magnetic field. We have shown that either the nucleons are mechanically unstable or the bag model calculations are not suitable for the conditions referred to above. In this work we have therefore studied the mechanical stability of a neutron or a proton placed in an ultra-strong magnetic field. On the other hand, in Ref.[21], Bander and Rubinstein have studied the stability of these objects from the effective mass point of view and showed that neutrons are much more stable energetically than protons in this situation.

This paper is organized in the following manner: in section 2, we review very briefly the MIT bag Lagrangian approach of color confinement. In section 3, we discuss the collapse of nucleons following the ideas of Martínez et al[16,17]. In section 4, on basis of the model proposed by Kohri et al [22] in the context of anisotropic e^+e^- pressure, we show that nucleons get flattened in the transverse direction of the magnetic field. The conclusions and discussions are presented in the last section.

2 Color Confinement- a Brief Overview

To study the mechanical stability of neutron/proton bags in the presence of ultra-strong magnetic fields, taking a flat space-time coordinate, we have considered the MIT bag model of quark confinement[23,24,25]. We have taken into account both the gluonic interaction of quarks and the bag pressure B to confine quarks within the bag. Before we go into the detailed discussion on the mechanical instability problem of nucleonic bags in the presence of intense magnetic fields, we give a brief overview of the MIT bag model lagrangian approach to obtain the pressure balance at the surface of the nucleons. The usual form of the MIT bag lagrangian density is

$$\begin{aligned}\mathcal{L}_{MIT} = &\left(i\left\{\bar{\psi}\gamma^\mu\partial_\mu\psi - (\partial_\mu\bar{\psi})\gamma^\mu\psi\right\} + g\bar{\psi}\frac{\lambda_a}{2}\gamma^\mu V_\mu^a\psi - \bar{\psi}m\psi\right.\\ &\left. - \frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} - B\right)\theta_v(x) - \frac{1}{2}\bar{\psi}\psi\Delta_s \qquad (1)\end{aligned}$$

where g is the strong interaction coupling constant, the λ_a's are the $SU(3)$ generators, with $a = 1, 2, ...8$, the gluonic color index, ${V_\mu}^a$ is the gluonic field four vector, $F_{\mu\nu}^a$ is the corresponding field tensor, m is the current mass of quarks, B is the bag constant, $\theta_v = 1$ inside the bag and $\theta_v = 0$ outside the bag, $\partial\theta_v/\partial x^\mu = n_\mu\Delta_s$, Δ_s is the surface delta-function and n_μ is the space-like unit vector normal to the surface. The sum over flavor and color quantum numbers carried by quarks has not been shown explicitly. To obtain the pressure balance at the bag surface, we consider the energy momentum tensor of the bag

$$\begin{aligned}T^{\mu\nu} &= -g^{\mu\nu}\mathcal{L} + \left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\partial^\nu\psi + \partial^\nu\bar{\psi}\frac{\partial\mathcal{L}}{\partial(\partial_\mu\bar{\psi})}\right)\\ &= -g^{\mu\nu}\mathcal{L} + \frac{i}{2}\left(\bar{\psi}\gamma^\mu\partial^\nu\psi - (\partial^\nu\bar{\psi})\gamma^\mu\psi\right)\theta_v\,. \qquad (2)\end{aligned}$$

Using the condition of energy-momentum conservation, $\partial_\mu T^{\mu\nu} = 0$, we have

$$B\Delta_s n^\nu + \frac{i}{2}\left(\bar{\psi}\gamma^\mu\partial^\nu\psi - (\partial^\nu\bar{\psi})\gamma^\mu\psi\right)n_\mu\Delta_s = 0 \qquad (3)$$

and

$$\partial_\mu \left(\bar{\psi}\psi\Delta_s \right) = 0\,. \tag{4}$$

Now, considering the surface boundary condition, given by (obtained from standard Euler-Lagrange equation)

$$in_\mu \gamma^\mu \psi = \psi \tag{5}$$

we obtain, on the bag surface,

$$Bn^\mu = \frac{1}{2}\frac{\partial}{\partial x_\mu}(\bar{\psi}\psi)\,. \tag{6}$$

This equation is nothing but the pressure balance equation. Since $n^\mu n_\mu = -1$, we have on the bag boundary

$$B = -\frac{1}{2} n_\mu \partial^\mu (\bar{\psi}\psi)\,. \tag{7}$$

In case of a spherical bag, $n^\mu \equiv (0, \hat{r})$, this pressure balance equation reduces to

$$B = -\frac{1}{2}\frac{\partial}{\partial r}(\bar{\psi}\psi) \tag{8}$$

which means that the outward pressure of the quarks is exactly balanced by the inward vacuum pressure B on the surface of the bag.

3 Collapse of Nucleonic Bags

Now we shall consider the nucleonic bag (either neutron or proton) as an interacting thermodynamic system in equilibrium. The constituents are valance quarks, sea quarks and gluons. Then, the total kinetic pressure of the system is given by

$$P_{in} = P_{in}^{(v)} + P_{in}^{(s)} + P_{in}^{(g)} \tag{9}$$

where v, s and g denote the valance quarks, sea quarks and gluonic contributions, respectively. As discussed before, this internal kinetic pressure has to be balanced by the external bag pressure to maintain the stability of the system. Then we can write down the effective thermodynamic potential per unit volume of the system as

$$-\Omega = P_{in} - B \tag{10}$$

and it should be zero. Following Martínez et al[16,17], in the presence of an ultra-strong magnetic field of strength B_m, the thermodynamic potential per unit volume reduces to[a]

$$T_\mu^\nu = \left(T\frac{\partial\Omega}{\partial T} + \sum_r \mu_r \frac{\partial\Omega}{\partial\mu_r} \right) g_\mu^4 g_4^\nu + 4F_{\mu\lambda}F^{\nu\lambda}\frac{\partial\Omega}{\partial F^2} - g_\mu^\nu \Omega\,. \tag{11}$$

Hence, the longitudinal component of pressure (along the direction of the field) is given by

$$T_{zz} = P_{||} = -\Omega = 0 \tag{12}$$

while the transverse part of the total pressure is

$$T_{xx} = T_{yy} = P_\perp = -\Omega - \mathcal{M}B_m = P_{||} - \mathcal{M}B_m \tag{13}$$

where $\mathcal{M}$ is the effective magnetic dipole moment density of the bag. Since $\Omega = 0$, nucleons will therefore be inflated or collapsed in the transverse direction in the presence of an ultra-strong magnetic field depending on the overall sign of $\mathcal{M}$. The system will collapse if $\mathcal{M}$ is positive; else it will be inflated in the transverse direction. In order to have an order of magnitude estimate of extra in/outward pressure, we choose the contribution to $\mathcal{M}$ from valance quarks only (in fact, the valance quarks contribute only in the evaluation of the magnetic dipole moment of the nucleons). The magnetic dipole moment density of the ith component ($i = u$ or d quarks) is given by

$$\mathcal{M}_i = -\frac{\partial\Omega_i}{\partial B_m} \tag{14}$$

and the total value is given by

$$\mathcal{M} = \sum_{i=u,d} \mathcal{M}_i \tag{15}$$

where

$$\Omega_i = \frac{g_i q_i B_m}{4\pi^2} \sum_{\nu=0}^{\nu_{\max}} \sum_{s=\pm1} \left[\mu_i(\mu_i^2 - M_{i,\nu,s}^2)^{1/2} - M_{i,\nu,s}^2 \ln\left(\frac{\mu_i + (\mu_i^2 - M_{i,\nu,s}^2)^{1/2}}{M_{i,\nu,s}} \right) \right] \tag{16}$$

is the thermodynamic potential density of the component i; in these expressions, g_i and q_i are respectively the degeneracy and charge of the ith species,

$$M_{i,\nu,s}^2 = \left\{ (p_\perp^2 + m_i^2)^{1/2} + sQ_iB_m \right\}^2 , \tag{17}$$

[a] We have chosen the gauge $A^\mu \equiv (0, -yB_m/2, xB_m/2, 0)$, so that B_m is a constant magnetic field along the Z-axis.

m_i is the current quark mass ($= 5$MeV), $p_\perp = (2\nu q_i B_m)^{1/2}$ is the transverse component of momentum and Q_i is the anomalous magnetic dipole moment of the ith quark species ($Q_u = 1.852\mu_N$ and $Q_d = -0.972\mu_N$, μ_N is the nuclear magneton). The maximum value of Landau quantum number is given by

$$\nu_{\rm max}^{(i)} = \left[\frac{(\mu_i^2 - sQ_iB_m)^2 - m_i^2}{2q_iB_m}\right] \tag{18}$$

where [ ] indicates an integer less than the decimal number inside the brackets. To obtain the chemical potentials for the u and d quarks, we have made the following assumptions. The ith quark species density within the nucleon is given by

$$n_i = \frac{g_iq_iB_m}{2\pi^2}\sum_{\nu=0}^{\nu_{\rm max}}\sum_{s=\pm1}(\mu_i^2 - M_{i,\nu,s}^2)^{1/2} = \frac{\mathrm{N0}(i)}{V} \tag{19}$$

where NO(i) is the number of ith quarks species in the system. Therefore, NO(i)=NO(u)=1 for neutrons and 2 for protons. Similarly, NO(i)=NO(d)=2 for neutrons and 1 for protons and V is the nucleonic volume. We further assume that $r = 0.8$fm as the radius of the nucleons. Solving numerically, we have obtained the chemical potentials μ_i's for both u and d quarks and hence we have evaluated the magnetic dipole moment per unit volume for the system. In Fig.(1) we have plotted $\mathcal{M}B_m$ for various values of B_m for both neutrons and protons. The product $\mathcal{M}B_m$ is always positive and oscillatory in the strong field regime ($\geq 10^{17}$G). The system will therefore collapse in the transverse direction and becomes ellipsoidal with cylindrical symmetry. The minor axes lengths b will therefore oscillate with the strength of the magnetic field in particular, above 10^{17}G. Now, in the study of the mechanical stability of strongly magnetized neutron stars in a quantum mechanical scenario, it has been shown that the system will either be inflated or collapsed if the magnetic dipole moment is negative or positive, respectively. It has further been shown that neutron matter always behaves like a paramagnetic material with $\mathcal{M} > 0$; as a result, in the quantum mechanical picture a strongly magnetized neutron star always collapses in the transverse direction. Therefore, we can infer that the conclusion drawn for such macroscopic objects like neutron stars is also valid in the microscopic level, e.g. neutrons or protons. We can then conclude that in a strong magnetic field, not only neutron stars, even their constituents, neutrons and protons, become mechanically unstable. Alternatively, one could conclude that the bag model is perhaps not applicable in such strange situation; in that case, the use of a bag model for magnetized quark stars is also questionable. Therefore, the investigations of this section show that both neutrons and protons become cigar-like and in the extreme case they may reduce to what is called black-strings.

4 Flattening of Nucleons

In this section we shall evaluate the longitudinal and transverse parts of the kinetic pressures following Ref.[22]. We choose the gauge $A^\mu \equiv (0, 0, xB_m, 0)$ so that $\vec{B}_m \equiv (0, 0, B_m)$. Then, the solution of the Dirac equation is given by

$$\psi = \exp(-iE_n t) \begin{pmatrix} \phi \\ \chi \end{pmatrix} \tag{20}$$

where ϕ and χ are the upper and lower components. The upper component is given by

$$\phi = \exp(ip_y y + ip_z z) f_n \zeta_s, \tag{21}$$

where $n = 0, 1, 2, ...$ are the Landau quantum numbers and $s = \pm 1$, the spin quantum numbers, so that

$$\zeta_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{22}$$

$$\zeta_{-1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{23}$$

$$f_n(x, p_y) = \frac{1}{(2^n n! \pi^{1/2})^{1/2}} \exp\left(-\frac{\xi^2}{2}\right) H_n(\xi) \tag{24}$$

$\xi = (q_i B_m)^{1/2}(x - p_y/(q_i B_m))$ and $H_n(\xi)$ is the Hermite polynomial of order n. The lower component of Dirac equation is given by

$$\chi = \frac{\vec{\sigma}.(\vec{p} - q_i \vec{A})}{E_n + m_i} \phi \,, \tag{25}$$

and the energy eigenvalue is $E_n = (p_z^2 + m_i^2 + q_i B_m (2n + 1 - s))^{1/2}$. Then, we have from the first part of Eq.(2),

$$T^\mu_\nu = \mathrm{diag}(E_n, -\hat{P}_x, -\hat{P}_y, -\hat{P}_z) \tag{26}$$

whereas all the off-diagonal terms are zero. Thus, it is very easy to show

$$\hat{P}_x = \hat{P}_y = \left(n + \frac{1}{2} - \frac{s}{2}\right) \frac{q_i B_m}{E_n}, \hat{P}_z = \frac{p_z^2}{E_n} \,; \tag{27}$$

these are called the dynamic pressure[22] components. The ensemble average of these pressures are given by (at $T = 0$)

$$P_x = P_y = \frac{(q_i B_m)^2}{2\pi^2} \sum_{\nu=0}^{[\nu_{\max}]} \nu(2 - \delta_{0\nu}) \ln \left\{ \frac{\mu_i + (\mu_i^2 - m_\nu^2)^{1/2}}{m_\nu} \right\} \tag{28}$$

which correspond to the transverse part, with $2\nu = 2n + 1 - s$ and $m_\nu^2 = m_i^2 + 2q_i B_m \nu$. Similarly, we have the longitudinal component

$$P_z = \frac{q_i B_m}{4\pi^2} \sum_{\nu=0}^{[\nu_{\max}]} (2 - \delta_{0\nu}) \left[\mu_i(\mu_i^2 - m_\nu^2)^{1/2} - \ln \left\{ \frac{\mu_i + (\mu_i^2 - m_\nu^2)^{1/2}}{m_\nu} \right\} \right] . \tag{29}$$

Following the same numerical techniques as followed in the previous section,

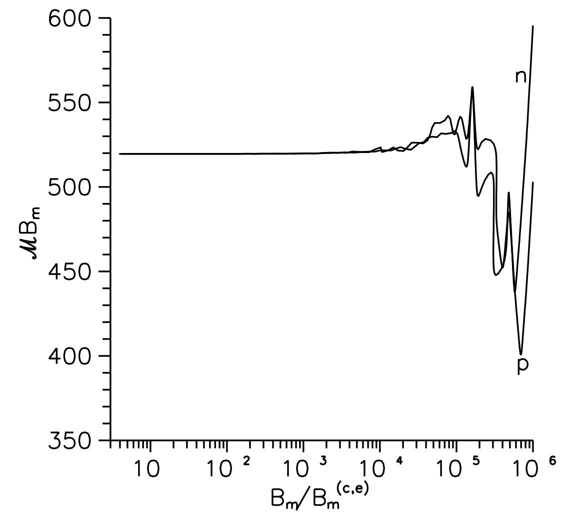

Figure 1. Variation of $\mathcal{M}B_m$ with $B_m/B_m^{(c)(e)}$ for neutrons (indicated by the symbol n) and protons (indicated by the symbol p).

in Figs.(2) and (3) we have plotted the longitudinal and transverse components of the kinetic pressure with various magnetic field strengths for protons and neutrons, respectively. The curves in both figures show that the longitudinal part of the kinetic pressure is zero and/or very low for a high magnetic field strength, whereas the transverse part is high for intense magnetic fields. These two components saturate to some constant value for a low or moderate

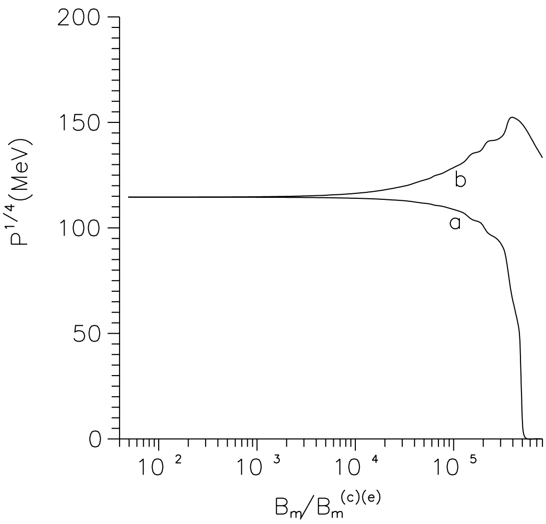

Figure 2. Variation of the kinetic pressure ($P^{1/4}$ in MeV) with $B_m/B_m^{(c)(e)}$ for protons. Curve a is for longitudinal and b is for transverse components of the kinetic pressures, respectively.

magnetic field strength, which indicates that the system reduces to a pressure isotropic configuration at low magnetic fields (as we generally see in conventional thermodynamic systems). Therefore, according to this model, at very high magnetic field strength, the system (neutron or proton) becomes oblate in shape and in the extreme case it reduces to a black disk.

5 Conclusions

In conclusion, we have studied the mechanical stability of neutrons and protons in a compact neutron star in the presence of a strong quantizing magnetic field. We have followed two entirely different approaches. In the so called quantum mechanical picture, in which the interaction of magnetic dipole moments of quark constituents with the external magnetic field has been considered, the shapes of both neutron and proton become of prolate-type from their usual spherical nature. The effect is more prominent at the high field limit ($> 10^{16}$G). On the other hand, in the classical picture, both systems acquire oblate shapes; the effect is again prominent for high magnetic

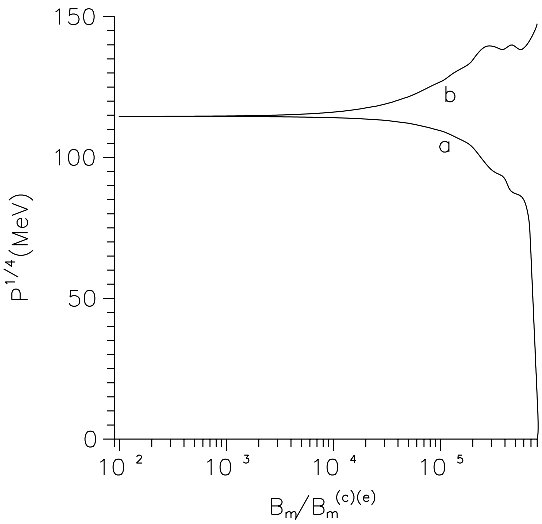

Figure 3. Variation of the kinetic pressure ($P^{1/4}$ in MeV) with $B_m/B_m^{(c)(e)}$ for neutrons. Curve a is for longitudinal and b is for transverse components of the kinetic pressures, respectively.

fields. In the classical picture, it has been observed that such anisotropy of the kinetic pressure is automatically removed at moderate ($\geq 10^{15-16}$G) values of the magnetic field strength and both systems become mechanically stable. However, in the quantum mechanical picture, there is always an extra in-word pressure in the transverse direction even for moderate values of the magnetic field strength. This is because of the non-zero finite values of $\mathcal{M}B_m$ in the systems, but the effect is not so significant. Therefore, the behavior of bulk objects like neutron stars and their constituents, e.g., neutrons and protons (which are of microscopic in size) are almost identical in an external strong magnetic field.

Acknowledgments

S.C. is thankful to the Department of Science and Technology, Gov. of India, for partial support of this work, Sanction number:SP/S2/K3/97(PRU).

References

1. R.C. Duncan and C. Thompson, Astrophys. J. Lett. **392**, L9 (1992); C. Thompson and R.C. Duncan, Astrophys. J. **408**, 194 (1993); C. Thompson and R.C. Duncan, MNRAS **275**, 255 (1995); C. Thompson and R.C. Duncan, Astrophys. J. **473**, 322 (1996).
2. P.M. Woods et al., Astrophys. J. **519**, L139 (1999); C. Kouveliotou, et al., Nature **393**, 235 (1998); Astrophys. J. **510**, L115 (1999).
3. S. Kulkarni and D. Frail, Nature **365**, 33 (1993).
4. T. Murakami et al, Nature **368**, 127 (1994).
5. K. Hurley, et al., Astrophys. Jour. **442**, L111 (1999); S. Mereghetti and L. Stella, Astrophys. Jour. **442**, L17 (1999); J. van Paradihs, R.E. Taam and E.P.J. van den Heuvel, Astron. Astrophys. **299**, L41 (1995); S. Mereghetti, astro-ph/99111252; see also A. Reisenegger, astro-ph/01003010.
6. D. Bandopadhyaya, S. Chakrabarty, P. Dey and S. Pal, Phys. Rev. **D58**, 121301 (1998); T. Ghosh and S. Chakrabarty, Phys. Rev. **D63**, 0403006 (2001); T. Ghosh and S. Chakrabarty, Int. Jour. Mod. Phys. **D10**, 89 (2001).
7. S. Chakrabarty, D. Bandopadhyay and S. Pal, Phys. Rev. Lett. **78**, 2898 (1997); D. Bandopadhyay, S. Chakrabarty and S. Pal, Phys. Rev. Lett. **79**, 2176 (1997).
8. C.Y. Cardall, M. Prakash and J.M. Lattimer, Astrophys. J. **554**, 322 (2001); A. Broderic, M. Prakash and J.M. Lattimer, Astrophys. J. **537**, 351 (2000).
9. E. Roulet, Astro-ph/9711206; L.B. Leinson and A. Pérez, Astro-ph/9711216; D.G. Yakovlev and A.D. Kaminkar, The Equation of States in Astrophysics, eds. G. Chabrier and E. Schatzman P.214, Cambridge Univ. Press, 1994; V.G. Bezchastnov and P. Haensel (astro-ph/9608090), Phys.Rev. **D54**, 3706 (1996).
10. V.G. Bezchastrov and P. haensel, Astro-ph/9608090. Esteban Roulet; JHEP, **9801**, 013 (1998) (hep-ph/9711206).
11. D.G. Yakovlev and D.A. Shalybkov, Sov. Astron. Lett. **16**, 86 (1990); D.G. Yakovlev and D.A. Shalybkov, Astrophys. Space Sc. **176**, 171 (1991); D.G. Yakovlev and D.A. Shalybkov, Astrophys. Space Sc. **176**, 191 (1991).
12. Sutapa Ghosh, Sanchayita Ghosh, Kanupriya Goswami, Somenath Chakrabarty, Ashok Goyal, Int. Jour. Mod. Phys. **D11**, 843 (2002).
13. K. Konno, T. Obata and Y. Kojima, Astron. Astrophys. **356**, 234 (2000).
14. K. Konno, T. Obata and Y. Kojima, astro-ph/0001397.

15. See also T. W. Baumgarte and S. L. Shapiro, astro-ph/0211339.
16. M. Chaichian et al, Phys. Rev. Lett. **84**, 5261 (2000) (hep-ph/9911218).
17. A. Pérez Martínez et al, hep-ph/0011399; astro-ph/0303213.
18. S. Chandrasekhar and E. Fermi, Astrophys. Jour. **118**, 116 (1953); see also D. Lai and S.L. Shapiro, Astrophys. Jour. **383**, 745 (1991).
19. S. Ghosh, S. Mandal and S. Chakrabarty, astro-ph/0205445.
20. S. Ghosh, S. Mandal and S. Chakrabarty, astro-ph/0207492.
21. M. Bander and H.R. Rubinstein, Nucl. Phys. Proc. Suppl. **31**, 248 (1992).
22. K. Kohri, S. Yamada and S. Nagataki, hep-ph/0106271.
23. Models of the Nucleon, From Quarks to Soliton, R.K. Bhaduri, Addiison-Wesley Publ. Comp. Inc., New York, 1988.
24. Quantum Chromodynamics, W. Greiner and A. Schäfer, Springer, 1989.
25. see also T. DeGrand, R.L. Jaffe, K. Johnson and J. Kiskis, Phys. Rev. **D12**, 2060 (1975).