NEUTRON STARS AND NATURALNESS IN EFFECTIVE NON-LINEAR RELATIVISTIC MODELS

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In this work we consider, similarly to the previous contributions to this proceedings, a discussion on neutron stars. In the first part of the work, we consider the fuzzy bag model and a new feature, the requirement of energy-momentum conservation over the finite smooth surface of the bag. After imposing center-of-mass and one-gluon exchange corrections, the model parameters are determined from a consistent fit of the baryon masses of the lowest baryon octet and decuplet. As one immediate and crucial consequence, the bag constant B acquires a radial dependence, whose functional form is completely determined from the internal consistency of the model. Together with chiral symmetry, respected from the coupling to the external pion cloud, which induces renormalization effects on the hadron wave functions and observables, our approach is a first step in modelling, in a consistent way, the transition of effective hadronic degrees of freedom towards a deconfined quark gluon plasma at very high densities. In the second part of this work, the structure of high density nuclear matter is studied in the framework of a mean-field description of non-overlapping neutron and proton bags with soft surfaces bound by the self-consistent exchange of the σ, ω and ϱ mesons. As an application we determine the structure of the EoS for neutron stars. We then consistently include in the formulation lepton degrees of freedom, chemical equilibrium and baryon number and electric charge conservations and investigate properties of neutron star matter. In the third part, we study dense hadronic matter in a generalized relativistic mean field approach which contains nonlinear couplings of the $\sigma, \omega, \varrho, \delta$ fields and compare its predictions for neutron star properties with results from different models found in the literature. In the forth part of the work, we confront results based on this multi-baryon lagrangian density modelling and compare its predictions with estimates obtained within a phenomenological naïve dimensional analysis based on the naturalness of the various coupling constants of the theory. Upon adjusting the model parameters to describe bulk static properties of ordinary nuclear matter, we discuss implications of the approach for dense hadronic matter, in particular for neutron stars.

Introduction

In the first part of this contribution we consider a recently developed, new version of the fuzzy bag model[1,2] which incorporates explicitly the conservation of energy and momentum over the smooth, finite surface of the bag causing the bag pressure B to acquire a radial dependence, $B = B(r)$, whose functional form is completely determined within the consistency of the model. This feature is missing in relativistic potential models, as well as in the original formulation of the fuzzy bag model[3] and its predecessor, the MIT bag model[4]: in both versions the non-perturbative QCD vacuum is parameterized by a constant B in the lagrangian density. As in previous approaches[5,6,7], we consider a bag with an interior region where the quarks are free, a surface region, where the quark wave function is gradually suppressed, and an exterior region, where the quark wave function vanishes. The physical picture underlying such a modelling is intuitively very appealing: it combines the interior region of the bag which corresponds to the perturbative vacuum of QCD with the exterior region of the non-perturbative vacuum of QCD. The finite surface region, with a typical scale of about half a fermi, models the smooth transition region between the two vacua, as expected from a gradual outside-inside restoration of chiral symmetry. We introduce center-of-mass and one gluon exchange corrections as well as the pion field to restore chiral symmetry. As a test of the consistency of the model, the masses of the low-lying baryons are fitted. In section 2, a simplified version of the model presented in the previous section is applied in the study of the structure of high density nuclear matter taking into account non-overlapping neutron and proton bags with soft surfaces bound by the self-consistent exchange of the σ, ω and ϱ mesons. As an application, we determine the structure of the EoS for neutron stars. We then consistently include in the formulation lepton degrees of freedom, chemical equilibrium and baryon number and electric charge conservation and investigate properties of neutron star matter[8].

In the continuation of the study of neutron star matter we introduce, in section 3, a recently developed[9] generalized relativistic mean field approach which contains nonlinear couplings of the $\sigma, \omega, \varrho, \delta$ fields and compare its predictions for neutron star properties with the results from current relativistic models. In section 4, we confront results based on the multi-baryon lagrangian density described in the previous section and compare its predictions with estimates obtained within a phenomenological naïve dimensional analysis based on the naturalness of the various coupling constants of the theory. Upon adjusting the model parameters to describe bulk static properties of ordinary nuclear matter, we discuss implications of the approach for dense hadronic matter, in particular for neutron stars.

1 The Modified Fuzzy Bag Model (MFBM): *fitting the masses of the low-lying baryons*

The main idea of the MFBM is to eliminate divergences of physical quantities as, for instance, in the self-energy of the nucleon, due to the characteristic sharp surface appearing in standard bag models. To solve this non-physical difficulty we follow the procedure introduced by Y. Nogami et al[3] making the surface of the bag fuzzy, i.e. we replace the step function $\theta(R-r)$ and the delta function $\delta(R-r)$ in the original version of the MIT-Lagrangian, respectively, by continuous suppression functions $F(r)$ and $G(r)$, which must represent distribution functions. Just like $\partial_r\theta(R-r) = -\delta(R-r)$, we also demand $dF(r)/dr = -G(r)$. In our model, the lagrangian density is[5]

$$\mathcal{L}_{FBM} = \frac{i}{2}[\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - \partial_{\mu}\bar{\psi}\gamma^{\mu}\psi] - B(r)F(r) - \bar{\psi}[m_q + (1+\gamma^0)V(r)]\psi\,. \quad (1)$$

In this expression $V(r) = \frac{1}{2}V_0 + V_c(r)$ is a given spherically symmetric confining potential for a single quark[5,6]. The quadri-divergence of the energy-momentum tensor $\partial_\mu T^{\mu\nu} = 0$ implies $\partial_r\,[B(r)\,F(r)] + \sum_q \bar{\psi}\,(1+\gamma^0)\,\psi\,\partial_r V_c(r) = 0$. We remark that, in contrast to the MIT bag, here the energy-momentum conservation is imposed, rather than derived from requiring the action to be stationary under deformations of the surface of the bag. Moreover, the quark wave functions and $F(r)$ must be self-consistently determined from a set of coupled equations for a given $V_c(r)$, which then enforces a radial dependence on the bag pressure B. We focus on baryons in their ground state (s-wave quarks); thus, the term $\bar{\psi}\,(1+\gamma^0)\,\psi$ exhibits only a radial dependence which reflects the underlying spherical symmetry; for excited states, the admixture of angular momenta quite naturally leads to an angular dependence in B. From the equations above it is possible to determine completely the form of $B(r)$ for a given choice of the suppression function and confining potential. This internal consistency avoids the high degree of arbitrariness, as shown in various approaches, and allows a very natural extrapolation from free baryons into dense hadronic matter, such as encountered in the investigation of neutron star properties.

By demanding the suppression function and the confining potential to be continuous for all values of r, we finally find[2], in the surface region (characterized by two radial parameters R_0, R_1 with $R_0 < r < R_1$), $B(r)$ satisfies

$$B(r) = \frac{1}{\mathcal{F}(r)} \left(B_0 - \frac{1}{2\,\pi} \sum_q \int_{R_0}^{r} \mathrm{d}r \, \frac{u(r)^2}{r^2} \, \frac{d\mathcal{V}_c(r)}{dr} \right) . \tag{2}$$

Due to the continuity of $B(r)$, then, at $R = R_0$, $B_0 = B(R_0)$. Requiring the product $B(r)\,F(r)$ to be finite for all r, from the condition $\mathcal{F}(R_1) = 0$, the expression inside the parenthesis in Eq. (2) has to vanish for $r = R_1$. Thus, as a consequence of energy-momentum conservation, $B(r)$ is completely determined for all values of r. The next step of our calculation is to consider the Euler-Lagrange equations and its solutions. After normalization of the solutions, we determine the baryon masses, given as the sum of the quark energies and the volume contribution

$$M = \sum_q E_q + \frac{2}{3} \sum_q \int_{R_0}^{R_1} \mathrm{d}r \, r \, u(r)^2 \, \frac{d\mathcal{V}_c(r)}{dr} \ . \tag{3}$$

For a more realistic estimate of the masses, we improve this result by including center-of-mass and one-gluon exchange corrections as well as the pion field to respect chiral symmetry. As a result, the pion-quark interaction induces renormalization effects on the hadronic wave functions and observables. Expecting only moderate corrections from the pion cloud, the corresponding calculations are readily performed in a framework similar to the non-relativistic Chew-Low model of the nucleon-pion interaction (here we neglect both baryon recoil and the influence from the vacuum due to the presence of anti-baryons)[2,6]. As a test of the consistency of the model, we provide in Table 1 our results for fitting the masses of the low-lying baryons[2,6]. In Table 2 we have listed the

Table 1. Results for the masses of the low-lying baryons (in MeV).

baryon	N	Δ	Λ	Σ	Σ^*	Ξ	Ξ^*	Ω^-
experim.	940	1232	1116	1193	1392	1318	1533	1672
theory	961	1232	1138	1195	1381	1312	1497	1671

Table 2. Separate contributions to the total mass of each baryon (in MeV).

baryon	N	Δ	Λ	Σ	Σ^*	Ξ	Ξ^*	Ω^-
E_B	960	960	1125	1125	1125	1289	1289	1453
E_{BF}	300	300	274	274	274	249	249	224
$(\Delta E_B)_{cm}$	-126	-126	-104	-104	-104	-88	-88	-77
$(\Delta E_B)_g^M$	-126	126	-126	-82	104	-100	85	71
$(\Delta E_B)_\pi$	-47	-28	-31	-18	-18	-38	-38	0

separate contributions which build up each baryon mass: E_B is the sum of the energy eigenvalues of each quark, E_{BF} is the volume energy due to the term $B(r)\,F(r)$ in the Lagrangian density, $(\Delta E_B)_{cm}$ is the center of mass correction, $(\Delta E_B)_g^M$ is the color magnetic energy, and $(\Delta E_B)_\pi$ is the self-energy due to the pion-quark interaction. Notice that there is a mean deviation of $\sim 13MeV$ between the experimental and the theoretical results in Table (1). This gap could be lowered by taking also into account the color-electric contribution to the energy due to one-gluon exchange.

In the following we consider the modelling of neutron star masses and baryon and lepton populations using the MFBM.

2 The Modified Fuzzy Bag Model (MFBM): *modelling the masses of neutron stars*

The starting point of our analysis is the lagrangian density model formulated in Eq. 1 combined with scalar-isoscalar, vector-isoscalar and vector-isovector meson-quark couplings[8]:

$$\begin{aligned}\mathcal{L}_{FBM}=&\frac{i}{2}[\bar{\psi}_q\gamma^\mu\partial_\mu\psi_q-\partial_\mu\bar{\psi}_q\gamma^\mu\psi_q]-B(r)F(r)-\bar{\psi}_q[m_q-g_\sigma^q\sigma+(1+\gamma^0)V(r)]\psi_q\\&+\frac{i}{2}[\bar{\psi}_q\gamma^\mu(ig_\omega^q\omega_\mu+ig_\rho^q\frac{1}{2}\boldsymbol{\tau}\cdot\boldsymbol{\varrho}_\mu)\psi_q]+\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma-\frac{1}{2}m_\sigma^2\sigma^2\\&-\frac{1}{2}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}m_\omega^2\omega_\mu\omega^\mu-\frac{1}{4}\mathbf{L}_{\mu\nu}\cdot\mathbf{L}^{\mu\nu}+\frac{1}{2}m_\varrho^2\boldsymbol{\varrho}_\mu\cdot\boldsymbol{\varrho}^\mu+\sum_\lambda\bar{\psi}_\lambda[i\gamma_\mu\partial^\mu-m_\lambda]\psi_\lambda\end{aligned} \quad (4)$$

with $F_{\mu\nu}=\partial_\mu\omega_\nu-\partial_\nu\omega_\mu$; $\mathbf{L}_{\mu\nu}=\partial_\mu\boldsymbol{\varrho}_\nu-\partial_\nu\boldsymbol{\varrho}_\mu$. In these expressions: ψ_q, σ, ω and ϱ represent respectively the quark, scalar-isoscalar, vector-isoscalar and vector-isovector meson fields and g_σ^q, g_ω^q, g_ϱ^q are the corresponding coupling constants; moreover, as leptons are an important ingredient for maintaining chemical equilibrium and charge neutrality in neutron stars, these particles are included in the formalism with ψ_λ representing lepton fields. In brief words, the main steps in our formulation involve: a) the determination of the Euler-Lagrange equations and their solutions for a given confining potential or, alternatively, for chosen suppression functions $F(r)$ and $G(r)$ (with appropriate distributional behaviors); b) the calculation of the components of the energy-momentum tensor in the mean-field approximation (in the following discussion in this section, the mean fields σ, ω and ϱ fields are denoted respectively by $\bar{\sigma}$, $\bar{\omega}$ and $\bar{\varrho}$; for the details see[8,10]); c) the determination of the EoS (EoS) for nuclear matter.

The Pauli principle induces Fermi motion of the nucleons. Thus the energy of a moving bag with momentum $\mathbf{k}$ is

$$\varepsilon(\mathbf{k}) = \sqrt{M_N^{*2} + \mathbf{k}^2} + 3g_\omega^q \bar{\omega} \pm \frac{1}{2} g_\varrho^q \bar{\varrho} \tag{5}$$

where (+) refers to protons (p) and (−) to neutrons (n) and M_N^* to the nucleon effective mass (see discussion below) ($N = n, p$).

The time-like mean-field component of the vector field is determined by imposing baryon number conservation from which we get $\bar{\omega} = g_\omega \rho_B / m_\omega^2$, with $g_\omega = 3g_\omega^q$, while the $\bar{\sigma}$ and $\bar{\varrho}$ mean-fields are determined from the thermodynamic condition:

$$\left(\frac{\partial E_{tot}}{\partial \bar{\sigma}}\right)_{R,\rho_B} = \left(\frac{\partial E_{tot}}{\partial \bar{\rho}}\right)_{R,\rho_B} = 0 \;\; ; \;\; \bar{\varrho} = \frac{g_\varrho}{2m_\varrho^2} \rho_3 \tag{6}$$

applied to the total energy per nucleon E_{tot} of nuclear matter[10]. We obtain

$$\bar{\sigma} = -\frac{\gamma}{(2\pi)^3 m_\sigma^2} \left(\int^{k_{F_p}} d\mathbf{k}_p \frac{M_p^*}{\sqrt{M_p^{*2} + \mathbf{k}_p^2}} \left(\frac{\partial M_p^*}{\partial \bar{\sigma}}\right)_R + \int^{k_{F_n}} d\mathbf{k}_n \frac{M_n^*}{\sqrt{M_n^{*2} + \mathbf{k}_n^2}} \left(\frac{\partial M_n^*}{\partial \bar{\sigma}}\right)_R \right) \tag{7}$$

and

$$E_{tot} = \frac{\gamma}{\rho_B (2\pi)^3} \left(\int^{k_{F_p}} d\mathbf{k}_p \sqrt{M_p^{*\,2} + \mathbf{k}_p^2} + \int^{k_{F_n}} d\mathbf{k}_n \sqrt{M_n^{*\,2} + \mathbf{k}_n^2} \right) + \frac{m_\sigma^2}{2\rho_B} \bar{\sigma}^2 + \frac{g_\omega^2}{2m_\omega^2} \rho_B + \frac{g_\varrho^2}{8m_\varrho^2 \rho_B} \rho_3^2 . \tag{8}$$

The pressure can be found from $p = \rho_B^2 (\partial E_{tot} / \partial \rho_B)$, from which we get the EoS of nuclear matter. In this expression, $\varrho_3 = \varrho_p - \varrho_n$, $m_\sigma = 550 MeV$, $m_\omega = 783 MeV$ and $m_\varrho = 770 MeV$. Moreover,

$$\left(\frac{\partial M_N^*}{\partial \bar{\sigma}}\right)_R = -g_\sigma \frac{E_{bag}}{M_N^*} \left[\left(1 - \frac{E_q}{E_{bag} R}\right) S(\bar{\sigma}) + \frac{m_q^*}{E_{bag}} \right] = -g_\sigma C_N(\sigma) \tag{9}$$

with $g_\sigma \equiv 3g_\sigma^q S_N(0)$, $C_N(\sigma) \equiv S_N(\sigma)/S_N(0)$ and

$$S(\bar{\sigma}) = \frac{E_q/2 + R m_q^*(E_q - 1)}{E_q(E_q - 1) + R m_q^*/2} . \tag{10}$$

E_{bag} is defined, from an equation similar to 3, as

$$E_{bag} = M_N = \sum_q E_q + \frac{2}{3} \sum_q \int_{R_0}^{R_1} \mathrm{d}r \, r \, u(r)^2 \, \frac{d\mathcal{V}_c(r)}{dr} - \frac{Z}{R} \tag{11}$$

where the Z parameter accounts for the energy of the zero-point motion and m_q^* is the quark effective mass $m_q^* = m_q - g_\sigma^q \bar{\sigma}$. Details on the quark substructure of the nucleons are thus entirely contained in the effective coupling $(\partial M_N^*/\partial\bar{\sigma})_R$. The coupling constants, $g_\sigma = 3g_\sigma^q$ and g_ω, are determined to fit the binding energy of nuclear matter (-16 MeV) at the saturation density, ρ_0=0.17 fm^{-3}. If the nucleon were simply made of three massive constituent quarks, the nucleon mass in vacuum, M_N, and that in the meson condensate, M_N^*, would have to satisfy $M_N \approx 3m_q$, $M_N^* \approx 3m_q^*$ and hence the effective nucleon mass would be[12] $M_N^* = M_N - g_\sigma\bar{\sigma}$ where we have defined $g_\sigma = 3g_\sigma^q$. Since one finds, from the expression of the nucleon effective mass, $(\partial M_N^*/\partial\bar{\sigma}) = -g_\sigma C_N(\sigma)$, the following self-consistent condition (SCC) then holds

$$g_\sigma\bar{\sigma} = -\frac{\gamma g_\sigma}{(2\pi)^3 m_\sigma^2}\left(\int^{k_{F_p}} d\mathbf{k}_p \frac{M_p^*}{\sqrt{M_p^{*2}+\mathbf{k}_p^2}}\left(-3g_\sigma^q S_p(\sigma) = -g_\sigma C_p(\sigma)\right)_R + \int^{k_{F_n}} d\mathbf{k}_n \frac{M_n^*}{\sqrt{M_n^{*2}+\mathbf{k}_n^2}}\left(-3g_\sigma^q S_n(\sigma) = -g_\sigma C_n(\sigma)\right)_R\right) \quad (12)$$

with the multiplicity $\gamma = 2$ for neutron matter and $\gamma = 4$ for nuclear matter. The effective nucleon mass is, in this case, given by $M_N^* = M_N - 3g_\sigma^q S_N(0)\,\bar{\sigma}$ or equivalently by $M_N^* = M_N - (C_N(\sigma)/S_N(\sigma))\,\bar{\sigma}$. Neutron star configurations may then be obtained by combining the resulting EoS for nuclear matter with the TOV equations for the general relativity metric[a], chemical equilibrium[b], baryon number and electric charge conservation[c]. The results indicate the consistency of our modelling as shown in Fig. 1. In particular, our results indicate a maximum mass for neutron stars smaller than typical QHD predictions but far yet from the most acceptable results. We expect to improve the predictions by considering a more appropriate parameterization of our modelling.

[a]The TOV equations describe the structure of a static, spherical and isotropic star with the pressure $p(r)$ and the energy density $\epsilon(r)$ correlated from the underlying nuclear model for an interior mass $M(r)$ inside a sphere of radius r; these equations involve various constraints and boundary conditions: they must be evaluated for the initial condition $\epsilon(0) = \epsilon_c$ (central star density) and $M(0) = 0$ at $r = 0$; the radius R of the star is determined under the condition that on its surface the pressure vanishes ($p(r)|_{r=R} = 0$).
[b]Chemical equilibrium conditions for neutron stars are: $\mu_i = b_i\mu_n - q_i(\mu_\ell - \mu_{\nu_\ell})$ where μ_i and μ_ℓ stand for the baryon and lepton chemical potentials, respectively; b_i is the baryon number; the baryon and lepton electrical charges are represented by q_i.
[c]Baryon number and electric charge conservation equations: $\rho_B = \sum_B k_{F,B}^3/3\pi^2$, and $\sum_B q_{e,B} k_{F,B}^3/3\pi^2 - \sum_\ell k_{F,\ell}^3/3\pi^2$.

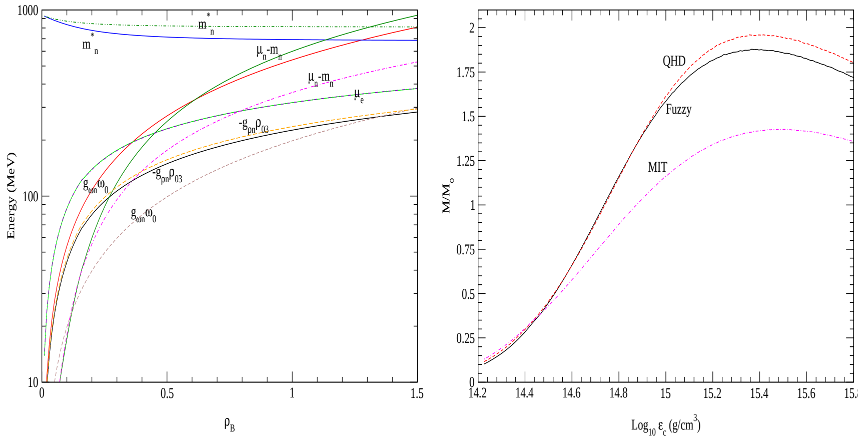

Figure 1. On the left panel: compared are the results of the field amplitudes and chemical potentials for neutron star matter (with the composition $n + p + e + \mu$) in chemical equilibrium; the results correspond to the MIT bag model (dashed lines) and the fuzzy bag model with $\lambda = 0.05 fm^{-3}$ (solid lines). On the right panel: maximum mass of neutron matter as predicted by the fuzzy bag model with $\lambda = 0.05 fm^{-3}$ (solid line) compared to the corresponding results obtained with the Glendenning and MIT bag models.

In the following section, to improve our predictions for global static properties of neutron stars, we introduce a new QHD model for nuclear star matter which contains nonlinear self-coupling interaction terms involving the σ and δ mesons.

3 Nonlinear σ, δ Couplings in a Relativistic Mean Field Theory for Neutron Stars

As the basic new feature of our approach, we expand the scalar sector of conventional QHD models, by including in addition to the scalar-isoscalar σ-meson, the scalar-isovector δ-meson. Such an extension is supported mainly from the large isospin-asymmetry in neutron matter. Guided by a previous treatment[11], we start in the baryon sector, with a new version of the derivative coupling model[11] which describes a system of baryons coupled to four mesons (σ, ω, ϱ and δ) and two leptons (e, μ) and which contains a gradient coupling involving baryons and the scalar-isoscalar and scalar-isovector meson fields.

	M	J	I	n	q	s
N	939	1/2	1/2	1	$0, 1$	0
Λ	1115	1/2	0	1	0	-1
Σ	1190	1/2	1	1	$-1, 0, 1$	-1
Ξ	1315	1/2	1/2	1	$-1, 0$	-2
σ	800	0	0	0	0	0
ω	782	1	0	0	0	0
ϱ	770	1	1	0	$-1, 0, 1$	0
δ	980	0	1	0	$-1, 0, 1$	0

B	τ_3	M_B^*
p	$+1$	$M_p^* = M_p \left(1 + \frac{g^\star_{\sigma p}\sigma + g^\star_{\delta p}\delta_3}{\alpha M_p}\right)^{-\alpha}$
n	-1	$M_n^* = M_n \left(1 + \frac{g^\star_{\sigma n}\sigma - g^\star_{\delta n}\delta_3}{\alpha M_n}\right)^{-\alpha}$
Λ	0	$M_\Lambda^* = M_\Lambda \left(1 + \frac{g^\star_{\sigma\Lambda}\sigma}{\alpha M_\Lambda}\right)^{-\alpha}$
Σ^+	$+1$	$M_\Sigma^* = M_\Sigma \left(1 + \frac{g^\star_{\sigma\Sigma}\sigma + g^\star_{\delta\Sigma}\delta_3}{\alpha M_\Sigma}\right)^{-\alpha}$
Σ^0	0	$M_\Sigma^* = M_\Sigma \left(1 + \frac{g^\star_{\sigma\Sigma}\sigma}{\alpha M_\Sigma}\right)^{-\alpha}$
Σ^-	-1	$M_\Sigma^* = M_\Sigma \left(1 + \frac{g^\star_{\sigma\Sigma}\sigma - g^\star_{\delta\Sigma}\delta_3}{\alpha M_\Sigma}\right)^{-\alpha}$
Ξ^-	-1	$M_\Xi^* = M_\Xi \left(1 + \frac{g^\star_{\sigma\Xi}\sigma - g^\star_{\delta\Xi}\delta_3}{\alpha M_\Xi}\right)^{-\alpha}$
Ξ^0	0	$M_\Xi^* = M_\Xi \left(1 + \frac{g^\star_{\sigma\Xi}\sigma}{\alpha \Xi}\right)^{-\alpha}$

Table 3. On the left, baryons and mesons considered in this work. The symbols represent: M: mass; J: spin; I: isospin; n: baryon number; q: electric charge; s: strangeness. On the right, third component of isospin and effective masses for the baryon fundamental octet.

The lagrangian density in our approach is

$$\begin{aligned}
\mathcal{L} = \sum_B \Bigg\{ \bar{\psi}_B \Bigg(\left[1 + \frac{(g_\sigma\sigma + g_\delta \vec{\tau}_B \cdot \vec{\delta})}{\alpha M_B}\right]^{\alpha} \gamma_\mu \left(i\partial^\mu - g_\omega \omega^\mu + \frac{1}{2} g_\varrho \boldsymbol{\tau}_B \cdot \boldsymbol{\varrho}^\mu \right) \\
- \left(1 - \frac{g_\sigma \sigma}{M_B}\right)^{\eta} M_B \Bigg) \psi_B + \frac{1}{2}\left(1 + \frac{(g_\sigma\sigma + g_\delta \vec{\tau}_B \cdot \vec{\delta})}{\beta M_B}\right)^{2\beta} \left(\frac{1}{2} {m_\omega}^2 \omega_\mu \omega^\mu - \frac{1}{4}\omega_{\mu\nu}\right) \\
+ \frac{1}{2}\left(1 + \frac{(g_\sigma\sigma + g_\delta \vec{\tau}_B \cdot \vec{\delta})}{\gamma M_B}\right)^{2\gamma} \left(\frac{1}{2} m_\varrho^2 \boldsymbol{\varrho}_\mu \cdot \boldsymbol{\varrho}^\mu - \frac{1}{4} \boldsymbol{\varrho}_{\mu\nu} \cdot \boldsymbol{\varrho}^{\mu\nu}\right) \Bigg\} \\
+ \left(\frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} {m_\sigma}^2 \sigma^2\right) + \left(\frac{1}{2}\partial_\mu \vec{\delta} \cdot \partial^\mu \vec{\delta} - \frac{1}{2} {m_\delta}^2 \vec{\delta}^2\right) \\
+ \sum_\lambda \bar{\psi}_\lambda [i\gamma_\mu \partial^\mu - m_\lambda] \psi_\lambda \qquad (13)
\end{aligned}$$

where $\omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ and $\boldsymbol{\varrho}_{\mu\nu} = \partial_\mu \boldsymbol{\varrho}_\nu - \partial_\nu \boldsymbol{\varrho}_\mu$.

The different steps of our formulation may be synthesized in the following.

B	τ_3	M_B^*
p	$+1$	$M_p^* \simeq M_p - g_{\sigma p}^{\star}\sigma - g_{\delta p}^{\star}\delta_3$
n	-1	$M_n^* \simeq M_n - g_{\sigma n}^{\star}\sigma + g_{\delta n}^{\star}\delta_3$
Λ	0	$M_\Lambda^* \simeq M_\Lambda - g_{\sigma\Lambda}^{\star}\sigma$
Σ^+	$+1$	$M_\Sigma^* \simeq M_\Sigma - g_{\sigma\Sigma}^{\star}\sigma - g_{\delta\Sigma}^{\star}\delta_3$
Σ^0	0	$M_\Sigma^* \simeq M_\Sigma - g_{\sigma\Sigma}^{\star}\sigma$
Σ^-	-1	$M_\Sigma^* \simeq M_\Sigma - g_{\sigma\Sigma}^{\star}\sigma + g_{\delta\Sigma}^{\star}\delta_3$
Ξ^-	-1	$M_\Xi^* \simeq M_\Xi - g_{\sigma\Xi}^{\star}\sigma + g_{\delta\Xi}^{\star}\delta_3$
Ξ^0	0	$M_\Xi^* \simeq M_\Xi - g_{\sigma\Xi}^{\star}\sigma$

B	τ_3	$g_{\kappa\epsilon B}^*$
p	$+1$	$g_{\sigma\alpha p}^* = g_\sigma \left(1 + \frac{g_{\sigma p}^{\star}\sigma + g_{\delta p}^{\star}\delta_3}{\alpha M_p}\right)^{-\alpha}$
n	-1	$g_{\sigma\alpha n}^* = g_\sigma \left(1 + \frac{g_{\sigma n}^{\star}\sigma - g_{\delta n}^{\star}\delta_3}{\alpha M_n}\right)^{-\alpha}$
Λ	0	$g_{\sigma\alpha\Lambda}^* = g_\sigma \left(1 + \frac{g_{\sigma\Lambda}^{\star}\sigma}{\alpha M_\Lambda}\right)^{-\alpha}$
Σ^+	$+1$	$g_{\sigma\alpha\Sigma^+}^* = g_\sigma \left(1 + \frac{g_{\sigma\Sigma}^{\star}\sigma + g_{\delta\Sigma}^{\star}\delta_3}{\alpha M_\Sigma}\right)^{-\alpha}$
Σ^0	0	$g_{\sigma\alpha\Sigma^0}^* = g_\sigma \left(1 + \frac{g_{\sigma\Sigma}^{\star}\sigma}{\alpha M_\Sigma}\right)^{-\alpha}$
Σ^-	-1	$g_{\sigma\alpha\Sigma^-}^* = g_\sigma \left(1 + \frac{g_{\sigma\Sigma}^{\star}\sigma - g_{\delta\Sigma}^{\star}\delta_3}{\alpha M_\Sigma}\right)^{-\alpha}$
Ξ^-	-1	$g_{\sigma\alpha\Xi^-}^* = g_\sigma \left(1 + \frac{g_{\sigma\Xi}^{\star}\sigma - g_{\delta\Xi}^{\star}\delta_3}{\alpha M_\Xi}\right)^{-\alpha}$
Ξ^0	0	$g_{\sigma\alpha\Xi^0}^* = g_\sigma \left(1 + \frac{g_{\sigma\Xi}^{\star}\sigma}{\alpha\Xi}\right)^{-\alpha}$

Table 4. On the left, third component of isospin and effective masses for the baryon octet considered in this work (taking $\alpha = 1$, $\eta = 0$ and $g_\delta = 0$, up to first order in σ, or considering alternatively $\alpha = 0$ and $\eta = 1$.). On the right, as in table 2, however for the scalar coupling constant (the other effective coupling constants follow a similar pattern).

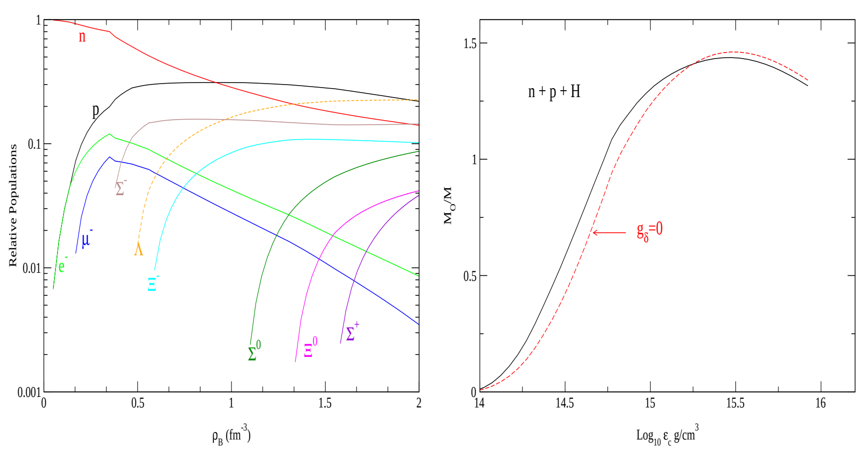

Figure 2. On the left, relative populations for nuclear matter and on the right, predictions for the maximum mass of neutron stars for $g_\delta = 2.39$.

We first introduce a re-scaling of the baryons and the ω and ϱ meson fields:

$$\phi \to \left[1 + \frac{(g_\sigma\sigma + g_\delta\vec{\tau}_B\cdot\vec{\delta})}{\alpha M_B}\right]^{-\alpha/2} \phi \tag{14}$$

with $\phi = \psi_B, \omega^\mu, \boldsymbol{\varrho}^\mu$. Then we combine these expressions with the previous ones and define new coupling constants: $g^\star_{\alpha\sigma B} \equiv m^\star_{\alpha B} g_\sigma$; $g^\star_{\beta\omega B} \equiv m^\star_{\beta B} g_\omega$; $g^\star_{\gamma\varrho B} \equiv m^\star_{\gamma B} g_\varrho$; $g^\star_{\alpha\delta B} = m^\star_{\alpha B} g_\delta$, with $m^\star_{\kappa B} \equiv (1 + (g_\sigma\sigma + g_\delta \tau_{3B}\delta_3)/\kappa M)^{-\kappa}$ $(\kappa = \alpha, \beta, \gamma)$, where τ_3 is the third component of the isospin vector $\vec{\tau}$ associated to baryon B and σ and δ_3 represent the mean field components of the σ and δ. We obtain

$$\begin{aligned}\mathcal{L} = \sum_B \bar{\psi}_B \Big\{ \gamma_\mu \Big(i\partial^\mu - g^\star_{\omega B}\omega^\mu - \frac{1}{2} g^\star_{\varrho B} \boldsymbol{\tau}_B \cdot \boldsymbol{\varrho}^\mu \Big) \\ - \Big(1 - \frac{g_\sigma \sigma}{M_B}\Big)^\eta m^\star_{\alpha B} M_B \Big\} \psi_B + \Big(\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}{m_\sigma}^2\sigma^2 \Big) \\ + \frac{1}{2}(\partial_\mu \vec{\delta}\cdot\partial^\mu\vec{\delta} - {m_\delta}^2\vec{\delta}^2) + \Big(\frac{1}{2}{m_\omega}^2\omega_\mu\omega^\mu - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} \Big) \\ + \Big(\frac{1}{2} m_\varrho^2 \boldsymbol{\varrho}_\mu \cdot \boldsymbol{\varrho}^\mu - \frac{1}{4}\boldsymbol{\varrho}_{\mu\nu}\cdot\boldsymbol{\varrho}^{\mu\nu} \Big) + \sum_\lambda \bar{\psi}_\lambda \left(i\gamma_\mu\partial^\mu - m_\lambda \right)\psi_\lambda \,. \end{aligned} \qquad (15)$$

From this expression, the effective nucleon mass is defined as

$$M^*_N = M_N \left(1 - \frac{g_\sigma\sigma}{M_N}\right)^\eta \left[1 + \frac{(g_\sigma\sigma + g_\delta \vec{\tau}_N \cdot \vec{\delta}))}{\alpha M_N}\right]^{-\alpha} . \qquad (16)$$

We notice that for $\alpha = 1$, $\eta = 0$ and $g_\delta = 0$, up to first order in σ, the rescaled lagrangian gives for the effective nucleon mass the well known Yukawa minimal coupling

$$M^*_N \simeq M_N - g_\sigma\sigma \,. \qquad (17)$$

Similarly, taking $\alpha = 0$ and $\eta = 1$ the same result holds.

The inclusion of the δ meson deserves some comments. Its mass is very high when compared to the nucleon mass, corresponding to a length scale of the strong interaction of the order of less than 0.4 Fermi. In most models found in the literature, the relevant physical phenomena are restricted to longer length scales ($> 0.5 Fermi$) with the dynamics at shorter length scales, — after integrating out the contributions corresponding to heavier meson degrees of freedom —, implicitly taken into account in the various coupling parameters of the theory.

Assuming the coupling constants are *natural* (see discussion in the next section), it would be possible in a consistent way to truncate in any desired order any expansion of the lagrangian density just by counting powers of the expansion parameters. A crucial aspect in order to accomplish convergence is to assume the expansion parameters are small in the desired physical domain.

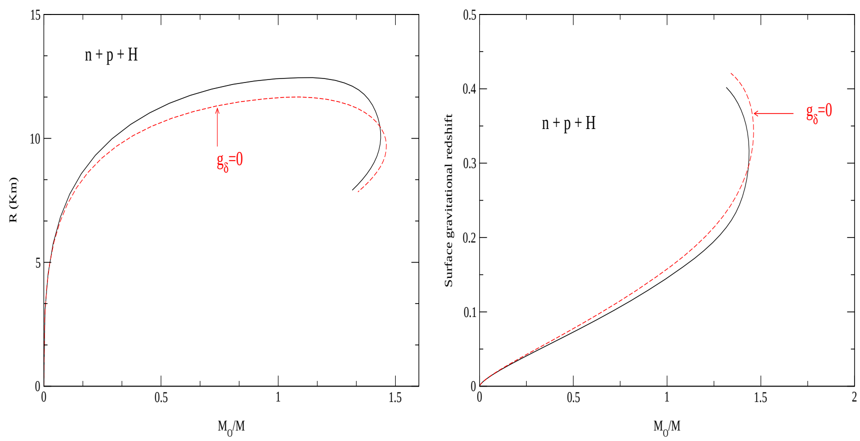

Figure 3. On the left, mass-radius relation for neutron stars and on the right, predictions for the stellar redshift for $g_\delta = 2.39$.

The inclusion of the δ meson in this formalism may represent, — as it may test the dynamics at shorter length scales, — a more accurate indication of the degree of naturalness of current effective models. Moreover, similarly to the σ meson, the δ meson couples to the baryon masses introducing baryonic splittings whose signals depend on the value of the third component of the baryonic isospin. These splittings correspond to mixed attractive and repulsive strong interaction components into the formalism and may cause a negative contribution to the symmetry energy of nuclear matter, reducing the gap between the Fermi energies of baryon isospin multiplets (n and p; Σ^+, Σ^0 and Σ^-; $\Xi^-$and Ξ^0). Its presence may thus imply in an attenuation of the repulsive interaction with increasing baryon density contributing for the stability of nuclear matter. Typical results in our formulation are shown in Figs. 2 and 3. In particular, our predictions for neutron star properties are in good agreement with the results of Vela X-1.

In the next section we consider the naturalness of effective field theory as a possible theoretical constraint to the prediction of neutron star observables in relativistic effective field theory.

4 Naturalness in the Nonlinear σ, δ Self-Coupling Sectors in a Relativistic Mean Field Effective Theory for Neutron Stars

In building up an effective field theory for the nuclear many-body pro-

Table 5. Values of coupling constants, nucleon effective mass and compression modulus of nuclear matter at saturation density (models I (with the δ meson) and II (without the δ meson) (see Ref.[8]). (saturation density: $\rho_0 = 0.16 fm^{-3}$; binding energy at saturation density: $B(E) = -16 MeV$; asymmetry coefficient $a_4 = 32.5 MeV$.)

Model	$(\frac{g_\sigma}{m_\sigma})^2$ (fm^2)	$(\frac{g_\omega}{m_\omega})^2$ (fm^2)	$(\frac{g_\varrho}{m_\varrho})^2$ (fm^2)	$(\frac{g_\delta}{m_\delta})^2$ (fm^2)	$M^\star/M$	K (MeV)
I	7.50	2.62	10.63	2.39	0.85	224.5
II	7.50	2.62	4.76	0.00	0.85	224.5

blem, a natural way to classify their contributions is to expand the lagrangian density in terms of the characteristic scales of QCD[13,14,15]. By restricting the system to a certain limited physical domain, the relevant physical phenomena are in general dominated by a few specific long-range degrees of freedom, while the dynamics at shorter length scales, which corresponds to heavier physical degrees of freedom, may be integrated out and thus not explicitly taken into account into the formalism. The effects of these heavier degrees of freedom are, however, implicitly absorbed in the various coupling parameters of the theory. While there is strong evidences, from studies of ordinary nuclear matter, that the expansion in the nonlinear mesonic couplings quickly converges, — keeping only cubic and quartic order self-couplings of the σ meson field for instance provides a semi-quantitative fit to nuclear matter data[15] —, a controlled and useful extension to significantly higher densities ($\rho \geq 5\rho_0$) requires some assumption on the ordering of the expansion coefficients.

For the sake of consistency, expanding the lagrangian density of the effective theory order by order, we assume the unknown dimensionless coupling parameters of the theory are *natural*, i. e. of order unity. This assumption (or a similar one), *naturalness*, makes it possible, in a proper and accurate way, to formally sum up in a compact form, to any desired order, an expansion of the lagrangian density, — which in principle would contain an infinite number of terms. Assuming naturalness of the expansion scheme, it necessarily converges for sufficiently low nuclear densities and can be analytically continued to any desired density of hadronic matter.

Evidently, different expansion *organizing* schemes are possible: focusing on quarks degrees of freedom, a fundamental scale in QCD is the renormalization invariant parameter $\Lambda_{QCD} \sim 200 MeV$, or the number of colors of quarks, N_c, reminiscent of the SU(3) symmetry group structure of QCD. However, when focusing on meson and baryons as effective low energy degrees of freedom (equivalently realized in the large N_c limit as a result of chiral symmetry breaking), the appropriate scales are the low-energy chiral parameters of QCD, i. e. the weak pion decay constant f_π or equivalently the chiral parameter Λ: $f_\pi = 93MeV$, $\Lambda_\chi \sim 1GeV \leq 4\pi f_\pi$.

For the accurate development of any effective theory, there is a crucial and non trivial aspect which should be addressed to as a first step: how to assign dimension scales to each contribution in the lagrangian density in order to keep the various coupling constants of the theory both dimensionless and *natural*? We follow here the naïve dimensional analysis (NDA), a procedure proposed by Manohar and Georgi[13,16], which assigns a mass scale of appropriate size to any term in an effective lagrangian density. The NDA procedure when applied in the formulation of a lagrangian density with nucleons and strongly interacting mesonic fields may be synthesized as follows: the amplitude of each strongly interacting field in the lagrangian, i.e. the meson fields, becomes dimensionless when divided by the pion decay weak constant. Furthermore, to obtain the correct dimension ((energy)^4) for the lagrangian density, an overall normalization scale $f_\pi^2 \Lambda_\chi^2 \simeq f_\pi^2 M^2$, with M denoting the nucleon mass, has to be included. Finally, for identical meson fields self-interacting terms of power n, a symmetrization factor $n!$, for proper counting, should be included.

A corresponding interaction lagrangian which involves the isoscalar-scalar meson field σ and the isoscalar-vector meson field ω coupled to the nucleon field is then defined in effective field theory as [14,15,16]

$$L_{eff} = \sum_{i,k} \frac{c_{i,k}}{i!k!} (\frac{\sigma}{f_\pi})^i (\frac{\omega}{f_\pi})^k \left(\frac{\partial \, or \, m_\pi}{M} \right) (\frac{\bar{\psi}\Gamma\psi}{f_\pi^2 M})^\ell f_\pi^2 \Lambda^2 \tag{18}$$

with unknown expansion coefficients (coupling constants) $c_{i,k}$ (a direct generalization of this expression may involve additional mesonic fields such as the π, ϱ and δ meson fields as well as the photon). In the expression above ψ represents a baryon field, Γ is a Dirac matrix and derivatives are denoted by ∂. The coupling constants are dimensionless and of order $\mathcal{O}(1)$ if naturalness holds. In the *natural* limit, using the Goldberger-Treiman relation and the Brown-Rho scaling, the effective field theory gives rise to the so called exponential coupling:

$$L_{eff}(c_{i,k} = 1) \longrightarrow \exp(\frac{\sigma}{M}) \exp(\frac{\omega}{M}) \left(\frac{\partial \, or \, m_\pi}{M} \right) (\frac{\bar{\psi}\Gamma\psi}{f_\pi^2 M})^\ell f_\pi^2 \Lambda^2 \,. \tag{19}$$

To proceed with our analysis of naturalness, with a full QCD calculation presently beyond any reach, we consider the *natural limit* of the model developed in the previous section and confront its predictions, for different values of the parameters α, β, γ and κ, with the corresponding ones in the natural limit:

$$g^{\star}_{\kappa\lambda B}|_{\kappa\to\infty} \to \exp\,(\frac{-g_\sigma\sigma + g_\delta\vec{\tau}_{3B}\cdot\vec{\delta}_3}{M_B})g_{\lambda B} \tag{20}$$

with $\lambda = \sigma, \omega, \varrho, \delta$, which gives in the mean-field approach

$$\begin{aligned}\mathcal{L} &= \sum_B \bar{\psi}_B \exp\,(\frac{-g_\sigma\sigma + g_\delta\tau_{3B}\delta_3}{M_B})\left\{\gamma_\mu\left(i\partial^\mu - g_{\omega B}\omega^\mu - \frac{1}{2}g_{\varrho B}\boldsymbol{\tau}_B\cdot\boldsymbol{\varrho}^\mu\right)\right.\\ &- \left.\left(1-\frac{g_\sigma\sigma}{M_B}\right)^\eta M_B\right\}\psi_B + \left(\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}{m_\sigma}^2\sigma^2\right)\\ &+ \left(\frac{1}{2}\partial_\mu\vec{\delta}\cdot\partial^\mu\vec{\delta} - \frac{1}{2}{m_\delta}^2\vec{\delta}^2\right) + \left(\frac{1}{2}{m_\omega}^2\omega_\mu\omega^\mu - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu}\right)\\ &+ \left(\frac{1}{2}m_\varrho^2\boldsymbol{\varrho}_\mu\cdot\boldsymbol{\varrho}^\mu - \frac{1}{4}\boldsymbol{\varrho}_{\mu\nu}\cdot\boldsymbol{\varrho}^{\mu\nu}\right) + \sum_\lambda \bar{\psi}_\lambda\left(i\gamma_\mu\partial^\mu - m_\lambda\right)\psi_\lambda\,. \end{aligned}\tag{21}$$

The results shown in Figs. 4 and 5 clearly indicate that, if naturalness represents a fundamental constraint in relativistic field theory, it may help to discriminate current models of neutron star matter. Our results also indicate that our modelling represents a potential candidate, concerning naturalness, to constrain neutron star matter observables.

5 Conclusion

The approaches presented in sections 1 and 2 represent first steps in current attempts to improve the modelling of quark dynamics and confinement beyond standard bag model frameworks, bridging the gap to sigma, color-chromo-dielectric or conventional quark-meson coupling models (for details see references in[2]). From its very nature, — in particular its consistency on formulating the dynamics of the bag surface and its incorporation of chiral symmetry,— it is a promising step for further detailed extensions and applications are for example the investigation of baryon form factors at large momentum transfers, or the immersion of the quark bag into hadronic matter towards a more realistic investigation of the transition of effective (mesons and baryons) to constituent (effective) quark and gluons degrees of freedom in the high density domain of nuclear matter. The results of our nonlinear approach presented in sections 3 and 4 show that, if naturalness holds, it may represent an important constraint in effective field theory for the determination of the mass and other properties of neutron stars. On this respect, our results indicate, in the comparison with other QHD models found in the literature,

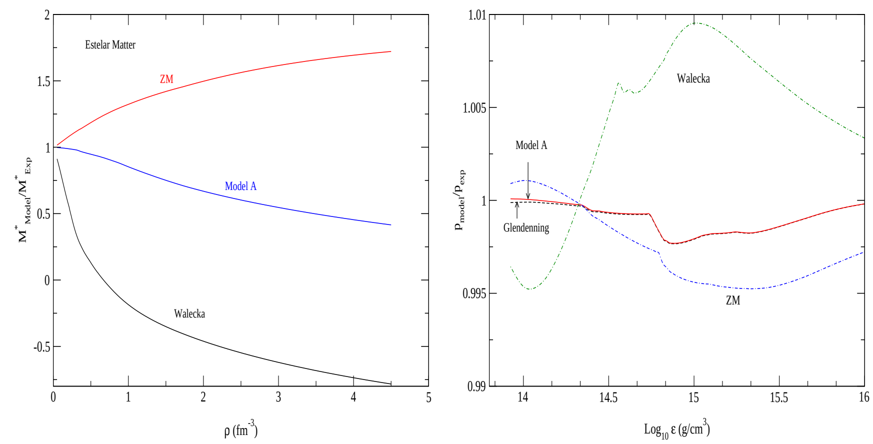

Figure 4. On the left panel: ratios between the predictions involving values for the effective nucleon mass as obtained in different models (M^*_{Model}) and the exponential one (M^*_{Exp}). The figure shows the results corresponding to our model (Model A) in comparison to the results of the ZM and Walecka models. On the right panel: ratios between the predictions involving values for the EoS of nuclear matter as obtained in different models (P_{Model}) and the exponential one (P_{Exp}). The figure shows the results corresponding to our model (Model A) in comparison to the results of the ZM, Glendenning and Walecka models.

the potentialities of our modelling to constrain neutron star matter and for making predictions, in a more consistent way, on global static properties of neutron stars.

Acknowledgements

César A. Zen Vasconcellos acknowledges the *Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES)*, Brazil and the *Instituto de Cibernética, Matemática y Física (ICIMAF)*, Cuba, for financial support for his participation on the *International Workshop on Strong Magnetic Fields and Neutron Stars*. The authors wish also to thank Hugo Pérez Rojas and Herman Mosquera Cuesta for the invitation to participate in this event as well as Aurora, Hugo, Zochil, Betty, Armando, Maria, Joanna and Eloísa for their kind hospitality.

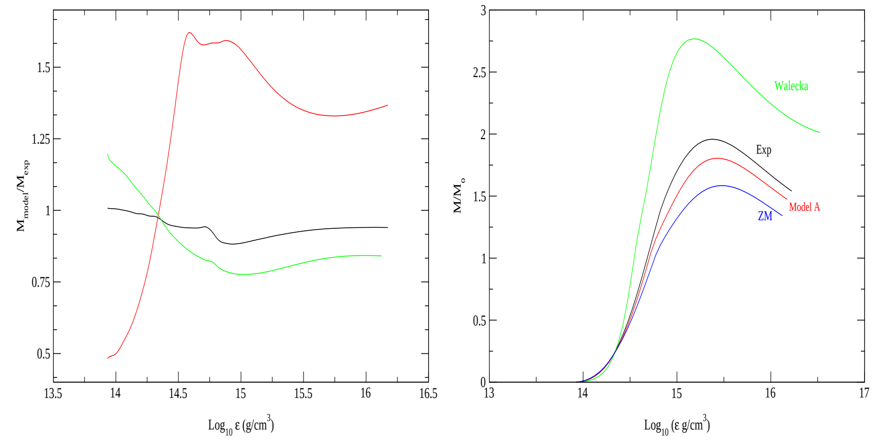

Figure 5. On the left, ratios between the predictions involving values for the mass of a neutron star as obtained in different models (M_{Model}) and the exponential one (M_{Exp}). The figure shows the results corresponding to our model (Model A) in comparison to the results of the ZM, Glendenning and Walecka models. On the right, predictions for the mass of a neutron star. The figure shows the results corresponding to our model (Model A) in comparison to the results of the ZM, Exponential and Walecka models.

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