SCALAR FIELDS AND MODELS OF THE UNIVERSE

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In this work, a few models of the Universe which include, in particular, dark energy are presented. In some of them, dark energy is considered to be only a slowly rolling scalar field (quintessence), while in others, its presence is simulated through a negative cosmological constant. The scalar field is minimally coupled to gravity and does not interact with matter, so these models are to be used after decoupling of radiation and matter. However, this ansatz, and a proper normalization, allow to find new general classes of solutions for the cosmological equations. The inclusion of the negative cosmological constant results in the possibility of the future gravitational collapse of the universe; this inclusion solves the horizon problem which prevents the consistent formulation of string theory.

1 Introduction

From 1998 on, several important discoveries in astrophysics gave rise to the so called New Cosmology[1,2]. Based on observation, this branch of cosmology states, among other aspects, a flat (with a critical density) and accelerating Universe. Moreover, it states that the energy budget is, for dark energy, 2/3 and for matter, 1/3 (mostly dark matter).

The leading candidates to be identified with dark energy are the vacuum energy, parameterized by a cosmological constant, quintessence, simulated by a slowly rolling scalar field (the quintessence field), and a network of light and frustrated topological defects.

The cosmological constant can be incorporated into the quintessence potential as a constant which shifts the minimum of the potential, where the quintessence field rolls towards. Conversely, the minimum of the potential can itself also be regarded as a part of the cosmological constant. The most common procedure for separating these ingredients is to incorporate the, — probably nonzero —, minimum of the potential into the cosmological constant and then set that minimum equal to zero. The cosmological constant can be provided by assuming different modelings for the vacuum energy, such as the vacuum energy of quantum fields, the potential energy of classical fields and it may also even be associated to the intrinsic geometry of the Universe. So far, there are no sufficient reasons to set the cosmological constant (or the minimum of the quintessence potential) equal to zero.

The main point of this work is to draw attention to the fact that, as shown in Ref.[3], the ultimate fate of the Universe is, in a rather general context, more sensitive to the presence of the cosmological constant Λ field (the minimum of the potential) than to any other matter content. We have proved this assertion in some of the models we have work out in our research team in Las Villas Central University, Cuba.

On the other hand, the conception of an eternally accelerating Universe seems to be, because of the impossibility of formulating a corresponding S-matrix, not completely consistent with the predictions of String Theory. In a *de Sitter* space, the presence of an event horizon, corresponding to causally disconnected regions of space, implies the absence of asymptotic particle states which are needed to define transition amplitudes in order to build up the components of the S matrix. The inclusion of a negative cosmological constant in our models warrants that the present stage of accelerated expansion will be, eventually, followed by a period of collapse into a final cosmological singularity (AdS universe), thus eliminating this problem.

This contribution contains, basically, the main aspects related to our approach to this problem (for the details see Refs.[4,5]).

2 The Model

Our modelling is, in general grounds, a 3-component pie:

- Pressureless matter or dust (baryons and cold dark matter).
- Scalar field (quintessence with an exponential potential).
- A cosmological constant Λ.

The scalar field is minimally coupled and non-interacting with matter, so the action is:

$$S = \int d^4x \, \sqrt{-g} \{ \frac{c^2}{16\pi G}(R - 2\Lambda) + \mathcal{L}_\phi + \mathcal{L}_m \} \tag{1}$$

where Λ is the cosmological constant, $\mathcal{L}_m$ is the Lagrangian for the matter degrees of freedom and the Lagrangian for the quintessence field is given by

$$\mathcal{L}_\phi = -\frac{1}{2}\phi_{,n}\phi^{,n} - V(\phi) \, . \tag{2}$$

This model cannot be used to simulate the very beginning of the Universe, but only the period after decoupling of radiation and dust. Thus, we do not take into account here aspects as inflation, creation of matter, nucleosynthesis, among others. In the theoretical treatment, we apply the technique of *adimensional variables* (see Ref.[4]) which allows to determine the integration constants without any additional assumption. We use the dimensionless time variable $\tau = H_0 t$, where t is the cosmological time and H_0 is the present value of the Hubble parameter. In this case, $a(\tau) = a(t)/a(0)$ is the scale factor. Then we have that, at present ($\tau = 0$)

$$a(0) = 1 \ ; \ \dot{a}(0) = 1 \ ; \ H(0) = 1 \,. \tag{3}$$

We assume a homogeneous, flat and isotropic Universe. The field equations derivable from (1) are

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{2}{9}\sigma^2 \left\{ \frac{\rho_{m_0}}{a^3} + \frac{1}{2}\dot{\phi}^2 + V(\phi) \ + \frac{3}{2}\frac{\Lambda}{\sigma^2} \right\} \tag{4}$$

$$2\left(\frac{\ddot{a}}{a}\right) + \left(\frac{\dot{a}}{a}\right)^2 = -\frac{2}{3}\sigma^2 \left\{ \frac{1}{2}\dot{\phi}^2 - V(\phi) \ - \frac{3}{2}\frac{\Lambda}{\sigma^2} \right\} \tag{5}$$

and

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + V'(\phi) = 0 \tag{6}$$

where the mathematical symbol *dot* means derivative with respect to τ, while the *prime* means derivative with respect to ϕ. Moreover, in these expressions

$$V(\phi) = B^2 e^{-\sigma\phi} \tag{7}$$

$$\sigma^2 = \frac{12\pi G}{c^2} \tag{8}$$

with ρ_{m_0} denoting, in the previous expression, the present density of matter and B^2 a generic constant. We stress that the particular choice of σ shown in equation (8) allows for general exact integrations of the previous differential equations. Indeed, this choice has been used in the context of an inflationary theory[6,7,8,9], and in the Rubano-Scudellaro model[4,10]. Applying the Noether Symmetry Aproach[6,7,11,12], it can be shown that, with new variables, defined as

$$a^3 = uv \tag{9}$$

and

$$\phi = -\frac{1}{\sigma}\ln(\frac{u}{v}) \tag{10}$$

we can simplify the field equations and their formulations became similar to the ones used in Ref.[10]. With these new variables, the field equations (4-6) may be written as the following pair of equations (from now on we use, in our formulation, the energy density Ω_Λ instead of the cosmological constant Λ):

$$\ddot{u} = \frac{9\Omega_\Lambda}{4} u \tag{11}$$

and

$$\ddot{v} = \frac{9\Omega_\Lambda}{4} v + \sigma^2 B^2 u \,. \tag{12}$$

In the following, we analyze two cases.

Case I: $\Lambda = 0 \Rightarrow \Omega_\Lambda = 0$.

The solutions of the equations (11) and (12) are

$$u(\tau) = u_1 \tau + u_2 \tag{13}$$

and

$$v(\tau) = \frac{\sigma^2 B^2}{6} u_1 \tau^3 + \frac{\sigma^2 B^2}{2} u_2 \tau^2 + v_1 \tau + v_2 \,. \tag{14}$$

To find the integration constants we use Eqs. (3) and the field equations evaluated at $\tau = 0$, introducing the deceleration parameter of the Universe q_0. Finally, using $\Omega_{m_0} + \Omega_{Q_0} + \Omega_\Lambda = 1$, the above integration constants can be written in the following way:

$$u_2^{(\pm)} = \pm \frac{\sqrt{3}}{2} \sqrt{\frac{2(2 - q_0) - 3\Omega_{m_0}}{\sigma^2 B^2}} \;; \tag{15}$$

$$v_2^{(\pm)} = \frac{1}{u_2^{(\pm)}} \;\; ; \;\; u_{1\,[\pm]}^{(\pm)} = \frac{3 + [\pm]\frac{\sqrt{3}}{2}\sqrt{2(1 + q_0) - 3\Omega_{m_0}}}{2} u_2^{(\pm)} \;; \tag{16}$$

and

$$v_{1\,[\pm]}^{(\pm)} = \frac{3 - [\pm]\frac{\sqrt{3}}{2}\sqrt{2(1 + q_0) - 3\Omega_{m_0}}}{2u_2^{(\pm)}} \tag{17}$$

where $q_0 = -(1 + \dot{H}(0))$ is the present value of the deceleration parameter and $(\pm)$ and $[\pm]$ allow, in principle, for the four different branches of the solution.

Case II: $\Lambda \neq 0 \Rightarrow \Omega_\Lambda \neq 0$.

The solutions of the field equations (11) and (12) are, in this case,

$$u(\tau) = u_1 \sin(1.5\sqrt{-\Omega_\Lambda}\tau) + u_2 \cos(1.5\sqrt{-\Omega_\Lambda}\tau) \tag{18}$$

and

$$\begin{aligned} v(\tau) = & \left\{ v_2 - \frac{\sigma^2 \bar{B}^2}{9\Omega_\Lambda} u_2 - \frac{\sigma^2 \bar{B}^2}{3\sqrt{-\Omega_\Lambda}} u_1 \tau \right\} \cos(1.5\sqrt{-\Omega_\Lambda}\tau) \\ & + \left\{ v_1 - \frac{\sigma^2 \bar{B}^2}{9\Omega_\Lambda} u_1 + \frac{\sigma^2 \bar{B}^2}{3\sqrt{-\Omega_\Lambda}} u_2 \tau \right\} \sin(1.5\sqrt{-\Omega_\Lambda}\tau) \end{aligned} \tag{19}$$

where u_1, u_2, v_1 and v_2 are the integration constants. Applying the same procedure of case I, the integration constants result in

$$u_2^{(\pm)} = \pm\sqrt{\frac{3(2 - q_0 - 1.5\Omega_{m_0} - 3\Omega_\Lambda)}{2\sigma^2 \bar{B}^2}}\,; \tag{20}$$

$$v_2^{(\pm)} = \frac{1 + \frac{\sigma^2 \bar{B}^2}{9\ \Omega_\Lambda}\ u_2^2}{u_2^{(\pm)}}\,; \tag{21}$$

$$u_{1\ [\pm]}^{(\pm)} = \frac{\{\sqrt{3} - [\pm]\sqrt{1 + q_0 - 1.5\Omega m_0}\}}{\sqrt{-3\ \Omega_\Lambda}} u_2^{(\pm)}\,, \tag{22}$$

and

$$v_{1\ [\pm]}^{(\pm)} = \frac{2 - \sqrt{-\Omega_\Lambda}\ v_2^{(\pm)} u_{1\ [\pm]}^{(\pm)}}{\sqrt{-\Omega_\Lambda}\ u_2^{(\pm)}} \tag{23}$$

respectively; here the null subscripts indicate present values, and we recall that Ω_i are component densities (in units of the critical density; m stands for matter, Q for the quintessence field and Λ for the cosmological constant).

3 Analysis of the Results

We see from the expressions for the integration constants that, in both cases, our solutions have several branches. We have chosen here the "all-pluses" branch, in which upper plus signs are preferred over lower minus ones.

Because $\sqrt{1+q_0-1.5\Omega_{m_0}}$ must be real, the following constrain on the present value of the deceleration parameter holds:

$$q_0 \geq -1 + 1.5\Omega_{m_0}\,. \tag{24}$$

It can be noticed that the constants (and, consequently, the solutions) depend on 4 parameters: Ω_{m_0}, Ω_Λ, q_0 and B^2. As can be seen from Eq. (8), σ^2 is fixed and equal to 3/2 in our units ($8\pi G = c = 1$). In this normalization, we have considered the present value of the critical density of the Universe, $\rho_{c_0} = 9H(0)^2/2\ \sigma^2 = 3$. We have chosen $\Omega_{m_0} = 0.3$ and $q_0 = -0.44$, which values are perfectly acceptable for most available models. Though we have made calculations for several values of Ω_Λ in the range -0.01 a -0.30, for simplicity we present here results for $\Omega_\Lambda = -0.15$, bearing in mind that our predictions change just a little for other values of this quantity. Concerning B^2, it was shown in analytical studies, that the relevant cosmological magnitudes we have studied are independent of this quantity. That is the case of the scale factor, the Hubble and deceleration parameters and the energy density, pressure and the state parameter of the quintessence field. However, it can be easily shown that B^2 can be of the order of the critical density of the Universe. It can also be shown a relation between this parameter and the present value of the scalar field ϕ_0:

$$\phi_0 = -\frac{1}{\sigma} \ln \frac{2 - q_0 - 1.5\Omega_{m_0} - 3\Omega_\Lambda}{\bar{B}^2}\,. \tag{25}$$

Above considerations lead us to choose for subsequent calculations $B^2 = 1$, which just means a determined rescaling in ϕ_0. We postpone for further investigations the question of whether we need finely tuned initial conditions to get a determined value of ϕ_0.

Now we mainly explore the dynamics of this model for case II, in which we have a non vanishing cosmological constant, making the comparison with case I (one should notice however, that the following plots correspond to a non-zero Λ: case II).

Fig. 1 shows the evolution of the scale factor for $\Omega_\Lambda = -0.15$. For the above values of the other parameters, we have obtained a collapsing Universe, independently of the value of Ω_Λ. We also see that with the decrease (modular increase) of Ω_Λ, the time of collapse diminishes. In case I, for which $\Omega_\Lambda = 0$, just a look at the polynomial solutions (13) and (14), bearing in mind Eq. (9), make us suspect that it occurs an eternal acceleration of the Universe, which is not shown here for the sake of brevity. Fig. 2 shows the behavior of the deceleration parameter as a function of the redshift z. In agreement with Turner and Riess[13] and other authors, this figure shows that the acceleration

Figure 1. This plot of scale factor versus time shows a collapsing universe. Conversely, if $\Lambda = 0$, the accelerated expansion is eternal

is a relatively recent phenomenon, with the transition from the decelerated phase to the accelerated one occurring at redshift near 0.5. However, as follows from Fig. 1, acceleration is not eternal: in the future we will have $q > 0$ again, which would give rise to the collapse of the Universe. Fig. 3 shows the energy densities of matter and dark energy (quintessence field plus cosmological constant, i.e., effective quintessence). In the literature it is widely accepted that using an exponential potential leads to a dark energy density which scales like matter, which implies a constant ratio of quintessence to matter energy density, at least in the matter domination regime. But this is a consequence of assuming the state parameter ω of dark energy almost perfectly constant, which in our case is far from being true, as seen in Fig. 3. We appreciate that matter dominates in a redshift interval by 0.4 to 1.6, which is roughly consistent with the decelerated universe shown in Fig. 2. For higher redshifts, dark energy dominates, but Fig. 4 (state parameter of effective quintessence versus redshift) shows that in that epoch its state parameter is positive. This points at a past epoch in the evolution when gravity of the dark

Figure 2. The acceleration is a rather recent phenomenon, as can be seen in this graphic of deceleration parameter versus redshift.

energy was attractive, which is consistent with the deceleration, and with the increase of the deceleration parameter at higher redshifts (given the fact that then both matter and dark energy have attractive gravity).

4 Conclusions

In a recent paper [3] it was pointed out that the ultimate fate of the evolution of our Universe is much more sensitive to the presence of the cosmological constant than any other matter content. In particular, the universe with a negative cosmological constant will always collapse eventually, even though the cosmological constant may be nearly zero and undetectable at all at the present time. Our results support the very general assertions of Ref.[3], we have shown that in our model, for a determined region of the parameter space, the universe collapses if there is a non-vanishing cosmological constant, at difference of the eternal accelerated expansion for the case in which $\Lambda = 0$. This

Figure 3. Matter energy density dominates in a redshift interval from 0.4 to 1.6 (roughly).

also favors the consistent formulation of string theory, as explained in the introduction.

We continue working on this kind of models and pretend to check them with several astrophysical observations, such as supernovae, CMB, structure formation, etc. Some preliminary results are encouraging. It is also our interest to check the stability of the solutions found, especially in the case in which we have the cosmological constant, which implies a three component system. This implies the realization of a phase-space analysis, instead of the phase-plane for a two component system. Finally, it is our intention to include radiation in these models to be able to explore the early Universe, before decoupling of matter and radiation.

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Figure 4. The state parameter of dark energy (quintessence field plus cosmological constant) is far from being constant.

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