ELECTRICALLY CHARGED NEUTRON STARS.

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We study electric charge effects in relativistic stars assuming that the charge distribution is proportional to the mass density. As the pressure and density of matter and gravitational fields inside the stars are large, a strong electric field can be present. The hydrostatic equation which arises from general relativity, is modified considerably to meet the requirements of the inclusion of electric charge. We make a detailed study of the effects of electric charge on a polytropic equation of state and show in particular the balance between the gravitational and Coulomb forces. We conclude that in order to see any appreciable effect on the phenomenology of compact stars, electrical fields have to be huge ($\sim 10^{21}$ V/m), which implies that the total electric charge is Q$\sim 10^{20}$ Coulomb. We also discuss the stability of charged particles and raised the question about the validity of isolating the forces acting only on them when the contribution from charged particles not only affect the Coulomb force, but also are helping to soften the gravity part of the pressure gradient to prevent collapse.

1 Introduction

The study of the effect of electric charge and electric field in a gravitationally bound system has been done previously by some authors. Rosseland[1] (see also Eddington [2]) studied the possibility of a self-gravitating star on Eddington's theory to contain a net charge where the star is modeled by a ball of hot ionized gas. In such a system, the electrons (lighter particles) tend to rise to the top because of the difference in the partial pressure of electrons compared to that of ions (heavier particles). The motion of electrons to the top and further escape from the star is stopped by the electric field created by the charge separation. The equilibrium is attained after some amount of electrons escaped leaving behind an electrified star whose net positive charge is of about 100 Coulomb per solar mass, and building an interstellar gas with a net negative charge. As shown by Bally and Harrison[3], this result applies to any bound system whose size is smaller than the Debye length of the surrounding media. The conclusion is that a star formed by an initially neutral gas cannot acquire a net electric charge larger than about 100C per solar mass.

In fact, one should expect a star like the sun to hold some amount of net charge due to the much more frequent escape of electrons than that of protons. Moreover, one should also expect that the escape would stop when the electrostatic energy of an electron $e\Phi$ is of the order of its thermal energy kT. This gives for a ball of hot matter with the sun radius, a net charge $Q \sim 6.7 \times 10^{-6} T$ (in Coulomb). Hence, the escape effect cannot lead to a net electric charge much larger than a few hundred Coulomb for most of the gaseous stars.

In the case of cold stars, the electrostatic energy eQ/r is balanced by the gravitational energy mM/r, which for a spherical ball gives a charge of approximately 100C per solar masses (see Glendenning[4]). This is so for Newtonian stars. For very compact stars, however, the high density and the relativistic effects must be taken into account (see e.g. Bekenstein[5]). In a strong gravitational field, the general relativistic effects are felt and the star needs more charge to be in equilibrium. Moreover, for very compact stars, the induced electric field can be substantially higher than in the case of the sun. For instance, the same amount of charge yields an electric field approximately 10^9 times larger at the surface of a neutron star than at the surface of the sun. Therefore, even a relatively small amount of net charge on compact stars can induce intense electric fields whose effects may become important to the structure of the star. This fact deserves further investigation.

From the theoretical point of view however, it has been long known that the classical theory of gravitation and electrostatics admits equilibrium configurations of charged fluids where the charge density ρ_{ch} can be as large as the mass density ρ, in appropriate units. A static continuous distribution of charged dust matter (with zero pressure) will be in equilibrium everywhere if $\rho_{ch} = \pm\rho$ (in geometric units). This gives, for a ball of dust with the mass of the sun, a net charge of about 1.7×10^{20} Coulomb. If such a huge charge were present in the sun, the electric field at the surface would be huge ($\sim 10^{11}$V/cm) but still smaller than the critical field for pair creation, 10^{16}V/cm. In the case of charged dust with $\rho_{ch} = \rho$, the equilibrium configuration is independent of the shape of the body, so that we can build a dust star with radius large enough (and small density) to generate a smaller electric field.

The general relativistic analog for charged dust stars was discovered by Majumdar[6] and by Papapetrou [7], and further discussed by Bonnor[8] and several other authors[9]. Study for the stability of charged fluid spheres have been done by Bekenstein[5], Zhang et al.[10], de Felice & Yu[11], Yu & Liu[12], de Felice et al.[13], Anninos & Rothman[14] and others. Bekenstein argued about the possibility of the production of a huge field ($\sim 10^{20} - 10^{21}$ Volts/m) in order that the charge density would make an appreciable effect on the system, but his estimates were rather speculative and do not show any guarantee that the star would be stable. This was indirectly verified by Zhang et al.[10] who found that the structure of a neutron star, for a degenerate relativistic fermi gas, is significantly affected by the electric charge just when the charge density is close to the mass density (in geometric units). In the investigations by de Felice et al., and by Anninos & Rothman, they assumed that the charge distribution followed particular functions of the radial coordinate, and they were mostly interested in the extreme $Q = M$ case.

Our basic consideration to incorporate electric charge into the system is in the form of trapped charged particles where the charge goes with the positive value. The effect of charge does not depend on its sign by our formulation. The energy density which appears from the electrostatic field will *add up* to the total energy density of the system, which in turn will help in the *gaining* of the total mass of the system. The fundamental difference with the standard Tolman-Oppenheimer-Volkoff (TOV) equation with that of the case of charged system is the presence of the Maxwell-Einstein stress tensor. We solve the modified TOV equation for polytropic equations of state assuming that the charge density goes with the matter density and discuss the results. We do not discuss here how this extra charge is formed in the star. A mechanism to generate charge asymmetry for charged black holes has been suggested recently by Mosquera Cuesta et al. (2003)[15] and the same may be applied for compact stars too.

In this work, we are not claiming that compact stars always have so large charge and strong electric field, but even then the charged fluid element still remains at rest, taking into account only the hydrostatic equilibrium equation. In that case we see some effect of the strong electric field on the mass-radius relation of the star. One may raise the question what happens to a single charged particle inside the system. It will face huge electrostatic force and will soon escape the system and the whole system will drop down to the classical limit of 100 C. However, this classical limit, obtained from the balance of forces for a proton or an electron at the surface of the star, is considered when the particles are free to move around and there are no other effects to constrain the system. But in our case of relativistic stars, the particles inside the medium are subject to a very high global pressure and density. Here the gravitational and the Coulomb forces are coupled to each other in every respect and it is very difficult to disentangle the effects of the forces only on the charged particles and hence it is not guaranteed that they will leave the star. As we will see, the electric field will reduce considerably the gravitational part of the pressure gradient, making it difficult to separate the forces on the charged particles. In our assumption, the ratio of charge to matter density is maintained everywhere in the star, so, the problem of charged particle escaping at the surface, where the matter density is zero, is also ruled out.

If these highly charged compact stars exist, they need to be isolated systems since outside the star, the Coulomb force will overwhelm the gravitational one. Any oppositely charged particle in the stellar atmosphere will be attracted and similarly charged particles will be repulsed by the charged star. Our main objective here is to study the effect of charge in static relativistic compact stars made by a fluid that obeys a polytropic equation of state and to analyze the phenomenology of these charged stars.

2 Electrical Neutrality of Stars

The basic argument to assume the charge neutrality of stars is based on the fact that the total charge of the stars should lie below a certain limit where the coulomb repulsive force overwhelms the gravitational attractive force at the surface of the star. This limiting value can be viewed as the Coulomb force acting on a proton at the surface and we have the limiting range as :

$$\frac{(Ze)e}{R^2} \leq \frac{GMm}{R^2} \leq \frac{G(Am)m}{R^2} \tag{1}$$

where, Ze, R and M are the net charge of the star, its radius and mass, respectively and m and e are the mass and charge of a proton. In the above equation, the mass of the star is considered to be smaller than Am because of the gravitational binding of the system. In gravitational units,

$$\frac{Z}{A} < \left(\frac{m}{e}\right)^2 < 10^{-36}. \tag{2}$$

If we take into account that there are approximately $A \simeq 10^{57}$ baryons in a neutron star ($M \simeq M_{\odot}$), so, for protons, $Z < 10^{21}$, which gives

$$Q \simeq Ze \simeq 100 \text{ Coulomb}. \tag{3}$$

If the star had a net negative charge, then the electrons being the carrier of charge, the value of the limit would be reduced by the factor m_e/m (see Ref.[4]).

A system like a neutron star has very highly dense matter and the effect of the medium is very much felt on the particles in the form of kinetic pressure and Pauli blocking effects. In fact we will show that the maximum allowed charge is obtained when $\rho_{ch} \simeq \sqrt{G}\rho$ in natural units from the assumption that more mass can hold more charge ($\rho_{ch} \propto \rho$). Because of the large density found in neutron stars ($\rho \simeq 10^{15} g/cm^3$) we will expect a large charge if ρ_{ch} is of the same order as ρ. Thus $Q(r) \simeq \sqrt{G}M(r)$ and at the surface of a neutron star with $M \simeq M_\odot$, the total charge is $Q \simeq \sqrt{G} M_\odot \simeq 10^{20}$ Coulomb. Similarly, if we calculate the intensity of the electric field at the surface of the star, it gives a value of $10^{21} - 10^{22}$ V/m. In the case of maximum charge, the relation $Q(r)=\sqrt{G}M(r)$ at the surface of the star shows that the ratio becomes $Z/A \simeq 10^{-18}$ as compared to 10^{-36} in Eq.(2). This explains why we are having charges 10^{18} times larger than that in relation (3). This Z is the net charge in the star and is the difference of the charges of the charged particles with opposite sign ($Z \sim Z_{net} \sim |Z_+ - Z_-|$). However, the number of charges of equal and opposite sign is not at all limited.

For relativistic stars, the presence of the electric field will increase the mass of the star and also decrease the pressure by the electrostatic energy density associated with that field. Furthermore, the repulsive Coulomb force will be modified by the metric which depends on the mass. The point we want to stress is that, in such dense systems of relativistic stars, it is quite difficult to disentangle the effects of forces on charged particles when the mass changes the Coulomb force, the electric field (charge) can considerably change the gravitational force. In relativistic mean field theory, the repulsion term of the pressure coming from the nuclear interaction due to the meson exchange forces has origin in the density of all the system (neutral and charged particles), not allowing any partial pressure analysis. So only a global balance of all the forces acting in an element of the charge fluid seems to be plausible inside the star. The gravitational attraction does not distinguish charge coming from the field acting on the mass density of the *neutral* and the charged particles will act against the repulsive nuclear pressure and Coulomb forces.

3 General Relativistic Formulation

We take the metric for our static spherical star as

$$ds^2 = e^{\nu} c^2 dt^2 - e^{\lambda} dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2)\,. \tag{4}$$

The stress tensor T^{μ}_{ν} will include the terms from the Maxwell's equation and the complete form of the Einstein-Maxwell stress tensor will be :

$$T^{\mu}_{\nu} = (P+\epsilon)u^{\mu}u_{\nu} + P\delta^{\mu}_{\nu} + \frac{1}{4\pi}\left(F^{\mu\alpha}F_{\alpha\nu} - \frac{1}{4}\delta^{\mu}_{\nu}F_{\alpha\beta}F^{\alpha\beta}\right) \tag{5}$$

where P is the pressure, ϵ is the energy density ($=\rho c^2$) and u-s are the 4-velocity vectors. For the time component, one easily sees that $u_t = e^{-\nu/2}$ and hence $u^t u_t = -1$. Consequently, the other components (radial and spherical) of the four vector are absent.

Now, the electromagnetic field is taken from the Maxwell's field equations and hence they will follow the relation

$$\left[\sqrt{-g}F^{\mu\nu}\right]_{,\nu} = 4\pi j^{\mu}\sqrt{-g} \tag{6}$$

where j^{μ} is the four-current density. Since the present choice of the electromagnetic field is only due to charge, we have only $F^{01} = -F^{10}$, and the other terms are absent. In general, we can derive the electromagnetic field tensor $F^{\mu\nu}$ from the four-potential A_{μ}. So, for a non-vanishing field tensor, the surviving potential is $A_0 = \phi$. We also considered that the potential has a spherical symmetry, i.e., $\phi = \phi(r)$.

The non-vanishing term in Eq.(6) is taken for ν=r. This gives the electric field as

$$^{t}_{t} \sim {}^{r}_{r} \ : \ \frac{1}{4\pi}\left(F^{\mu\alpha}F_{\alpha\nu} - \frac{1}{4}\delta^{\mu}_{\nu}F_{\alpha\beta}F^{\alpha\beta}\right) = \frac{-\mathcal{U}^2}{8\pi}$$

where,

$$\mathcal{U}(r) = \frac{1}{r^2}\int_0^r 4\pi r^2 \rho_{ch} e^{\lambda/2} dr. \tag{7}$$

This leads us to define the total charge of the system as

$$Q = \int_0^R 4\pi r^2 \rho_{ch} e^{\lambda/2} dr \tag{8}$$

where R is the radius of the star.

With the metric (4) one can easily get the Einsteins field equations as

$$^{t}_{t} \ : \ \frac{e^{-\lambda}}{r^2}\left(r\frac{d\lambda}{dr} - 1\right) + \frac{1}{r^2} = \frac{8\pi G}{c^4}\left(\epsilon + \frac{\mathcal{U}^2}{8\pi}\right) \tag{9}$$

$$^{r}_{r} \ : \ \frac{e^{-\lambda}}{r^2}\left(r\frac{d\nu}{dr} + 1\right) - \frac{1}{r^2} = \frac{8\pi G}{c^4}\left(P - \frac{\mathcal{U}^2}{8\pi}\right). \tag{10}$$

The first of Einstein's equations is used to determine the metric e^{λ}. The mass of the star is now due to the total contribution of the energy density of matter and electric energy ($\frac{\mathcal{U}^2}{8\pi}$) density. The mass takes the new form as

$$M_{tot}(r) = \int_0^r 4\pi r^2 \left(\frac{\epsilon}{c^2} + \frac{\mathcal{U}^2}{8\pi c^2} \right) dr \tag{11}$$

and the metric coefficient is given by

$$e^{-\lambda} = 1 - \frac{2GM_{tot}(r)}{c^2 r} \,. \tag{12}$$

From the conservation of the Einstein's field tensor ($T^{\mu}_{\nu\ ,\mu} = 0$) one gets the form of the hydrodynamical equation. To this end, we obtain the modified TOV as

$$\frac{dP}{dr} = -\frac{G\left[M_{tot}(r) + 4\pi r^3 \left(\frac{P}{c^2} - \frac{\mathcal{U}^2}{8\pi c^2}\right)\right](\epsilon + P)}{c^2 r^2 \left(1 - \frac{2GM_{tot}}{c^2 r}\right)} + \rho_{ch}\mathcal{U}e^{\frac{\lambda}{2}} \,. \tag{13}$$

The first term on the r.h.s. comes form the gravitational force with an effective pressure and density which we will discuss later, and the second term from the Coulomb force that depends on matter though the metric coefficient.

Numerical solutions in Fortran require the integral forms of Eqs. (7), (11) and (12) to be expressed in their differential forms. We can write the corresponding differential forms as

$$d\mathcal{U} = -\frac{2\mathcal{U}dr}{r} + 4\pi\rho_{ch}e^{\lambda/2}dr \tag{14}$$

$$dM_{tot} = 4\pi r^2 \left(\frac{\epsilon}{c^2} + \frac{\mathcal{U}^2}{8\pi c^2} \right) dr \tag{15}$$

and

$$d\lambda = \left[\frac{8\pi G}{c^2} r e^{\lambda} \left(\frac{\epsilon}{c^2} + \frac{\mathcal{U}^2}{8\pi c^2} \right) - \left(\frac{e^{\lambda} - 1}{r} \right) \right] dr \,. \tag{16}$$

So, the final four equations needed to be solved are Eqs. (13), (14), (15) and (16). The boundary conditions for the solution are, at the centre, where $r = 0$, $\mathcal{U}(r) = 0$, $e^{\lambda(r)} = 1$, $P(r) = P_c$ and $\rho(r) = \rho_c$, and at the surface where $r = R$, $P(r) = 0$. The inputs in the equations are the pressure P, the energy density ϵ and the charge density ρ_{ch}. The metric coefficient λ and the electric field $\mathcal{U}$ are inter-dependent. This gives us a set of four coupled differential equations which we solve simultaneously to get our results. We note here that the form of the equations does not change with the sign of the charge because the electric field appears in the mass term (15), and the pressure gradient term (13) in squares and in the Coulomb part; the product $\rho_{ch}\mathcal{U}$ is also invariant.

4 Charged Stars in a Polytropic Equation of State

4.1 Mass radius relations and structure

We now examine the effects of charge in a polytropic equation of state(EoS), which is a more general approach than considering any model dependent EoS. We assume the charge go with the mass density (ϵ) as

$$\rho_{ch} = f \times \epsilon \tag{17}$$

where $\epsilon = \rho c^2$ is in [MeV/fm^3]. With this assumption, the charge fraction f has dimension $\frac{1}{[km]}[fm^3/MeV]^{1/2}$ and the charge density ρ_{ch} is in $[MeV/fm^3]^{1/2}\frac{1}{[km]}$. This kind of 'mixed units' appear in our dimensions of f and ρ_{ch} because one can see from Eqs. (7) and (13) that the electric field is proportional to the square root of the pressure (in units of MeV/fm^3) and the integration over the radius r is carried out in kilometers. In geometrical units, this can be written as

$$\rho_{ch} = \alpha \times \rho \tag{18}$$

where charge is expressed in units of mass and charge density in units of mass density. This α is related to our charge fraction f as

$$\alpha = f \times \frac{0.224536}{\sqrt{G}} = f \times 0.86924 \times 10^3 \,. \tag{19}$$

Our choice of charge distribution is a reasonable assumption in the sense that large mass can hold large amount of charge.

The polytropic EoS is given by

$$P = \kappa\rho^{1+1/n} \tag{20}$$

where n is the polytropic index and is related to the exponent Γ as $n = \frac{1}{\Gamma-1}$. In the relativistic regime, the allowed value of Γ is $\frac{4}{3}$ to $\frac{5}{3}$. We have considered the adiabatic case of $\Gamma = \frac{5}{3}$ and the corresponding value of n is 1.5. Primarily, our units of matter density and pressure are in MeV/fm^3. Hence choosing a value of κ as 1.695 $(MeV/fm^3)^{\frac{2}{5}}$, we can fit quite well the EoS for the nonlinear Walecka model with hyperons[17,16] making our choice of a polytropic star more realistic. Thus we have an EoS which we analyze for different cases of charge fraction f and study the nature and behaviour of the system.

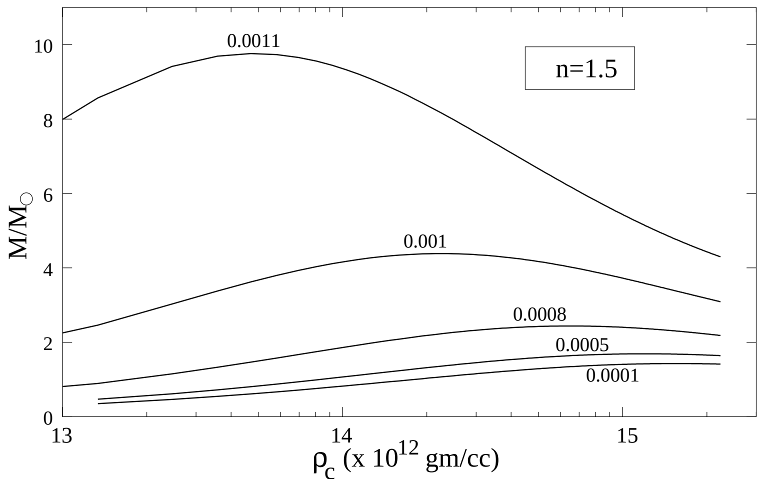

Figure 1. Central density against mass for different values of the factor f.

We have plotted the mass as a function of the central density ρ_c in Fig.(1) for different values of the charge fraction f. The stars which are on the higher density regime and have lower mass, are unstable because $\frac{dM}{d\rho_c} < 0$. Those falling in the lower density regime and have increasing mass are all allowed. The effect of the charge for $f = 0.0001$ on the structure of the star is not profound and it is comparable with that of the chargeless star. This value of f is however 'critical' in the sense that further increase in the value shows effect on the structure of the star. With the increase of the charge fraction from $f = 0.0001$ to $f = 0.0005$ the structure changes by 20% increase in the value; from $f = 0.0005$ to $f = 0.0008$, the increase is 35% and from $f = 0.0008$ to $f = 0.001$, the change is almost 90%, thus showing that the change in the structure is non-linear with the change in the charge fraction as can be seen in Fig.(1).

In Fig.(2) we plotted the mass-radius relation. Due to the effect of the repulsive force, the charged stars have large radius and larger mass as we should expect. Even if the radius is increasing with the mass, the M/R ratio is also increasing, but much slower. For the lower charge fractions, this increase in the radius is very small, but a look at the structure for the fraction $f = 0.001$ reveals that for a mass of 4.3 $M_\odot$, the radius goes as high as 35 kms. Though the compactness of the stars are retained, they are now better to be called as 'compact charged stars' rather than 'charged neutron stars'. The charge

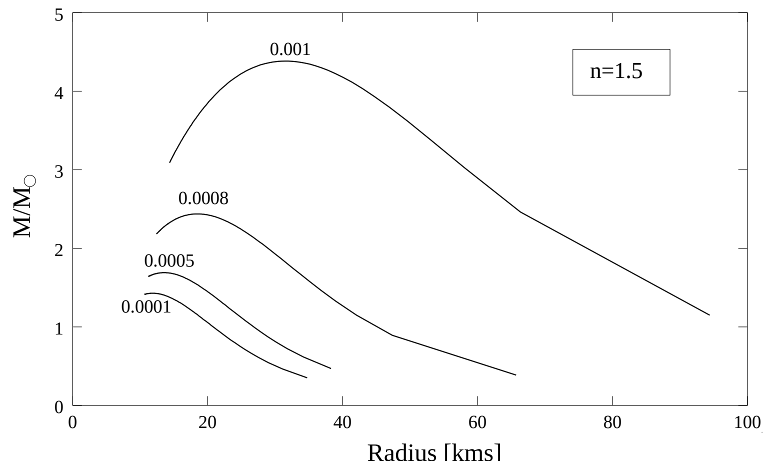

Figure 2. Mass-Radius relation for different values of the factor f.

fraction in the limiting case of maximally allowed value goes up to $f = 0.0011$, for which the maximum mass stable star forms at a lower central density even smaller than the nuclear matter density. This extreme case is not shown in Fig.(2) because the radius of the star and its mass is very high (68 km and 9.7 $M_\odot$ respectively) which suppress the curves of the lower charge fractions due to scaling. For this star, the mass contribution from the electric energy density is 10% than that from the mass density. It can be checked by using relation (19) that this charge fraction $f = 0.0011$ corresponds to $\rho_{ch} = 0.95616\rho$ in geometrical units.

In Fig.(3), we have plotted the metric coefficient e^λ as a function of radius for the maximum mass stars for each of the charge fractions. A quick comparison shows that the nature of e^λ is the same for all the stars with different charges. There is a slight increase in its value for higher charge fractions thus showing the gain in the compactness $\frac{\mathrm{M}}{\mathrm{R}}$ of the star with charge. This can be verified from the values of e^λ at the surface for two cases of charge fractions $f = 0.0001$ and $f = 0.001$

$$\left(\frac{2M}{R}\right)_{0.0001} = 1 - \frac{1}{1.55} = 0.3548 \quad \text{and} \quad \left(\frac{2M}{R}\right)_{0.001} = 1 - \frac{1}{1.7} = 0.4118.$$

This reconfirms that the compactness of the star increases despite the enormous increase in the radius with the increase of the charge fraction.

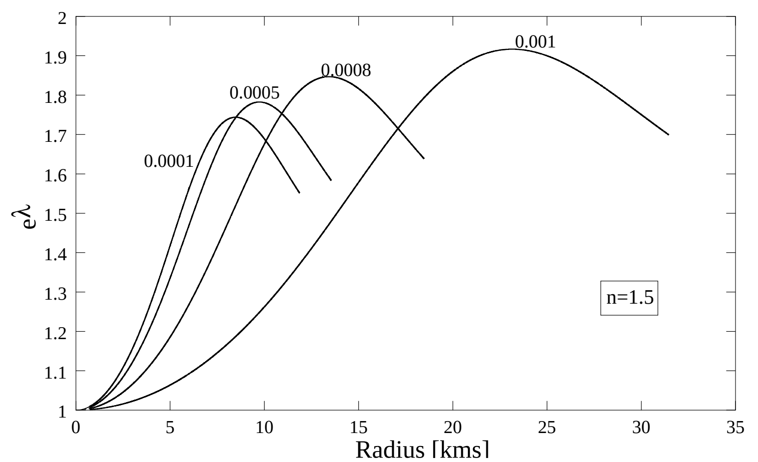

Figure 3. Variation of the metric e^{λ} with the radius in the maximum mass stars for different cases of charge distribution with varying f.

4.2 Effects of Charge and Electric field.

In this subsection we discuss the effect of the electric charge in the stellar profile.

In the Q× M diagram in Fig.(4), we plotted the mass of the stars against their surface charge. We have made the charge density proportional to the energy density and so it was expected that the charge, which is a volume integral of the charge density, will go in the same way as the mass, which is also a volume integral over the mass density. The slope of the curves comes from the different charge fractions. The nature of the curves in fact reflect that charge varies with mass (with the turning back of the curves all falling in the 'unstable zone' and not taken into consideration). If we consider the maximum allowed charge estimated by the condition ($\mathcal{U} \simeq \sqrt{8\pi P} < \sqrt{8\pi\epsilon}$) for $\frac{dP}{dr}$ to be negative (Eq. (13)), we see that the curve for the maximum charge in Fig.(4) has a slope of 1:1 (in a charge scale of 10^{20} Coulomb). This scale can easily be understood if we write the charge as $Q = \sqrt{G}M_{\odot}\frac{M}{M_{\odot}} \simeq 10^{20}\frac{M}{M_{\odot}}$Coulomb. It is worth mentioning that Q is the charge at the surface of the star where the pressure and also $\frac{dP}{dr}$ are already very small (ideally zero). So, at the surface, the Coulomb force is essentially balanced by the gravitational force and the relation of the charge and mass distribution we found is exactly the

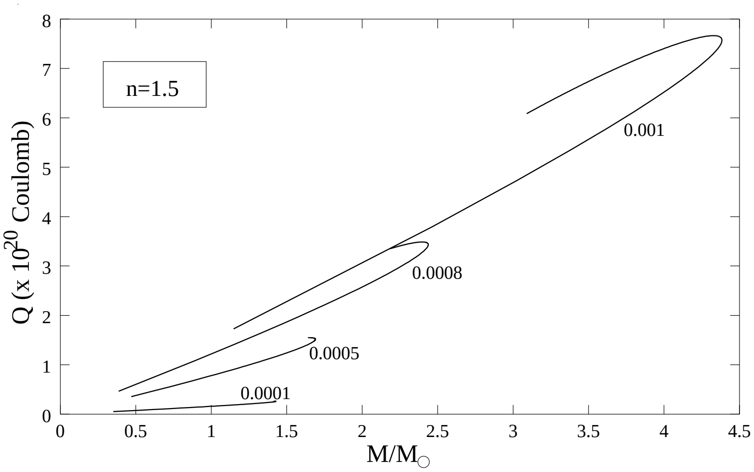

Figure 4. The variation of the charge with mass for different f.

same for the case of charged dust sphere discussed earlier by Papapetrou[7] and Bonnor[8], as we referred in the introduction.

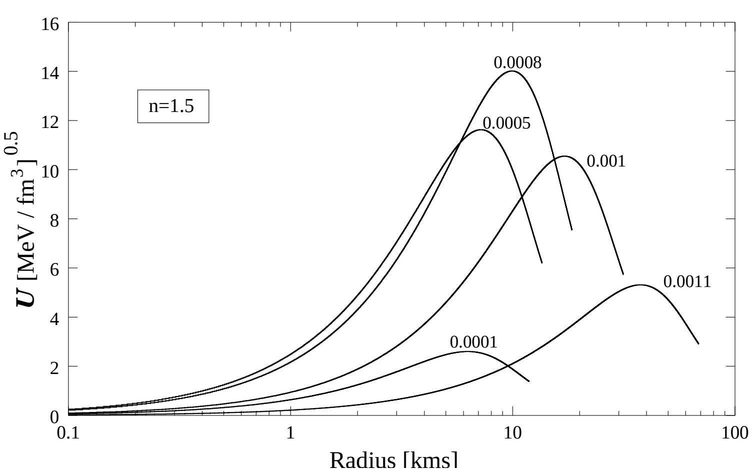

Figure 5. Variation of $\mathcal{U}$ with the radius in the maximum mass stars for different cases of charge distribution with varying f.

In Fig.(5), the electric field ($\mathcal{U}$) is plotted as a function of the stellar radius for the maximum star mass for different charge fractions f. The radius is shown in the log-scale. From the expression for the electric field (Eq.(7)), it is clear that the profile of the field will depend on the charge fraction f, on the metric coefficient e^{λ} and also on the radius of the star. The value of the field increases for charge fraction up to $f = 0.0008$ but then falls down. The decrease in the field for the higher charge fraction is attributed to the formation of the stable star at a very small density (Fig.(1)) for that particular charge fraction f.

4.3 The modified EoS inside the stars

From Eqs. (11) and (13), the effective energy density and pressure of the system can be viewed as

$$P^* = P - \frac{\mathcal{U}^2}{8\pi} \text{ and, } \epsilon^* = \epsilon + \frac{\mathcal{U}^2}{8\pi}. \tag{21}$$

In Fig.(6) we show two different charge fractions $f = 0.0005$ and $f = 0.0008$, the effective pressure (P^*) and energy density (ϵ^*). The effective pressure drops down to a negative value, but even in this case, the first part of Eq.(13) preserves its overall negative value because ($\mathrm{M}_{\mathrm{tot}} + 4\pi \mathrm{r}^3 \mathrm{P}^*$) is positive. Thus the overall sign of the pressure gradient $\frac{dP}{dr}$ is still negative as long as the attractive gravitational term is larger than the repulsive Coulombian one. These figures show that the effective EoS becomes stiffer due to the inclusion of charge and consequently allowing more mass in the star.

Also, the effective pressure directly reduces the value of the negative part of $\frac{dP}{dr}$, but the effective energy density increases the same through $\mathrm{M}_{\mathrm{tot}}$. This goes on until the effective pressure becomes so much negative that it overcomes the value of $\mathrm{M}_{\mathrm{tot}}$ and this limits the formation of star with higher charge fraction f.

4.4 Balance between the Coulomb and gravitational forces

In this section, we discuss the effects brought in the pressure gradient from the matter energy and the Coulomb energy. As mentioned previously, the total mass of the system $\mathrm{M}_{\mathrm{tot}}$ increases with increasing charge because the electric energy density 'adds on' to the mass energy density. This change in the mass is low for smaller charge fraction and going up to 7 times the

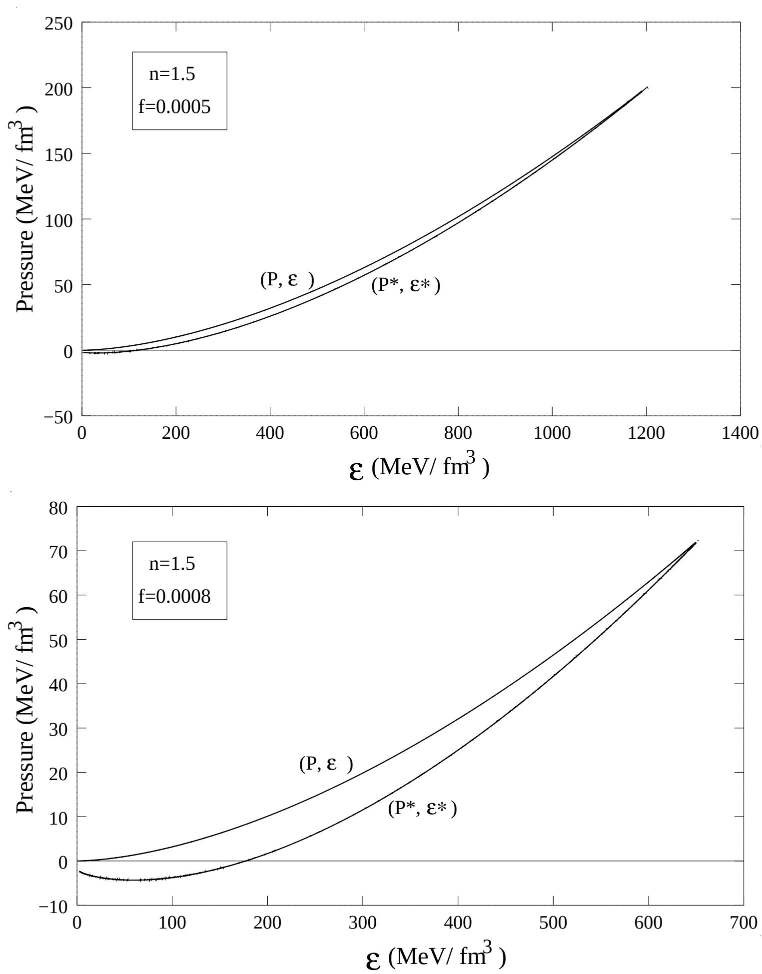

Figure 6. Pressure versus energy density plot for chargeless case and for charge fraction $f = 0.0005$ (left) and $f = 0.0008$ (right).

value of the chargeless case for maximum allowed charge fraction $f = 0.0011$. This effect however does not change the metric coefficient considerably when compared to the chargeless case (see Fig.(3)). So, the ruling term in Eq.(13) is the factor $(\mathrm{M_{tot}} + 4\pi \mathrm{r}^3\mathrm{P}^*)$. With the increase of charge, the value of P^* decreases, and hence the gravitational negative part of Eq.(13) decreases. The

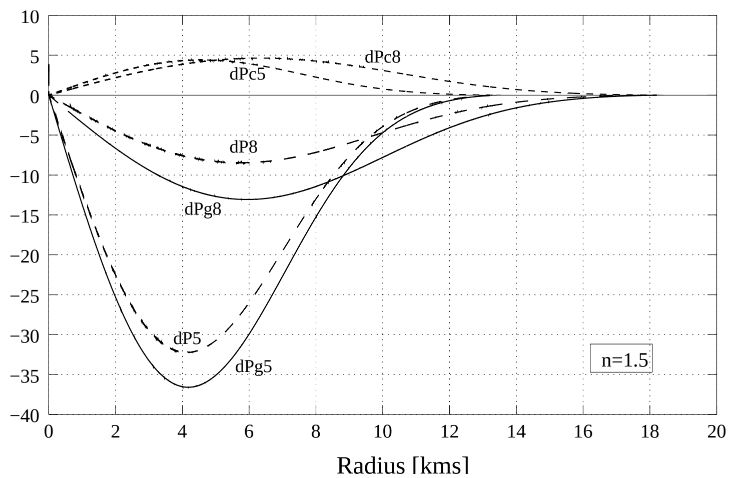

Figure 7. The Pressure gradient with the positive Coulomb part and the gravitational negative part is shown here for two different values of the charge factor f (=0.0005 & 0.0008). The total, $\frac{dP}{dr}$, is also plotted for the same. The annotations for the component of pressure for two charges f = 0.0005 and 0.0008 coming from the *matter part* are dPg5 and dPg8 respectively, those from *Coulomb part* are dPc5 and dPc8 respectively and the corresponding totals are dP5 and dP8.

central pressure is very high. So, with the softening of the pressure gradient, the system allows more radius for the star until it reaches the surface where the pressure (and the pressure gradient too) goes to zero. We should stress that because $\frac{\mathcal{U}^2}{8\pi}$ cannot be too much larger than the pressure in order to maintain $\frac{dP}{dr}$ negative as discussed before, we have a limit on the charge. It is interesting that this limit comes from the relativistic effects of the gravitational force and not just only from the repulsive Coulombian part.

This effect is illustrated in Fig.(7) where we have plotted both the positive Coulomb part and the negative matter part of the pressure gradient. The plots are for two values of the charge fraction f = 0.0005 and f = 0.0008. The positive part of $\frac{dP}{dr}$ maintains its almost constant value because the charge fraction f is the controller of the same, and in our case, they differ by a very small percentage. In the negative part, the changes are drastic and are mainly brought by the effective pressure as we already discussed.

5 Conclusions

Our analysis shows that the amount of charge contained in a dense system like a neutron star is very high and several orders of magnitude larger than those calculated by classical balance of forces at the surface of a star. This amount of charge mainly comes from the high density of the system since $\rho_{ch} \propto \rho$. We showed that the charge can be as high as 10^{20} Coulomb to bring in any change in the mass-radius relation of the star, yet remaining stable. In our study, we used a polytropic equation of state for our compact star with a choice of parameters such that the system is close to the realistic neutron stars. We showed that in the critical limit of the charge contained in the system, the maximum mass stable star forms in a lower density regime, however compactness keeps on increasing. We have also studied the change in the pressure gradient due to the effect of the charge contained in the system. It was expected from the classical picture of forces involved, that the repulsive force of the charged particles will add up to the internal pressure of the system and the entire repulsive force will be balanced by the gravitational force of the system. However, we can see from Fig.(7) that the contributions from the charged particles are helping to soften the gravity part of the pressure gradient, thus allowing more matter to stay in the system. The second term in the right hand side of Eq.(13) is positive always and does not depend on the nature of the charge whether it is positive or negative. The pressure gradient must be a negative term and hence $\frac{dP}{dr}$ is softened by the effect of the presence of charge. The net effect is the gravitational force which tries to *collapse* the system and is held further away by the Coulomb force which, in the absence of gravity, would have *exploded* the system.

We have studied the electric field inside the maximum mass star allowed by a certain charge fraction. We found that the field attains a maximum value for certain amount of charge fraction and then decreases for higher charge fractions (Fig.(5)). This is also interesting because normally it was expected that the increase in the charge fraction would increase the field also. But, as our charge distribution varies directly with the matter density, so the formation of the maximum mass star in the lower density regime for high charge fractions reduces the electric field. The mass of the stars (Eq.(11)) in this critical field limit is very high as compared to the chargeless case and have a contribution from the electrostatic energy density, but this is small compared to the contribution of the matter density.

As pointed by Bekenstein[5], this amount of charge will produce a huge electric field favoring pair creations and the system will destabilize. However, we can say that the critical field limit calculation has been done for a vacuum and we are not sure how the field will behave for a dense system. Recent observations have revealed that there are Magnetars which have magnetic fields as high as 10^{18} to 10^{20} Gauss. The equivalent critical field limit for magnetic field for pair creation in vacuum is 10^{13} Gauss. So, if such high fields really exist in a highly dense star and the stars are stable, then the critical field limit needs to be modified for high density matter. Putting aside this debate, we have checked only the behaviour of the system in the presence of a very high charge. Finally, the existence of these highly charged compact stars needs them to be isolated systems since outside the star, the Coulomb force will overwhelm the gravitational one.

6 Appendix : Conversion to Real Units

It is important to mention here the basic units used in our approach and their conversions from these units to the real units of the charge and fields like Coulomb and Volt/meter respectively. We used the charge density (ρ_{ch}) proportional to the energy density of the system, which in turn is in MeV/fm^3, with a factor f which has a dimension. From the fine structure constant $\alpha = \frac{e^2}{\hbar c} = \frac{1}{137}$, we get a relation for the charge and MeV/fm^3 which comes out as

$$1C \simeq 0.75 \times 10^{19} [MeV fm]^{1/2}. \tag{22}$$

Also, the dimensions of $\mathcal{U}^2/8\pi$ has to be the same as the dimension of pressure. Initially, we have the dimension of pressure as [MeV/fm^3], which indicates that $\mathcal{U}$ will be $[MeV/fm^3]^{1/2}$. Working out the relations, we find

$$1\left[\frac{MeV}{fm^3}\right]^{1/2} = 1.2 \times 10^{21}\mathrm{V/m}\ ;\ \left[\frac{MeV}{c^2 fm^3}\right] = 1.7826627 \times 10^{12} g/cm^3\,.$$

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