ENERGY TRANSPORT BY AXIONS IN A MAGNETAR BIRTH: CAN IT POWER A GAMMA-RAY BURST?

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The birth of a magnetar in a supernova explosion can be heralded by an axion burst. The "bremstrahlung" between neutrons in a supernova core provides a source of axions, whose "luminosity" depends on the axion mass. The range of permitted masses for axions 4×10^{-3} eV $\geq m_a \geq 10^{-5}$ eV is studied. The high intensity magnetic fields associated to magnetars affect the mean life of process $a \longrightarrow e^+e^-$, to become $\sim 10^6 - 10^{16}$ s. We estimate the fraction of the axionic flux that can be converted into pairs as function of the magnetic field intensity. Although not enough to power a long GRB, this mechanism provides the transportation of energy from the core to the external side of a magnetar and may be relevant in a number of events.

1 Introduction

1.1 The very basics of Gamma-Ray Bursts

Gamma-Ray Bursts (GRB) are a 30-old mystery being solved nowadays. They are extremely energetic bursts (10^{49} - $10^{54.5}$ erg if emission is isotropic) of hard photons ($0.1 - 1.5\, MeV$), during 0.01 - 100 seconds, some of them followed by afterglows in X-rays, optical and radio wavelengths (see [1,2] for a short review on GRB properties). The engine that powers such titanic bursts, however, remains to be uncovered. Several proposals have been made, and among the most popular we may count the hypernova [3], the collapsar [4], the binary neutron star merger [5], the neutron star-black hole merger [6], and the supranova [7] models. GRBs were also related to just-born (millisecond period) magnetars (see Section 1.2) in at least two models: production of a relativistic wind through particle acceleration [8] and spontaneous e^+e^- pair creation [9].

Loeb [10] was the first to propose that axion emission by supernova (SN) events could be the responsible for the transfer of the internal energy from the proto-neutron star to a external region, where they could be converted in electron-positron pairs or photons, producing the "fireball" commonly associated to a GRB. Another model including axions was proposed by [11]. Both models, however, ar independent of the intense magnetic field of a magnetar, which is known to act as a catalyzer for axion decay [12]. As we will show in

Section 3, this effect reduces the mean time for axion decay in more than 30 orders of magnitude, making possible for a fraction of the energy transported by axions to become available to a hot plasma created through axion decay (fireball). The issue is whether this big difference is enough to power a GRB.

Loeb [10] also remarked that if GRBs are produced in SN, only $\sim 10^{-4}$ of the latter are required to explain the GRB rate. The connection between SN and GRB is still being debated (see [13] and [14]), although observations of optical afterglows seem to confirm the association [15]. In this work we will argue that GRBs may be in fact related to a special kind of SNs, namely the ones which give origin to a magnetar.

1.2 The very basics of magnetars

Neutron stars are believed to be born starting from the core implosion of massive stars ($> 8\,M_\odot$), like supernova types II, Ib, Ic and Id. Their magnetic fields (10^{11} - 10^{13} G) and rotational periods (0.01 - 2 s) provide the means of detecting these objects as radio pulsars, due to the "lighthouse" effect (see [16] for a review on pulsars). Magnetars differ from ordinary pulsars because they have magnetic fields stronger than the critical Schwinger field $B_S = m_e^2/q_e = 4.41 \times 10^{13}$ G (m_e is the electron mass; q_e is the electron charge). They were first discussed in [17], and associated to Soft Gamma-ray Repeaters [18] and Anomalous X-Ray Pulsars [19] some time afterwards. These objects were not yet detected as radio pulsars, possibly due to photon splitting process that can occur at such high-intensity fields [20] quenching the emission. Recent information on magnetars can be found on Hurley [21].

1.3 The very basics of axions

Axions are pseudo-Goldstone bosons, introduced by Peccei and Quinn [22] to solve the CP problem of strong interactions. The Peccei-Quinn energy scale f_a is a free parameter of the theory, to be determined by observations. To simplify matters, thorough this work we will consider only three families of elementary particles (as presently observed).

The most popular models of axion models are the DFSZ model [23,24], where axions can couple to charged leptons at the tree level, and the KSVZ model [25,26], with couplings to nucleons and photons only. Both models are referred as "invisible axion" models, because the coupling constants to fermions are related to f_a as

$$g_{af} = C_f \frac{m_f}{f_a} \,, \tag{1}$$

and the coupling constant to photons is

$$g_{a\gamma} = \frac{\alpha}{2\pi f_a}\xi \,, \tag{2}$$

where m_f is the fermion mass, C_f is a constant between -0.6 and 0.4, $\xi = 0.748\pm0.033$, and α is the fine structure constant. Since $f_a \gg m_f$, interactions are very rare. The axion mass (m_a) is related to f_a through

$$m_a = \frac{6\times10^{15}}{f_a}\,eV^2 \,, \tag{3}$$

The range of axion masses allowed by observations and theoretical considerations is 4×10^{-3} eV $\geq m_a \geq 10^{-5}$ eV [27,28] The constants C are parameterized by $\cos^2\beta$, with an arbitrary β. The constant for electrons is $C_e = (\cos^2\beta)/3$ in the DFSZ model and $C_e = 0$ in the KSVZ model. The constant for neutrons is $C_n = -0.181 + 0.393\cos^2\beta$ in the DFSZ model, and $C_n = -0.04$ in the KSVZ model. The coupling constants from eq.(1) can be written for electrons and neutrons as

$$g_{ae} = C_e\frac{m_e}{f_a} = 2.84\times10^{-14}\cos^2\beta\left(\frac{m_a}{10^{-3}\,eV}\right) \tag{4}$$

$$g_{an} = C_n\frac{m_n}{f_a} = 6.15\times10^{-11}\left(\cos^2\beta - 0.46\right)\left(\frac{m_a}{10^{-3}\,eV}\right) \,, \tag{5}$$

for the DFSZ model respectively, and

$$g_{ae} = 0 \tag{6}$$

$$g_{an} = -6.29\times10^{-12}\left(\frac{m_a}{10^{-3}\,eV}\right) \,, \tag{7}$$

for the KSVZ one. The coupling constant to photons (eq. 2) can also be written as a function of m_a

$$g_{a\gamma} = 1.44\times10^{-22}\left(\frac{m_a}{10^{-3}\,eV}\right)eV^{-1} \,. \tag{8}$$

More detailed discussions about axions can be found on the excellent book by Raffelt [27] and references therein.

2 Axion emission in supernova cores

Several works discussed the axion emission in supernova cores. Some of them constrained the axion mass/coupling constants using the neutrino data from the SN1987A event. Since there was no motivation to suppose that a strong magnetic field could be generated in protoneutron stars (and specially in that particular event), no attempt was made to include effects of the magnetic field. However, two conclusions from those works are of interest here: (1) the nucleon-nucleon bremsstrahlung ($NN \longrightarrow NNa$) is the dominant axion emission process for the typical conditons of newly-born neutron stars ($T \sim 30 - 80$ MeV, $\rho \sim (6-10) \times 10^{14}$ g cm^{-3} (Mayle et al. 1988); (2) The non-degenerate nucleon limit works better than the degenerate one in that conditions. Actually, the best approximation is the one which results in *lower* emissivity (Brinkmann & Turner 1988). The medium in the supernova core will be described as non-degenerate, non-relativistic neutrons and protons, and ultra-relativistic, non-degenerate electrons, and all processes considered in this work apply to that situation.

We will consider the bremsstrahlung of neutrons in a hot supernova core as the main source of axions ($nn \longrightarrow nna$), although the np scattering ($np \longrightarrow npa$) can contribute at similar levels, depending on the exact chemical composition of the medium. The emissivity per volume unit is [27]

$$Q_{DFSZ} = 9.0 \times 10^{35}\, Y_n^2 \left(\cos^2\beta - 0.46\right)^2 \left(\frac{m_a}{4\times 10^{-3}\, eV}\right)^2 \left(\frac{\rho}{6\times 10^{14}\, g\, cm^{-3}}\right)^2 \left(\frac{T}{50\, MeV}\right)^{7/2} erg\, cm^{-3}\, s^{-1} \qquad (9)$$

for the DFSZ model and degenerate neutrons, and

$$Q_{KSVZ} = 9.3 \times 10^{33}\, Y_n^2 \left(\frac{m_a}{4\times 10^{-3}\, eV}\right)^2 \left(\frac{\rho}{6\times 10^{14}\, g\, cm^{-3}}\right)^{1/3} \left(\frac{T}{50\, MeV}\right)^{6} erg\, cm^{-3}\, s^{-1} \qquad (10)$$

for the KSVZ model and degenerate neutrons. The corresponding emissivities in the non-degenerate limit are

$$Q_{DFSZ} = 2.1 \times 10^{35}\, Y_n^2 \left(\cos^2\beta - 0.46\right)^2 \left(\frac{m_a}{4\times 10^{-3}\, eV}\right)^2 \left(\frac{\rho}{6\times 10^{14}\, g\, cm^{-3}}\right)^2 \left(\frac{T}{50\, MeV}\right)^{7/2} erg\, cm^{-3}\, s^{-1} \qquad (11)$$

for the DFSZ model, non-degenerate neutrons, and

$$Q_{KSVZ} = 2.2 \times 10^{33}\, Y_n^2 \left(\frac{m_a}{4\times 10^{-3}\, eV}\right)^2$$

$$\left(\frac{\rho}{6\times10^{14}\,g\,cm^{-3}}\right)^{2}\left(\frac{T}{50\,MeV}\right)^{7/2} erg\,cm^{-3}\,s^{-1} \tag{12}$$

for the KSVZ model, non-degenerate neutrons. In all cases T is the temperature of the newly-born neutron star, ρ is its density, and Y_n is the relative neutron abundance in the medium.

The spectral distribution of axions also matters, and it is different in the degenerate and non-degenerate approximations. A general expression of the emissivity is [27]

$$\frac{1}{Q}\frac{dQ}{dx} = \frac{x^2 s(x) e^{-x}}{\int_0^\infty x^2 s(x) e^{-x}} \tag{13}$$

where $x = E_a/T$, and

$$s_{nd}(x) \simeq \sqrt{1+\frac{x\pi}{4}} \tag{14}$$

$$s_d(x) \simeq \frac{x\left(x^2+4\pi^2\right)}{4\pi^2(1-e^{-x})} \tag{15}$$

are the functions for the non-degenerate and degenerate cases, respectively.

3 Axion decay times in strong magnetic fields

As mentioned in the previous Section, the strong magnetic field affects the emission of axions for both axion models considered here, increasing the probabilities, and even opening new channels for axion emission. As a corollary, the same high values of B also affect the reverse processes, that is, the decay of axions, increasing the probability of decay into electron-positron pairs ($a \longrightarrow e^+e^-$).

The decay of axions in strong magnetic fields was studied by Mikheev and Vassilevskaya [12] and Mikheev et al. [29], from which we quote the following lifetimes of axions with energy E_a:

$$\tau_{DFSZ} = 2.5\times10^{6}\cos^{-4}\beta\left(\frac{m_a}{4\times10^{-3}\,eV}\right)^{-2} \left(\frac{E_a}{1\,MeV}\right)\left(\frac{B}{B_S}\right)^{-1}\sqrt{1-\left(\frac{2m_e}{E_a\sin\theta}\right)^{2}}\; s \tag{16}$$

$$\tau_{KSVZ} = 1.9\times10^{6}\left(\frac{m_a}{4\times10^{-3}\,eV}\right)^{-2} \left(\frac{E_a}{1\,MeV}\right)\left(\frac{B}{B_S}\right)^{-1} s \tag{17}$$

whenever the following condition

$$E_a^2 \ll \left(\frac{m_e}{\sin\theta}\right)^2 \frac{B}{B_S} \tag{18}$$

is fulfilled, and

$$\tau_{DFSZ} = 1.2\times10^7 \cos^{-4}\beta \left(\frac{m_a}{4\times10^{-3}\,eV}\right)^{-2} \left(\frac{E_a}{100\,MeV}\right)^{1/3} \left(\frac{B\sin\theta}{B_S}\right)^{-2/3} s \tag{19}$$

$$\tau_{KSVZ} = 1.0\times10^{10} \left(\frac{m_a}{4\times10^{-3}\,eV}\right)^{-2} \left(\frac{E_a}{100\,MeV}\right)^{-1/3} \left(\frac{B\sin\theta}{B_S}\right)^{-4/3} s \tag{20}$$

in the opposite limit

$$E_a^2 \gg \left(\frac{m_e}{\sin\theta}\right)^2 \frac{B}{B_S}\,, \tag{21}$$

with θ being the angle between the axion moment and the magnetic field vector.

The spatial region where the magnetic field is above the critical value, a condition needed to allow the decay of axions into electrons and positrons, is restricted to the radius

$$R_S = 10 \left(\frac{R}{10\,km}\right)\left(\frac{B_0}{B_S}\right)^{1/3} km \tag{22}$$

R being the magnetar radius, and B_0 the surface magnetic field. Here we adopted the simplest dipole configuration for the magnetic field (as usual for neutron stars) even though its realization is questionable. The axion velocity can be taken as c for our purposes. The time in which any axion would cross R_S is about 10^{-4} s (taking $B_0 \sim 10B_S$), far less than any of the lifetimes obtained from eqs. (16) to (20), indicating that less than 10^{-10} of the axions would be able to generate a pair.

However, at face value in eq. (16), $E_a \sin\theta$ can be chosen arbitrarily close to m_e, making the lifetime to approach zero. In fact [???] showed that a minimum value different from zero is reached, because of the axion dispersion relation in the magnetic field. This minimum limit is expressed as

$$\tau_{min} = 1.2\times10^{-3} \cos^{-8/3}\beta \left(\frac{m_a}{4\times10^{-3}\,eV}\right)^{-4/3} \left(\frac{E_a}{1\,MeV}\right)\left(\frac{B}{B_S}\right)^{-2/3} s \tag{23}$$

which is comparable to the crossing time, indicating that a important fraction of the axions can give origin to e^+e^- pairs. In the next Section we will calculate how much energy is deposited in axions, and which amount can be converted into pairs.

4 Estimate of the energy transferred by axions

Admitting that the neutron star have no spatial or time variation of temperature, density, and axion emissivity, all which constitute gross approximations (see [30] for a more realistic view), though helpful to simplify calculations, we define $dN_a(t_0)/dE_a$ as the number of axions that leave the neutron star surface at any generic instant t_0 emitted with energies between E_a and $E_a + dE_a$.

The total energy emitted in axions is just

$$\epsilon_a(R) = \int_0^{\Delta t}\int_0^R\int_0^\infty 4\pi r^2 \frac{dQ}{dE_a}\, dt\, dr\, dE_a \tag{24}$$

where Δt s is the total emission time in which axions are emitted. Eq.(24) can be written as

$$\epsilon_a = \int_0^{\Delta t}\int_0^{\pi/2}\int_0^\infty E_a \sin\theta \frac{dN_a}{dE_a}\, dt\, d\theta\, dE_a \tag{25}$$

Outside of the magnetar, the number of axions is reduced by pair creation, catalyzed by the magnetic field. We may describe the decay by

$$\frac{d}{dt}\left(\frac{dN_a}{dE_a}\right) = -\frac{1}{\tau}\frac{dN_a}{dE_a} \tag{26}$$

Axions emitted radially from the magnetar surface at time t_0 will be at distance $r = R + c(t - t_0)$ from the magnetar center at instant t. This means that eq.(26) can be written as a function of distance instead of time, like

$$\frac{d}{dr}\left(\frac{dN_a}{dE_a}\right) = -\frac{1}{c\tau}\frac{dN_a}{dE_a} \tag{27}$$

Integrating from the magnetar surface (R) until a distance r, within the region where $B > B_S$ ($r < R_S$), we immediately find

$$\frac{dN_a}{dE_a}(r) - \frac{dN_a}{dE_a}(R) = -\int_R^r \frac{1}{c\tau}\frac{dN_a}{dE_a}\, dr \tag{28}$$

Comparing with eq.(25), it is easy to convert eq.(28) into

$$\epsilon_a(r) = \epsilon_a(R) - \int_R^r\int_0^{\pi/2}\int_0^\infty \frac{E_a\sin\theta}{c\tau}\frac{dN_a}{dE_a}\, dr\, d\theta\, dE_a \tag{29}$$

which shows how to evaluate the energy still carried by axions at a distance r from the magnetar center, provided that $R < r < R_S$. The difference $\epsilon_a(R) - \epsilon_a(R_S)$ gives the total axion energy that is deposited into pairs, and expressed as

$$\epsilon_{pair} = \int_R^{R_S} \int_0^{\pi/2} \int_0^{\infty} \frac{E_a \sin\theta}{c\tau} \frac{dN_a}{dE_a} \, dr \, d\theta \, dE_a \tag{30}$$

We can also calculate the axion-to-pairs conversion efficiency as

$$\eta = \frac{\epsilon_{pair}}{\epsilon_a(R)} = \frac{35}{128} \int_R^{R_S} \int_0^{\pi/2} \int_0^{\infty} \frac{x^2 s(x) e^{-x} \sin\theta}{c\tau} \, dr \, d\theta \, dE_a \tag{31}$$

through the use of eq.(13).

As the minimum lifetime from eq.(23) is $\sim 10^{-10}$ of the others, we will take just that one in consideration. It means the restrictions from eq. (18) and the limit condition

$$E_a = \frac{2m_e}{\sin\theta} \tag{32}$$

must be fulfilled. Additionally, the combination of those two conditions result in a third, $B > 4B_S$, or $R < 4^{1/3} R_S$. The condition of eq.(32) defines θ as function of E_a, up to a multiplicative factor

$$\delta\left[\theta - \arcsin\left(\frac{2m_e}{E_a}\right)\right]$$

in equations (31) and (30). Since the minimum energy an axion must possess to decay into a pair is $2m_e$, we must replace that value as the minimum energy in the energy integral lower limit, on those same equations. After these considerations, eq.(31) can be integrated on distance and angle, resulting in

$$\eta = 3.2 \times 10^{-6} \cos^{8/3}\beta \left(\tfrac{m_a}{4\times 10^{-3}\, eV}\right)^{4/3} \left(\tfrac{T}{50\, MeV}\right)^{-2} \left(\tfrac{B_0}{B_S}\right)^{2/3} \left[1 - \left(\tfrac{4B_S}{B_0}\right)^{1/3}\right] \int_{2m_e/T}^{\infty} s(x) e^{-x}\, dx \tag{33}$$

Since the axion decay is markedly anisotropic, half of the energy is deposited within a cone of aperture 2σ. The angle $sigma$ can be found numerically from the equation

$$\frac{\eta_{cone}}{\eta} = \frac{\int_{2m_e/T \sin\sigma}^{\infty} s(x) e^{-x}\, dx}{\int_{2m_e/T}^{\infty} s(x) e^{-x}\, dx} = \frac{1}{2} \tag{34}$$

where η_{cone} is the same as eq.(33), though integrated from $\theta = 0$ to $\theta = \sigma$ (the efficiency within the cone). The

The luminosity in pairs can be found by multiplying the emissivity by the correspondent efficiency (either degenerate case or non-degenerate), and by the volume of the magnetar. Proceeding this way, the luminosity in pairs for the non-degenerate case is

$$L_{nd} = 3.6 \times 10^{47}\, Y_n^2 \cos^{8/3}\beta \left(\cos^2\beta - 0.46\right)^2 \left(\frac{m_a}{4\times10^{-3}\, eV}\right)^{10/3}$$
$$\left(\frac{\rho}{6\times10^{14}\, g\, cm^{-3}}\right)^2 \left(\frac{T}{50\, MeV}\right)^{3/2} \left(\frac{B_0}{B_S}\right)^{2/3} \left[1 - \left(\frac{4B_S}{B_0}\right)^{1/3}\right] erg\, s^{-1} \qquad (35)$$

and the luminosity for the degenerate case turns out to be

$$L_{d} = 2.2 \times 10^{48}\, Y_n^2 \cos^{8/3}\beta \left(\cos^2\beta - 0.46\right)^2 \left(\frac{m_a}{4\times10^{-3}\, eV}\right)^{10/3}$$
$$\left(\frac{\rho}{6\times10^{14}\, g\, cm^{-3}}\right)^{1/3} \left(\frac{T}{50\, MeV}\right)^{4} \left(\frac{B_0}{B_S}\right)^{2/3} \left[1 - \left(\frac{4B_S}{B_0}\right)^{1/3}\right] erg\, s^{-1} \qquad (36)$$

which are displayed in Fig. 3.

5 Discussion

We have discussed in this work the main features of axion emission and decay as expected in magnetar birth events. Since bremsstrahlung of heavy nucleons is the main source of the axion luminosity, it should not be affected by the high magnetic field at least in a first approximation. The luminosity of that process has been rather well-known for years [31] and is expected to carry a $\sim 10\,\%$ fraction of the total luminosity dominated by neutrino emission. A huge magnetic field may, however, dramatically influence the probability of axion decay into pairs (which subsequently decay into photons), just in the region outside the magnetar where the escape is much easier. The emerging numbers are not terribly discouraging, although we believe it is unlikely that given the present constraints enough energy could be injected for a generic GRB to emerge. However, some "underluminous" bursts (notably the one associated with the supernova SN1998bw, which carried $\sim 10^{47} ergs$ at most) may arise in the very event of a magnetar formation. Of course this result could e changed if either even higher magnetic fields or lower temperatures are present (which appears unlikely), but also if additional axion processes contribute to lower the peak energy of the axion spectrum. Self-consistent detailed calculations need to be performed to address this novel application of axion physics.

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References

[] 1] M. Vietri, [astro-ph/9911523].
2] S.I. Blinnikov, [astro-ph/9911138].
3] B.Paczyński, Astrophys. J. 494, L45 (1998).
4] S. Woosley, Astrophys. J. 405, 273 (1993).
5] S.I. Blinnikov, I.D. Novikov, T.V. Perevodchikova and A.G. Polnarev, Soviet Ast. Lett. 10, 177 (1984).
6] B. Paczyński, Acta Astron. 41, 257 (1991).
7] M. Vietri and L. Stella, Astrophys. J. 507, L45 (1998).
8] V.V. Usov, Nature 357, 472 (1992).
9] J.A. de Freitas Pacheco and J.E. Horvath, unpublished (1999).
10] A. Loeb, Phys. Rev. D 48, 3419 (1993).
11] Z. Berezhiani and A. Drago, [astro-ph/9911333].
12] N.V. Mikheev and L.A. Vassilevskaya, Phys. Lett. B 410, 203 (1997).
13] J.C. Wheeler, I. Yi, I., P. Höflich and L. Wang, [astro-ph/9909293].
14] C.Graziani, D.Q. Lamb and G.H. Marion, Astron. Astrophys. Supp. 138, 469 (1999).
15] J.S. Bloom et al., [astro-ph/9905301].
16] R. N. Manchester, in Proceedings of IAU Symposium 199, 355 Eds. A. Pramesh Rao, G. Swarup, and Gopal-Krishna, (2002).
17] R.C. Duncan and C. Thompson, Astrophys.J. 392, L9 (1992).
18] C. Kouveliotou, S. Dieters, T. Strohmayer, J. van Paradjis, G.J. Fishman, C.A. Meegan, K. Hurley, J. Kommers, I. Smith, D. Frail and T. Murakami, Nature 393, 235 (1998).
19] C. Thompson and R.C. Duncan, Astrophys. J. 473, 322 (1996).
20] M.G. Baring and A.K. Harding, Astrophys. J. 507, L55 (1998).
21] K. Hurley, [astro-ph/9912061].
22] R.D. Peccei and H.R. Quinn, Phys. Rev. Lett. 38, 1791 (1977).
23] A. P. Zhitnitskii, Sov. J. Nucl. Phys. 31, 260 (1980).
24] M. Dine, W. Fischler and M. Srednicki, Phys. Lett. B 104, 199 (1981).
25] J. Kim, Phys. Rev. Lett. 43, 103 (1979).
26] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B

166, 493 (1980).
27] G.G. Raffelt, *Stars as Laboratories for Fundamental Physics* (The University of Chicago Press, Chicago 1996).
28] A.H. Córsico, O.G. Benvenuto, L.G. Althaus, J. Isern and E. García-Berro, New Astron. 6, 197 (2001).
29] N.V. Mikheev, O.S. Ovchinnikov, L.A. Vassilevskaya, Mod. Phys. Lett. A 13, 321 (1998).
30] A. Burrows and J.M. Lattimer, Astrophys. J. 307, 178 (1986).
31] R.P. Brinkmann and M.S. Turner, Phys. Rev. D 38, 2338 (1988).