EXPLODING BOSE-EINSTEIN CONDENSATES AND COLLAPSING NEUTRON STARS DRIVEN BY CRITICAL MAGNETIC FIELDS

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The problem of a condensate of a relativistic neutral vector boson gas composed by particles bearing a magnetic moment is discussed. Such a vector boson system is expected to be formed by parallel spin-pairing of neutrons or by neutral atoms in a sufficiently strong magnetic field under specific conditions of magnetic field strength and density. A strong self-magnetization arises due to a Bose-Einstein-like condensation. The system, which models the superfluid assumed to exist in the core of neutron stars, becomes more unstable under transverse collapse than the fermion gas. In the non-relativistic limit, the analogy with the behavior of exploding Bose-Einstein condensates (BECs) for critical values of the magnetic field, reported by several authors, is briefly discussed.

1 Introduction

The investigation of the properties of a condensate of vector neutral bosons having a magnetic moment is of very wide interest, its applications ranging from condensed matter (Bose-Einstein Condensates - BECs) to astrophysics (neutron stars).

Here we propose that a vector condensate arises from the Bose particles resulting from the coupling of spin up paired neutral particles. This would lead to an approximate model for ferromagnetic coupling among neutrons, which is expected to arise in presence of extremely strong magnetic fields. Because of this special configuration one expects that the resulting Bose particle to have a mass of order twice the fermion masses originating it. The idea of vector pairing (i.e. parallel spin-pairing in place of scalar or antiparallel pairing) is justified if we start from the essential property that the neutron ground state corresponds to spin alignment along the external field, and for fields large enough the difference between parallel and antiparallel pairing is a significant fraction of the energy of the pair. It is expected then that the spin parallel is a more energetically favorable coupling than the antiparallel or scalar one. Thus, the superfluid, which is the physical realization of the vector condensate, is also a ferromagnet.

For charged particles the situation is similar, and for electrons, this has lead to the suggestion of re-entrant superconductivity, for some condensed matter systems in magnetic fields of order 10^6 G. In this system a large population of parallel spin electrons is expected to occur in the Landau ground state $n = 0$ (see Boebinger and Passner [1] and references quoted therein).

2 Spectrum

We start by considering a relativistic neutral boson gas of particles with nonzero magnetic moment.

A variety of physical arguments allow for the bosons to be described as a neutral vector gauge field, even if it is the result of a pairing of fermions. Its spectrum can be derived from the poles of the inverse propagator for a massive gauge neutral boson that couples to an external magnetic field through the coupling constant $g = Mq$, where M is the mass of the particles and q has the dimensions of charge. The inverse propagator reads

$$D^{-1}(k_\mu, F_{\mu\nu}, M) = (k^2 + M^2)\delta_{\mu\nu} - k_\mu k_\nu - igF_{\mu\nu}\,, \tag{1}$$

where $F_{\mu\nu} = B(\delta_{\mu 1}\delta_{\nu 2} - \delta_{\mu 2}\delta_{\nu 1})$ is the electromagnetic field tensor describing the field B, which is pointing along the 3-rd axis in this our treatment. It is easy to find the spectrum from $DetD^{-1}(k_\mu, F_{\mu\nu}, M) = 0$ as

$$E(p, B) = \mp\sqrt{p_3^2 + p_\perp^2 + M^2 + \eta(\sqrt{p_\perp^2 + M^2})qB}\,, \tag{2}$$

where $\eta = \mp 1$ and the minus and plus signs in front of the square root indicate particles aligned respectively along and opposite to the magnetic field B. The ground state corresponds to the case $\eta = -1$. The magnetic moment of the particles is in this case defined as:

$$m = -\frac{\partial E(0, B)}{\partial B} = \frac{q}{\sqrt{M^2 - MqB}}$$

.

3 Thermodynamical properties

Starting from Eq.(2), one can write an expression for the thermodynamical potential $\Omega = -T \ln \mathcal{Z}$, where $\mathcal{Z}$ is the partition function. Once Ω is known, if μ is the chemical potential, the density is given by the formula $N = -\frac{\partial \Omega}{\partial \mu}$. Its expression reads

$$N = \frac{2\pi}{\hbar^3} \Sigma_\eta \int_{-\infty}^{\infty} p_\perp dp_\perp dp_3 [\frac{1}{e^{(E(\eta)-\mu)/T} - 1} - \frac{1}{e^{(E(\eta)+\mu)/T} - 1}] , \qquad (3)$$

and the thermodynamical potential turns out to be

$$\Omega = -\frac{T}{4\pi^2} \sum_{\eta=1,-1} \int_0^\infty p_\perp dp_\perp dp_3 \left[\ln(f^- f^+) + T^{-1} E(\eta) \right] , \qquad (4)$$

where $f^\mp = (1 - e^{-(E(\eta)\mp\mu)/T})$, and the second distribution term in Eq.(3), as well as the second logarithmic term in Eq.(4), account for the antiparticle contribution, whereas the term $T^{-1}E(\eta)$ accounts for the quantum vacuum term. Its contribution is independent of T and is expected to be significant for fields of order $qB \sim M$, a case we will not consider in what follows. Also, the antiparticle terms, which become interesting only at very high temperatures, will not be considered in this approach, because we are focusing our attention in a model of a canonical neutron star, usually said to be a "cold" degenerate neutron gas. One can show that for fixed N, μ is a decreasing function of T. Thus, by decreasing T (or by increasing the density) μ increases, and reaches its maximum value at the ground state energy (magnetic moment up) $\mu = \sqrt{M^2 - MqB}$, at some critical temperature T_c. In other words, the neutral particle temperature Green's function has an infrared pole at

$$p_4 = i\omega = \pm i(E(0, B) - \mu) = i(\sqrt{M^2 - MqB} - \mu) = 0$$

.

As in the usual theory of Bose-Einstein condensation, for temperatures $T < T_c$, the integral $N(T) < N$ gives the number of excited particles, the condensate being $N_0 = N - N(T) = N(1 - (T/T_c)^{3/2}$. Obviously, due to the magnetic field the system is subject to strong magnetostrictive forces expressed in anisotropic pressures [5]: they are different along and perpendicular to **B**.

The anisotropic pressures for the gas of magnetized particles were obtained for charged particles in previous papers [2,3], and for neutral particles in a recent one [4].

The pressure along the field is $P_3 = -\Omega$, and perpendicular to it is

$$P_\perp = -\Omega - B\mathcal{M}$$

, where $\mathcal{M}$ is the magnetization. Such an anisotropy can be compensated by deforming the body, but in some limits (discussed in Refs. [3,4] and extended here to BECs), it leads to a collapse of the system of particles.

One can estimate that near the condition for condensation, by calling $\bar{M} = \sqrt{M^2 - MqB}$, the density, Eq.(3), of excited particles is approximately given by

$$N \sim T\bar{M}\sqrt{\bar{M}^2 - \mu^2}/4\pi\,, \tag{5}$$

and decreases as $\mu \to \bar{M}$. The isotropic pressure decreases more strongly, as $P_0 = -\Omega \sim \frac{T}{6\pi}(\bar{M}^2 - \mu^2)^{3/2}$. In other words, the condensate does not contribute to the thermodynamical potential, or what is the same, to the longitudinal pressure. The thermodynamic potential of the condensate is thus zero, but its (rest) energy is non-zero, since $U_0 = \mu N = \bar{M}N$. The condensate contributes, however, negatively to the transverse pressure, as we shall see below.

4 Magnetic Behavior

In general, it is expected to have a background of excited bosons and fermions with the total transverse pressure having contributions of both fermions and excited bosons. Meanwhile, the magnetization contains in addition the contribution from the boson condensate, which may be very large. For bosons, after the phase transition to a condensate, the magnetization must be written as $\mathcal{M}_b = -\partial\Omega/\partial B + N_0 m$, where the second term contains the contribution to the magnetization due to the condensate energy, $N_0 \partial\bar{M}/\partial B = N_0 m$. In our present case, the total transverse pressure is then

$$P_{t\perp} = -(\Omega_b + \Omega_f) - B(\mathcal{M}_b + \mathcal{M}_f)$$

[3,4], where t refers to total, and b and f refer, respectively, to bosons and to fermions. If there is a large amount of bosonization and most of the particles are in the condensate, i.e., when $-(\Omega_b + \Omega_f)$ is equal or smaller than the negative pressure term $-B(\mathcal{M}_b + \mathcal{M}_f)$, either the total pressure perpendicular to the field vanishes or becomes negative and the system collapses. We consider two cases: the relativistic neutron star, and the non-relativistic gas of few millions of atoms, that is a BEC.

For a neutron star, and in the case where there is neutrons' spin-pairing parallel to B, which leads to an effective spin one boson particle as the one described above, having an effective mass as that of neutron m_n. Thus, even assuming temperatures of $\sim 10^8$K, since $m_n/T \sim 10^5$, the system must be considered as highly degenerate, that is, below the critical temperature ($T_c \geq 10^{11}$ K), and the density of excited particles is negligible small as compared with the condensate density. Also, the Bose-Einstein condensate would lead to a more energetically favorable state than the fermion gas, as is seen by starting from the general expression for the internal energy density, $U = \mu N + TS + \Omega$.

As for the condensate it is $\Omega = 0$ and $S = -\partial\Omega/\partial T = 0$, its internal energy is $U_b \sim \mu_b N_b$, where $\mu_b \leq 2M_f$ and $N_b \leq N_f/2$, whereas for the Fermi gas it is $U_f \sim \mu N_f + \Omega_f$, where Ω_f is positive and of order μN_f, and $\mu_f > M_f$. Thus, $U_b < U_f$. As pointed out above, the Bose condensate bears the properties of a superfluid and also of a ferromagnet. Therefore, most of the system must be in the condensate state.

If initially a strong magnetic field is created locally in the star (say, of order $B_i \sim 10^{12-13}$ G, as inherited from the implosion of the progenitor (a supergiant red star) by the mechanism of flux conservation, that field would be amplified by the Bose-Einstein condensation. As the particles in the condensate are described by the spectrum of Eq.(2), then in the limit of zero momentum, and for $\eta = -1$, the condensate becomes polarized. Thence, one can estimate the magnetization as given as $\mathcal{M}_f = N_0 m$ (the thermodynamical potential of the excited particles is obviously zero).

By taking $N_0 \sim 10^{39}$, $m \sim 10^{-23}$, one gets that the system generates a spontaneous magnetization $\mathcal{M}_f \sim 10^{16}$ G, leading to a self-consistent field

$$B_f = 4\pi\mathcal{M}_f \sim 10^{17}G$$

. In this sense, the bosonization we are describing is a simplified version of a more realistic superfluid or ferromagnetic coupling in which the particles interact usually with several neighbors, whereas we are assuming in our bosonization model that it results from two-particle vector pairing. However, by calling B_f, B_i the final and initial fields, respectively, we see that (in the present case) it leads to an internal field parameter of order $x = B_f/B_i \sim 10^{3-4}$. Then, the pressure perpendicular to $\mathbf{B}$ would be negative and of order 10^{33-34} erg cm^{-3}, hence driving a transversal collapse.

5 The Exploding BECs of Atoms

The non-relativistic limit can be discussed straightforwardly, since the sort of condensation we are discussing is a low-momentum phenomenon. In the case of BECs in the laboratory various experimental groups estimate the BECs density as aorund $N \sim 10^{14}$ and $m \sim 10^{-20}$. For laboratory magnetic fields of order 10^3 G, this combination of physical properties leads to a negative pressure of ~ 1 dyn cm^{-2}, which means a compression of the condensate perpendicular to the field. This compression may lead itself to a collapse. Collapse (implosion) of BECs has been observed by several groups (see Ref.[6] and references therein). Although such phenomenon is attributed to an effect produced by tuning the magnetic field so as to make the interatomic distance to be in a region of repulsive potential, it seems to us that the negative pressure contribution might play a significant if not a fundamental role in driving the exploding BEC.

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