INTERACTION OF ELECTROMAGNETIC RADIATION WITH SUPERCRITICAL MAGNETIC FIELD

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It is pointed, that for the asymptotically strong - super-Schwinger - magnetic field B, exceeding the critical value of $B_{cr} = (m^2c^3/eh) = 4.4 \cdot 10^{13}$ Gauss, the vacuum polarization effects become important not only in the γ-range, but also for softer electromagnetic quanta, including X-rays, optic photons and electromagnetic waves of radio frequencies. This is a consequence of the linearly growing term $\asymp (B/B_{cr})$ present in the vacuum polarization in the asymptotically strong magnetic field. The results may be essential in studying reflection, refraction and splitting of X-rays, light and radio waves by magnetic fields of magnetars, as well as in considering emission of such waves by charged particles .

1 Introduction

Although it is long since the refracting and bi-refriging properties of a strong magnetic field in the vacuum have been realized, up to now their only essential consequences considered in a realistic astrophysical context remain the photon splitting effect [1,2] and the effect of photon capture[3−9]

Both effects are currently discussed mostly as applied to e-m radiation belonging to the γ-range. They depend crucially on the deviation of the photon dispersion curve from the customary shape it has in empty vacuum, $k_0^2 = |\mathbf{k}|^2$, where k_0 is the photon energy, and $\mathbf{k}$ is its momentum. As long as one restricts oneself to considering the magnetic fields B below the Schwinger critical value $B \leq B_{cr} = (m^2/e) = 4.4 \cdot 10^{13}$ Gauss, the only essential source of this deviation is the singular behavior of the polarization operator $\Pi_{\mu\nu}(k)$ near the creation thresholds of mutually independent electron and positron on Landau levels n, n' by a photon (the cyclotron resonance) [3,4,5] or a still stronger singular behavior of $\Pi_{\mu\nu}$ near the thresholds of a mutually bound e^+e^--pair (the positronium atom) formation[6−10]. To reach (at least the lower of) these thresholds the photon should belong to the γ-ray range, with its energy above or of the order of 1 MeV. For this reason the effect of photon capture, with its transformation into electron-positron pair, derived from the singular behavior of $\Pi_{\mu\nu}(k)$, concerns mostly the γ-quanta, as long as their propagation in a pulsar magnetosphere of traditional pulsars is concerned. It was estimated, that the fields about $B = 0.1B_{cr}$ are sufficient to provide this effect[5] and to protect the positronium atom, into which the captured γ-quantum is transformed, against ionization by the accelerating electric field in the polar gap and by the thermal photons[7−10].

Also the Adler effect[1] of photon splitting $\gamma \to \gamma\gamma$ for such fields is usually discussed relating to γ- quanta[11−14]. There are two reasons why, again, the γ-range is important. The first one is that the photon splitting becomes possible in the magnetic field, because the deviation of the dispersion curve from the $k_0^2 = \mathbf{k^2}$ law opens a kinematical aperture for this process - the wider, the stronger the deviation (and the deviation is strong near the thresholds). Besides, there is a strong birefringence for the photons in the γ-range, since only one eigenvalue $\kappa_2(k)$ of the tensor $\Pi_{\mu\nu}$ is singular near the lowest ($n = n' = 0$) thresholds, while the other two eigenvalues $\kappa_{1,3}(k)$ remain finite, until the next thresholds ($n = 0$, $n' = 1$ or $n = 1$, $n' = 0$) are reached. This implies that the photons of only one polarizational mode are essentially affected by the medium. This birefringence leads to polarization selection rules in the photon splitting process, which are well pronounced. The second reason is dynamical. The matrix elements of the photon splitting are subject to the same resonant behavior near the thresholds as the polarization operator. The aforesaid explains why mainly the γ-range is the first to be affected by the magnetized vacuum.

The situation changes considerably after one passes to super-Schwinger magnetic fields $B \gg B_{cr}$, expected to be existing in soft gamma-ray repeaters and anomalous X-ray pulsars, whose magnetic field is estimated that large (I refer here to the talk, given by Prof. A.Reisenegger in this Workshop[15], and also to Ref.[16]). The point is that in such asymptotic regime a linearly growing term, proportional to B/B_{cr}, appears[14,18,17] in one of the eigenvalues, κ_2, of the polarization operator, thus providing an extra (apart from the cyclotron resonance) large contribution into the refraction of the vacuum.

In Section 3 we study the consequences of this phenomenon for the photon propagation, basing on the first three leading contributions into asymptotic expansion of the polarization operator eigenvalues for large B, obtained within the one-loop approximation. One of these consequences is a frequency-independent, but direction-sensitive, large refraction index for propagation non-parallel to the magnetic field in one (out of three) polarization modes in the kinematical domain far from the threshold. The corresponding strong polarization- and direction-sensitive refraction occurs for electromagnetic radiation of any frequency range, including X-ray, optic and radio range.

This study is preceded by Section 2, where exact results concerning the electromagnetic radiation propagation in the magnetized vacuum are described. These follow exceptionally from the general properties of relativistic, gauge, charge invariances [19] and Onsager theorem [20]. The results, presented in Section 2, are valid irrespective of any approximation and the field strength, unless the opposite is explicitly indicated.

2 Exact Facts about Electromagnetic Eigenmodes in an External Magnetic Field

There are three propagating eigenmodes, corresponding to the vacuum excitations with photon quantum numbers in an external magnetic field B. The dispersion law, *i.e.* the dependence of the energy k_0 of the quantum (or the frequency in the wave) on its momentum $\mathbf{k}$, is given for each mode by a solution of the equation

$$k^2 = \kappa_i(k_0^2 - k_\parallel^2, k_\perp^2), \qquad i = 1, 2, 3. \tag{1}$$

Here $k_\parallel$ and $k_\perp$ are the momentum components along and perpendicular to the magnetic field $\mathbf{B}$, respectively, and k^2 is the photon four-momentum squared, $k^2 = k_\perp^2 + k_\parallel^2 - k_0^2$. The κ's in the right-hand sides in Eqs.(1) are eigenvalues of the polarization operator[3,4,19].

It is a general consequence of the relativistic covariance that the eigenvalues depend upon the two combinations of momenta, specified in Eq. (1). This implies, that solutions of the dispersion Eqs. (1) have the following general structure

$$k_0^2 = k_\parallel^2 + f_i(k_\perp^2), \qquad i = 1, 2, 3, \tag{2}$$

and that the direction of the group velocity in each mode, $\mathbf{v} = \partial k_0/\partial \mathbf{k}$, does not coincide (as long as $k_\perp \neq 0$) with that of the phase velocity $\mathbf{k}/k_0$. To see this, calculate the components of the group velocity $v_{\perp,\parallel}$ across and along the magnetic field $\mathbf{B}$, resp., on solutions (2) of each dispersion equation (1)

$$v_\perp \equiv \frac{\partial k_0}{\partial k_\perp} = \frac{k_\perp}{k_0}\frac{\partial k_0^2}{\partial k_\perp^2} = \frac{k_\perp}{k_0}\frac{1 - \frac{\partial \kappa_i}{\partial k_\perp^2}}{1 + \frac{\partial \kappa_i}{\partial (k_0^2 - k_\parallel^2)}} = \frac{k_\perp}{k_0}\frac{\mathrm{d} f_i(k_\perp^2)}{\mathrm{d} k_\perp^2}\,,$$

$$v_\parallel \equiv \frac{\partial k_0}{\partial k_\parallel} = \frac{k_\parallel}{k_0}\,. \tag{3}$$

It follows from Eq. (3) the angle θ the direction $\mathbf{v}$ of the electromagnetic energy propagation makes with the external magnetic field satisfies the relation

$$\frac{v_\perp}{v_\parallel} \equiv \tan\theta = \left(1 - \frac{\partial \kappa_i}{\partial k_\perp^2}\right)\left(1 + \frac{\partial \kappa_i}{\partial (k_0^2 - k_\parallel^2)}\right)^{-1} \tan\vartheta, \tag{4}$$

where ϑ is the angle between the photon momentum (phase velocity) and the external field, $\tan\vartheta \equiv k_\perp/k_\parallel$. The following statement takes place: if the phase velocity exceeds the velocity of light c, *i.e.* if $k_\perp^2 + k_\parallel^2 > k_0^2$, (or $f_i(k_\perp^2) < k_\perp^2$ in (2)), but the group velocity (3) does not, $v_\perp^2 + v_\parallel^2 \leq 1$, then $\tan\theta < \tan\vartheta$. The conditions of this statement are fulfilled for the dispersions laws found within the approximation-dependent calculations of the κ's. For the super-Schwinger fields, treated within one-loop approximation, this fact follows explicitly from equations of Section 3 below. Therefore, the photon tends to deviate closer to the magnetic field line, as compared to its momentum direction.

It follows from the gauge invariance that

$$\kappa_i(0,0) = 0, \qquad i = 1,2,3. \tag{5}$$

This property implies that for each mode there always exists a dispersion curve with $f_i(0) = 0$, which passes through the origin in the $(k_0^2 - k_\parallel^2, k_\perp^2)$-plane. However, out of these three solutions only two may simultaneously correspond to physical massless particles - the photons. The third one is a non-physical degree of freedom, characteristic of gauge theories: in a magnetic field a photon has two degrees of freedom, the same as in the empty vacuum. Which of the modes becomes non-physical, depends upon the direction of propagation and the specific form of the function $f_i(k_\perp^2)$ in (2). We shall further discuss this matter for the super-Schwinger field limit in the next Section. Massive branches of solutions of Eqs. (1), with $f_i(0) > 0$, may also exist, despite condition (5). For them the number of physical degrees of freedom is three, so all the three equations (1) may have physical solutions simultaneously (see, *e.g.* the positronium branches found in [8,22,23])

The refraction index n_i in mode i is

$$n_i \equiv \frac{|\mathbf{k}|}{k_0} = \left(1 + \frac{\kappa_2}{k_0^2}\right)^{\frac{1}{2}} = \left(1 + \frac{k_\perp^2 - f_i(k_\perp^2)}{k_0^2}\right)^{\frac{1}{2}}. \tag{6}$$

The gauge invariance property (5) implies that the refraction index (6) for parallel propagation, $k_\perp = 0$, be exactly equal to unity for the massless, $f_i(0) = 0$, branches in every mode

$$n_i^\parallel = 1. \tag{7}$$

The electromagnetic wave propagating strictly along the external constant and homogeneous magnetic field does this with the velocity of light in the vacuum c, the phase and group velocities coinciding in this case.

If, within a certain approximation, the eigenvalue κ_i is a linear function of its arguments with the condition (5) met, the refraction index (6) for the corresponding mode depends on a single combination of the photon energy and momenta, which is the direction of propagation ϑ. This happens in a non-resonant situation, for instance, as described in the next section.

The polarizations of the modes are described in an approximation- independent way[4,19] by the following relations, where $\mathbf{e}^{(\mathbf{i})}$ and $\mathbf{h}^{(\mathbf{i})}$ are the electric and magnetic fields in the wave belonging to mode number $i = 1, 2, 3$

$$\mathbf{e}^{(1)} = -\frac{\mathbf{k}_\perp}{k_\perp} k_0, \qquad \mathbf{h}^{(1)} = (\frac{\mathbf{k}_\perp}{k_\perp} \times \mathbf{k}_\|), \tag{8}$$

$$\mathbf{e}_\perp^{(2)} = \mathbf{k}_\perp k_\|, \quad \mathbf{e}_\|^{(2)} = \frac{\mathbf{k}_\|}{k_\|}(k_\|^2 - k_0^2), \quad \mathbf{h}^{(2)} = -k_0 \left(\mathbf{k}_\perp \times \frac{\mathbf{k}_\|}{k_\|} \right), \tag{9}$$

$$\mathbf{e}^{(3)} = -k_0 \left(\frac{\mathbf{k}_\perp}{k_\perp} \times \frac{\mathbf{k}_\|}{k_\|} \right), \quad \mathbf{h}_\perp{}^{(3)} = -\frac{\mathbf{k}_\perp}{k_\perp} k_\|, \quad \mathbf{h}_\|{}^{(3)} = \frac{\mathbf{k}_\|}{k_{\ \|}} k_\perp. \tag{10}$$

Here, the cross stands for the vector product, and the boldfaced letters with subscripts $\|$ and $\perp$ denote vectors along the directions, parallel and perpendicular to the external magnetic field, resp. The electric field $\mathbf{e}$ in the wave of mode 1 is parallel to $\mathbf{k}_\perp$, in mode 2 it lies in the plane containing the vectors $\mathbf{k}, \mathbf{B}$, in mode 3 it is orthogonal to this plane, *i.e.* mode 3 is always transversely polarized.

Note, that the normalization is different for each line of Eq. (10), so one may judge about vanishing of some components as compared to others within one line, but not between different lines.

Concerning the direction of propagation, two cases are essentially different. If $k_\perp = 0$, we speak about longitudinal propagation. If not, a Lorentz boost along the external (constant and homogeneous) magnetic field exists, which does not change the value of the latter and does not introduce an extra electric field, but nullifies $k_\|$. Hence, the general case of non-parallel propagation $k_\perp \neq 0$, $k_\| \neq 0$ is reduced to purely transversal propagation, $k_\| = 0$ (in the corresponding reference frame). One should keep in mind, however, that the above transformation changes the photon energy k_0 and should be treated with precautions when one considers a field with curved lines of force.

For transversal propagation, $\mathbf{k} \perp \mathbf{B}$, ($k_\| = 0$), modes 2, 3 are transversely polarized $\mathbf{e}^{(2,3)} \perp \mathbf{k}$ in two mutually orthogonal planes, $\mathbf{e}^{(2)} \perp \mathbf{e}^{(3)}$, while mode 1 is longitudinally polarized, $\mathbf{e}^{(1)} \parallel \mathbf{k}$ with no magnetic field in it, $\mathbf{h}^{(1)} = 0$. It is expected not to correspond to a photon (depending on the dispersion law).

On the contrary, for longitudinal propagation, $\mathbf{k} \parallel \mathbf{B}$, ($k_\perp = 0$), modes 1, 3 are transversely polarized, $\mathbf{e}^{(1,3)} \perp \mathbf{B}$, and their electric field vectors lie in mutually orthogonal planes, $\mathbf{e}^{(1)} \perp \mathbf{e}^{(3)}$, as they always do, while mode 2 is longitudinally polarized, $\mathbf{e}^{(2)} \parallel \mathbf{B}$, and does not contain a magnetic field, $\mathbf{h}^{(2)} = 0$. This time mode 2 is expected not to correspond to a photon, whereas mode 1 is a physical electromagnetic wave, which matches the electromagnetic wave of mode 3: together they may form a circularly polarized transversal wave because of the degeneracy property

$$\kappa_1((k_0^2 - k_\parallel^2), 0) = \kappa_3((k_0^2 - k_\parallel^2), 0)\,. \tag{11}$$

The latter reflects the cylindric symmetry of the problem of a photon propagating along the external magnetic field.

Another remark of almost general character is in order. One might expect the possibility of the Cherenkov radiation by a charged particle moving in an optically dense medium formed by the magnetized vacuum, provided the particle moves faster than the group velocity of the radiation it emits. This effect, however, does not take place in known situations. Consider an emission of a photon by an electron in a magnetic field, not accompanied by the change of its Landau quantum number, $n = n'$ (otherwise, that would be cyclotron, and not Cherenkov, radiation). According to the kinematical analysis of the energy and momentum conservation, done in Ref.[21] (also to the study[21] of analyticity regions of the one-loop photon polarization operator in electron-positron plasma in a magnetic field, calculated in Ref.[20]), the Cherenkov photon may only belong to the right lower sector

$$k_0^2 - k_\parallel^2 \leq 0, \quad k_\perp^2 \geq 0 \tag{12}$$

in the $(k_0^2 - k_\parallel^2, k_\perp^2)$-plane. The substantial reason for this circumstance is the degeneration of the electron energy with respect to the center-of-orbit position in the transversal plane. No dynamical calculations, hitherto known, provide a penetration of photon dispersion curves into this sector. The only exception is a non-physical situation due to exponentially strong external fields, to be mentioned in Subsection 3.2 below. We conclude that under standard conditions, no Cherenkov effect is possible.

3 Photon Dispersion in Super-Schwinger Magnetic Field

3.1 Asymptotic expansion of polarization tensor eigenvalues

In the asymptotic region of super-critical magnetic fields, $B \gg B_{cr}$, and restricted energy of longitudinal motion $k_0^2 - k_\parallel^2 \ll (B/B_{cr})m^2$, the three eigenvalues $\kappa_{1,2,3}(k)$ of the polarization operator, calculated within the one-loop approximation in Ref.[19,25] have the following behavior, derived from equations of Ref.[4]

$$\kappa_1(k_0^2 - k_\parallel^2, k_\perp^2) = \frac{\alpha k^2}{3\pi}\left(\ln\frac{B}{B_{cr}} - C - 1.21\right)$$
$$-\frac{\alpha k^2}{\pi}\left(\exp\left(-\frac{k_\perp^2}{2m^2}\frac{B_{cr}}{B}\right) - 1\right), \tag{13}$$

$$\kappa_2(k_0^2 - k_\parallel^2, k_\perp^2) = \frac{\alpha k^2}{3\pi}\left(\ln\frac{B}{B_{cr}} - C\right) - \frac{\alpha}{3\pi}\left(1.21k_\perp^2 + 0.69(k_0^2 - k_\parallel^2)\right)$$
$$+\frac{\alpha}{3\pi}\left(-3k_\perp^2 + (k_0^2 - k_\parallel^2)\right)\left(\exp\left(-\frac{k_\perp^2}{2m^2}\frac{B_{cr}}{B}\right) - 1\right)$$
$$+\frac{\alpha B m^2(k_0^2 - k_\parallel^2)}{\pi B_{cr}}\exp\left(-\frac{k_\perp^2}{2m^2}\frac{B_{cr}}{B}\right)\int_{-1}^{1}\frac{(1-\eta^2)\mathrm{d}\eta}{4m^2 - (k_0^2 - k_\parallel^2)(1-\eta^2)}, \tag{14}$$

$$\kappa_3(k_0^2 - k_\parallel^2, k_\perp^2) = \frac{\alpha k^2}{3\pi}\left(\ln\frac{B}{B_{cr}} - C\right) - \frac{\alpha}{3\pi}\left(0.21k_\perp^2 - 1.21(k_0^2 - k_\parallel^2)\right)$$
$$-\frac{\alpha}{\pi}(2k_\perp^2 + k_\parallel^2 - k_0^2)\left(\exp\left(-\frac{k_\perp^2}{2m^2}\frac{B_{cr}}{B}\right) - 1\right). \tag{15}$$

Here $\alpha = 1/137$ is the fine structure constant and $C = 0.577$ is the Euler constant. Eqs. (13), (14), (15) are accurate up to terms decreasing with B like $(B_{cr}/B)\ln(B/B_{cr})$ and faster. The coefficients in front of the factor $\left(\exp\left(-\frac{k_\perp^2}{2m^2}\frac{B_{cr}}{B}\right) - 1\right)$, which does not grow with B, are calculated only taking into account contributions of the lowest Landau states of the virtual electron and positron. (This factor disappears when we are not close to the resonance and hence $k_\perp^2$ is not enormously large. We shall set the exponentials equal to unity in the most cases, considered below.)

The parts growing with B in (13), (14) and (15) were written in Ref.[18], their derivation from equations of Refs.[4] is traced in detail in Ref.[22,23]. The linearly growing term in Eq. (14) in a different way was obtained in Ref.[17] using a two-dimensional (one time, one space) diagrammatic technique developed to serve the asymptotic magnetic field regime. The logarithmic terms in the expressions above do not dominate over the constant terms unless one would like to include exponentially large magnetic fields into consideration. (That would be unreasonable not only because such fields are hardly expected to exist in nature, but mainly because their consideration is beyond the scope of Quantum Electrodynamics: the logarithmically growing terms in Eqs. (13), (14), (15) are associated with the absence of asymptotic freedom in QED (*cf.* analogous asymptotic behavior [24] in the Euler-Heisenberg effective Lagrangian).) The derivation of the terms, not growing with B in Eqs. (13), (14), (15), will be published elsewhere. The asymptotic expressions used in Ref.[14] do not coincide with ours, except for the linear-in-B term.

The limiting expressions (13), (14), (15) do satisfy the exact properties (11) and (5).

In this paper I shall only deal with the transparency region, $k_0^2-k_\parallel^2 \leq 4m^2$ (*i.e.* with the kinematic domain, where $\kappa_{1,2,3}$ are real), because I will be interested in photons with $k_0 < 2m$, even $k_0 \ll 2m$, which never reach the free pair creation threshold $k_0^2 - k_\parallel^2 = 4m^2$. In this domain the integral in Eq. (14) is well defined, and the last term in (14) is

$$-\frac{2\alpha m^2}{\pi}\frac{B}{B_{cr}}\exp\left(-\frac{k_\perp^2 B_{cr}}{2m^2 B}\right)$$
$$\times\left(1-\frac{4m^2}{(k_0^2-k_\parallel^2)^{1/2}(4m^2-k_0^2+k_\parallel^2)^{1/2}}\arctan\left(\frac{k_0^2-k_\parallel^2}{4m^2-k_0^2+k_\parallel^2)}\right)^{\frac{1}{2}}\right). \tag{16}$$

(Above the threshold, one should attribute infinitesimal imaginary part to m^2 and express the integral in terms of logarithms). The eigenvalue κ_2 (14) has a singular branching point in the complex plane of the variable $(k_0^2 - k_\parallel^2)$ near the lowest pair creation threshold $(k_0^2 - k_\parallel^2)_{thr} = 4m^2$. Thresholds of creation of e^+e^--pairs with the electron and the positron on excited Landau levels $n, n' \neq 0$

$$(k_0^2 - k_\parallel^2)_{thr}^{n,n'} = m^2\left[\left(1+n\frac{B}{B_{cr}}\right)^{\frac{1}{2}}+\left(1+n'\frac{B}{B_{cr}}\right)^{\frac{1}{2}}\right]^2 \tag{17}$$

are shifted in the asymptotic regime to infinitely remote region. For this reason the eigenvalues $\kappa_{1,3}$, responsible for photons of such polarizations that can only create e_+e_-- pairs with at least one charged particle in an excited Landau state, do not contain imaginary parts or singular branching points in this regime. On the other hand, the eigenvalue κ_2 has only one singular branching point, corresponding to the possibility of creation of an electron and a positron in the lowest Landau states by the photon, polarized as in mode 2. The singular threshold behavior of Eq. (14) near the point $k_0^2 - k_\parallel^2 = 4m^2 - \epsilon$, $\epsilon > 0$, $\epsilon \to 0$ is

$$\kappa_2(k) \sim \frac{2\alpha B m^3}{B_{cr}} \exp\left(-\frac{k_\perp^2}{2m^2}\frac{B_{cr}}{B}\right)\left(4m^2 - k_0^2 + k_\parallel^2\right)^{-\frac{1}{2}} . \tag{18}$$

As might be expected, this is the same as the behavior near this threshold of the exact one-loop expression for $\kappa_2(k)$ [4], before the limiting transition to large fields has been performed.

3.2 Propagation of eigenmodes in the super-Schwinger field limit

If Eq.(13) for κ_1 is taken as the r.-h. side of Eq. (1), the latter has only one solution, which is the trivial dispersion law $k^2 = 0$. With the relation $k^2 = 0$ satisfied, however, the 4-potential corresponding to the electromagnetic field of mode 1 becomes proportional to the photon 4-momentum vector k_μ, unless $k_\parallel = 0$ (see [4,22,23]). Therefore, for non-parallel propagation, mode 1 corresponds only the gauge degree of freedom we discussed in Section 2, with no real electromagnetic field associated with it.

The solution of equation (1) for the second mode $i = 2$ is dominated by the cyclotron resonance (18), that causes the strong deviation of the dispersion curves in Fig.1 from the shape $k^2 = 0$ (the light cone).

The behavior of the dispersion curves of mode 2 near the threshold for super-Schwinger magnetic fields $B \gg B_{cr}$ is the same as for the "moderate" fields $B \leq B_{cr}$, and therefore it also presents the photon capture effect for photons harder than $2m$, known for such fields[5]: if one calculates Eq.(4) near the threshold $k_0^2 - k_\parallel^2 = 4m^2$ using Eq.(18) as κ_2 to get

$$\tan\theta = \frac{k_\perp}{k_\parallel}\frac{B_{cr}}{Bm^2}(4m^2 - k_0^2 + k_\parallel^2) , \tag{19}$$

one concludes that the angle θ between the external magnetic field and the direction of the wave packet propagation in mode 2 tends to zero, the faster the stronger the field. If the photon energy k_0 is just a little less than $2m$, the photon may be close to the threshold when its $k_\parallel$ disappears. In this upper point the wave packet stops, since the group velocity length $v_\perp^2 + v_\parallel^2 = v_\parallel^2(1 + \tan^2\theta)$, equal to $k_\parallel^2/k_0^2$ according to the second line of Eqs.(3) and (19), disappears together with $k_\parallel$.

As applied to the conventional pattern of a pulsar magnetosphere, already described in the talk by Prof. Qiao[26], this effect acts as following[5]. A curvature γ-quantum emitted tangentially to the magnetic line of force, *i.e.* placed

Figure 1. A family of dispersion curves for mode 2 - solutions of equation (1), with Eq.(14) taken for the r.-h. side, below the threshold $k_0^2 - k_\parallel^2 = 4m^2$. The values of the external magnetic field, corresponding to the curves, are (from left to right) $B = 10B_{cr}, 100B_{cr}, 1000B_{cr}$. The straight line is the light cone - dispersion curve for $B = 0$. The dashed horizontal line marks the maximum to which the photon with the energy k_0 may proceed , provided that $k_0 < 2m$. The variables along the axes are plotted in units of $4m^2 \approx 1MeV^2$

initially in the origin of Fig.1, then evolves along its dispersion curve as it propagates in the dipole magnetic field with its line of force curved, since the components $k_\parallel$ and $k_\perp$ are changing. The maximum value of the ordinate $k_0^2 - k_\parallel^2$ occurs at $k_\parallel = 0$, and it is the photon energy squared, k_0^2. If the latter is greater than $4m^2$, the photon may achieve the horizontal asymptote in Fig.1. Here its group velocity across the magnetic field, $\mathrm{d}k_0/\mathrm{d}k_\perp$, disappears, $\mathrm{d}k_0/\mathrm{d}k_\perp \to 0$, and hence it propagates along the magnetic field and does not cross the threshold, since the other branch of the dispersion curve, which passes above the threshold, is separated by a gap from the initial branch. As a matter of fact, a mixed state: photon-pair is formed[5], analogous to polariton known in condensed matter physics. The massless part of its spectrum is presented by the dispersion curves in Fig.1. The photon gradually turns into the $e^+ - e^-$-pair and exists mostly in that form when finally it is propagating along the magnetic lines of force. This capturing effect is important for the formation of radiation of pulsars with the fields $B > 0.1B_{cr}$, since it prevents - once the binding of the electron-positron pair into a positronium atom is taken into account[6−10]- the screening of the accelerating electric field in the polar gap. It may be essential for magnetars with their fields $\sim 10^{14} - 10^{15}$ Gauss, as well.

The new features, introduced by super-Schwinger fields are that the dispersion curves for mode 2, presented in Fig.1, step aside from the light cone already far from the resonance region. This means that although the photons softer than $2m = 1MeV$ cannot proceed to the values of the ordinate in Fig.1 higher than their energy squared (corresponding to $k_\parallel = 0$), they can still reach the region where the transversal group velocity $\mathrm{d}k_0/\mathrm{d}k_\perp$ becomes much less than unity and are thus captured to the trajectory almost parallel to the magnetic field. This is how the capture effect extends to the photon energies below the border $k_0 = 2m$. The cyclotron singularity at the pair-creation threshold in such fields is so strong that even low energy photons that are unable to create a pair are sensitive to it, provided they belong to mode 2!

Besides the extension of the photon capture effect to softer photons, the inclusion of super-Schwinger fields into consideration has another impact. It leads to large direction-dependent refraction of mode 2 electromagnetic waves of low frequency. To see this, consider the limit

$$k_0^2 - k_\parallel^2 \ll 4m^2 \tag{20}$$

in Eq.(14), which reduces to neglecting $k_0^2 - k_\parallel^2$ in its integrand. Then this equation becomes (we neglected the constant terms in (14) as compared to the logarithmic and linearly growing with the field terms; the effect of the omitted terms can be readily retained in the equations below, if necessary)

$$\kappa_2(k) = \frac{\alpha}{3\pi} k^2 \ln \frac{B}{B_{cr}} + \frac{\alpha}{3\pi}(k_0^2 - k_\parallel^2)\frac{B}{B_{cr}} \exp\left(-\frac{k_\perp^2}{2m^2}\frac{B_{cr}}{B}\right). \tag{21}$$

The dispersion equation (1) for mode 2 ($i = 2$) then has the solutions expressing the photon energy k_0 as a function of its transversal and longitudinal momenta:

$$k_0^2 = k_\parallel^2 + k_\perp^2 \left(1 - \frac{\alpha}{3\pi}\ln\frac{B}{B_{cr}}\right)\left[1 - \frac{\alpha}{3\pi}\ln\frac{B}{B_{cr}} + \frac{\alpha}{3\pi}\frac{B}{B_{cr}}\exp\left(-\frac{k_\perp^2 B_{cr}}{2m^2 B}\right)\right]^{-1}. \tag{22}$$

The exponential factor in (22) cannot be essential within the region (20). Besides, the logarithmic term can be neglected unless one considers exponentially large B. Thus, we are left with the dispersion law

$$k_0^2 = k_\parallel^2 + k_\perp^2 \left(1 + \frac{\alpha}{3\pi}\frac{B}{B_{cr}}\right)^{-1}. \tag{23}$$

Eq.(23) presents analytically the straight line parts of the dispersion curves in Fig.1 adjoint to the origin for various values of B. The components of the group velocity (3) $v_{\perp,\parallel}$ calculated from (23) are

$$v_\perp = \frac{k_\perp}{k_0}\left(1+\frac{\alpha}{3\pi}\frac{B}{B_{cr}}\right)^{-1}, \qquad v_\parallel = \frac{k_\parallel}{k_0}\,. \tag{24}$$

The modulus of the group velocity squared is now

$$v_\perp^2 + v_\parallel^2 = \frac{1}{1+\frac{\alpha}{3\pi}\frac{B}{B_{cr}}} + \frac{\frac{\alpha}{3\pi}\frac{B}{B_{cr}}\cos^2\vartheta}{1+\frac{\alpha}{3\pi}\frac{B}{B_{cr}}\cos^2\vartheta}\,, \tag{25}$$

where ϑ is the angle between the photon momentum and the field, $\tan\vartheta = k_\perp/k_\parallel$. Eq.(25) has the maximum value of unity for the parallel propagation, $\vartheta = 0$, in accordance with the general statement of Section 2, and is minimum for perpendicular propagation, $\vartheta = \pi/2$.

The expression (4) for the angle θ between the direction of the electromagnetic energy propagation and the external magnetic field in the super-Schwinger limit for mode 2 becomes

$$\frac{v_\perp}{v_\parallel} = \tan\theta = \frac{k_\perp}{k_\parallel}\left(1+\frac{\alpha}{3\pi}\frac{B}{B_{cr}}\right)^{-1} = \tan\vartheta\left(1+\frac{\alpha}{3\pi}\frac{B}{B_{cr}}\right)^{-1}. \tag{26}$$

Once $\tan\theta < \tan\vartheta$, in the problem formulated above about the photon emitted tangentially to curved lines of force, it will bend towards these lines. This relates to low-frequency radiation, as well.

The refraction index (6) in mode 2 for $k_0^2 - k_\parallel^2 \ll 4m^2$, $B \gg B_{cr}$ is

$$n_2 = \left(\frac{1+\frac{\alpha}{3\pi}\frac{B}{B_{cr}}}{1+\frac{\alpha}{3\pi}\frac{B}{B_{cr}}\cos^2\vartheta}\right)^{\frac{1}{2}}. \tag{27}$$

The refraction index obtained depends upon the direction of the photon momentum, characterized by the angle ϑ, but does not depend upon its energy. In other words, there is no frequency dispersion in a wide range from slow radio waves and up to soft γ-rays with $k_0 \ll 2m$. This is a consequence of the fact, that in κ_2 only linear parts in momenta squared were as a matter of fact left (correspondingly, $f(k_\perp^2)$ in (2) is proportional to $k_\perp^2$ according to (23)).

The refraction index (27) is maximum for transversal propagation ($k_\parallel = 0$, $\vartheta = \pi/2$)

$$n_2^\perp = \left(1+\frac{\alpha}{3\pi}\frac{B}{B_{cr}}\right)^{\frac{1}{2}} = \left(1+7.7\cdot 10^{-4}\frac{B}{B_{cr}}\right)^{\frac{1}{2}}. \tag{28}$$

For $B \sim 10 \cdot B_{cr}$ the declination of the refraction index (28) from unity exceeds that value for gases at atmospheric pressure in optic range by an order of magnitude, for $B \sim 1000 \cdot B_{cr}$ it reaches the value, characteristic of transparent liquids and glass; the refraction index (28) becomes equal to that of diamond ($n = 2.4$) for $B = 27 \cdot 10^{16}$Gauss.

Contrary to the just considered case of mode 2, the polarization tensor eigenvalue κ_3 (15) does not contain a contribution linearly growing with the external field, as well as the resonance. It is, hence, reasonable to keep the constant and logarithmic contributions, but not the difference of the exponential factor from unity, in Eq.(15). Then for mode 3 the dispersion Eq.(1) has the solution

$$k_0^2 = k_\parallel^2 + k_\perp^2 \frac{Z - \frac{\alpha}{3\pi}}{Z}\,, \tag{29}$$

where

$$Z = 1 - \frac{\alpha}{3\pi}\left(\ln\frac{B}{B_{cr}} - C - 1.21\right)\,. \tag{30}$$

The known absence of asymptotic freedom in QED manifests itself in the negative sign in front of the logarithm in Eq.(30). This results in pathological consequences for the fields, as large as $\sim B_{cr}\exp(3\pi/\alpha)$. In this domain the coefficient of $k_\perp^2$ in Eq.(29) becomes, as the field grows, first less than zero and later greater than unity. The corresponding dispersion laws are non-physical, since they lead to the group velocity greater than unity. In the negative slope case in Eq.(29), $e > B\exp(-0.21 - C - 3\pi/\alpha)/B_{cr} > 1$, the dispersion curve enters the sector (12) acceptable for the Cherenkov radiation. But this is the Cherenkov emission of tachyons! It is also odd, that in the latter case e.-m. waves may only propagate inside the cone $0 < \tan\vartheta < -1 + \alpha/3\pi Z$ with its axis along the external field, irrespective of the way they are produced. This domain of exponentially large external fields is not of our interest in the present paper.

For the fields that are not exponentially large, with the logarithmic terms of the order of unity, one should treat all the terms, marked by the coefficient $\alpha/3\pi$ in (29), as small. Then, finally, the dispersion law for mode 3 becomes:

$$k_0^2 = k_\parallel^2 + k_\perp^2\left(1 - \frac{\alpha}{3\pi}\right)\,. \tag{31}$$

Notably, the field-containing logarithmic terms have cancelled from here. Thus the dispersion law (31) of mode 3 is saturated in the sense that, unlike Eq.(23) for mode 2, it has reached the universal form, independent of the external field in the super-Schwinger limit. The refraction index of mode 3 corresponding to (31) is

$$n_3 = 1 + \frac{\alpha}{6\pi} \sin^2 \vartheta. \tag{32}$$

Again, the same as in Eq.(28), the maximum refraction in mode 3 is achieved at perpendicular propagation, $\vartheta = \pi/2$:

$$n_3^{\perp} = 1 + 3.8 \cdot 10^{-4}. \tag{33}$$

This refraction index is of the order of that of gaseous ammonia and cannot be made larger by increasing the external field any further.

4 Conclusion

We have found that in the asymptotic case of an external magnetic fields B that can be orders of magnitude larger than the Schwinger value of 4.4×10^{13}Gauss, the refractive capacity of the magnetized vacuum grows unlimitedly with this field for electromagnetic radiation belonging to the polarization mode 2, but comes to a saturation at a moderate level of corrections $\sim \alpha/3\pi$ for mode 3. For the "parallel energy" of the photon not close to the cyclotron resonance, $k_0^2 - k_\parallel^2 \ll 4m^2$, the refraction effects for mode 2 essentially exceed the above small corrections, typical for non-asymptotic domain, already for $B \sim 10 \cdot Bcr$. A regime is established in the range of photon frequencies/energies, extending from zero and up to soft γ-rays, for which the dispersive properties of the magnetized vacuum are in each mode independent of the photon frequency/energy, but do depend upon the direction of its propagation. Apart from the fact that the refraction index in mode 2 for the propagation non-parallel to the external field numerically grows with the field, it is remarkable that the angle between the group velocity and the direction of the photon momentum grows, too, the wave packet being attracted by the line of force of the external field. In this way, the effect of γ-quantum capture by a strong magnetic field, known to exist due to resonance phenomena near the thresholds of free and bound pair creation, is extended to lower energy ranges. Therefore, not only hard γ-rays, but also X-rays, light and radio-waves undergo strong dispersive influence of the magnetized vacuum, when the magnetic fields are of the order of magnitude of those estimated to exist in magnetars. In view of this fact, the e-m energy canalization phenomena may become important not only within the traditional context described in Subsection 3.2 above, but also as applied to the scattering of e-m waves falling onto the magnetic field from outside[3]. These may be, for instance, the X-rays emitted from the accretion disk or from the pulsar surface outside the region where the magnetic field enters it. Recently the problem of the bending of electromagnetic radiation by the dipole magnetic field of a neutron star was addressed[27], and the competition of this process with the effects of gravity was considered. (The author is indebted to Prof. H. Mosquera Cuesta who attracted his attention to that work in the course of Workshop). We insist, however, that such effects cannot be adequately treated disregarding the refraction index dependence on the direction of propagation and using the quadratic-in-the-field expressions for it, only valid in the low field limit, as is the case in Ref.[27].

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References

1. S. L. Adler, J. N. Bahcall, G. G. Callan and M. N. Rosenbluth, *Phys. Rev. Lett.* **25**, 1061 (1970).
2. V. O. Papanian and V. I. Ritus, ZhETF **61**, 2231 (1971); **65**, 1756 (1973) (*Sov. Phys.- JETP* **34**, 1195 (1972); **38**, 879 (1974)).
3. A. E. Shabad, *Lettere al Nuovo Cimento* **2**, 457 (1972).
4. A. E. Shabad, *Ann. Phys.* **90**, 166 (1975).
5. A. E. Shabad and V. V. Usov, *Nature* **295**, 215 (1982), *Astrophys. Space Sci.* **102**, 327 (1984).
6. V. V. Usov and A. E. Shabad, *Pis'ma v ZhETF* **42**, 17 (1985) (*Sov. Phys.- JETP Letters* **42**, 19 (1985)).
7. H. Herold, H. Ruder and G. Wunner, *Phys. Rev. Lett.* **54**, 1453 (1985).

8. A. E. Shabad and V. V. Usov, *Astrophys. Space Sci.* **117**, 309 (1985), **128**, 377 (1986).
9. V. B. Bhatia, Namrata Chopra and N. Panchapakesan, *Astrophys. J.* **388**, 131 (1992).
10. V. V. Usov and D. B. Melrose, *Aust. Journ. of Phys.* **48**, 571 (1995).
11. V. V. Usov, *ApJ.* **L87**, 572 (2002).
12. M. G. Baring, *Astrophys. J* **L69**, 440 (1995).
13. A. C. Harding, M. G. Baring and P. L. Gonthier *Astrophys. J.* **246**, 476 (1997).
14. M. V. Chistyakov, A. V. Kuznetsov and N. V. Mikheev, *Phys. Lett.* **B434**, 67 (1998).
15. Andreas Reisenegger, *The present volume* (2003).
16. V. M. Kaspi, *Pulsar Astronomy - 2000 and Beyond. ASP Conference Series* **202** (1999) (M.Kramer, N.Wex and R.Wielebinski, eds.), arXiv:astro-ph/9912284.
17. V. V. Skobelev, *Izv. Vyssh. Uchebn. Zaved. Fiz.* No**10**, 142 (1975); Yu. M. Loskutov and V. V. Skobelev *Phys. Lett.* **A56**, 151 (1976).
18. A. E. Shabad, *Kratkie Soobtchenia po Fizike (Sov. Phys. - Lebedev Inst. Reps.)* **3**, 13 (1976).
19. I. A. Batalin and A.E.Shabad, *ZhETF* **60**, 894 (1971) (*Sov. Phys.- JETP* **33**, 483 (1971)).
20. H. Pérez Rojas and A. E. Shabad, *Ann. Phys.* **121**, 432 (1979).
21. H. Pérez Rojas, *ZhETF* **76**, 3 (1979) (*Sov. Phys.- JETP* **49**, 1 (1979)); H. Pérez Rojas and A. E. Shabad, *Ann. Phys.* **138**, 1 (1982).
22. A. E. Shabad, *Trudy Fizicheskogo Instituta im. P.N. Lebedeva* **192**, 5 (1988).
23. A.E.Shabad, *Polarization of the Vacuum and a Quantum Relativistic Gas in an External Field* (Nova Science Publishers, New York, 1991) *Proceedings of the Lebedev Physics Institute, Academy of Sciences of the USSR, Series editor N.G.Basov*, Volume **191**, edited by V.L.Ginzburg.
24. V. I. Ritus, *ZhETF* **69**, 1517 (1975); **73**, 807 (1977).
25. Wu-yang Tsai, *Phys. Rev. D.* **10**, 2699 (1974).
26. Guojun Qiao, *The present volume* (2003).
27. V. L. Denisov, I. P. Denisova and S. I. Svertilov, *Doklady Akademii Nauk* **380**, 325 (2001); arXiv:astro-ph/0110705.