R-MODES AND STEADY GRAVITATIONAL RADIATION FROM MILLISECOND PULSARS

A. REISENEGGER AND A. BONAČIĆ

Departamento de Astronomía y Astrofísica, Facultad de Física,
Pontificia Universidad Católica de Chile, Santiago, Chile
areisene,abonacic@astro.puc.cl

Millisecond pulsars (MSPs) are generally agreed to originate in low-mass X-ray binaries (LMXBs), in which mass transfer onto the neutron stars spins them up to their observed, fast rotation. The lack of MSPs and LMXBs rotating near break-up and the similar rotation periods of several LMXBs have been attributed to the accretion torque being balanced, at fast rotation, by gravitational radiation, perhaps caused by an oscillation mode made unstable through the so-called Chandrasekhar-Friedman-Schutz mechanism. Recently, Wagoner has argued that internal dissipation through weak interaction processes involving Λ^0 and Σ^- hyperons may cause LMXBs to evolve into a quasi-steady state, in which the neutron star has a nearly constant rotation rate, temperature, and mode amplitude. We take this hypothesis one step further, showing that MSPs descending from these LMXBs spend a long time in a similar state, in which a low-amplitude r-mode turns them into extremely steady sources of both gravitational waves and thermal X-rays, while they spin down due to a combination of gravitational radiation and the standard magnetic torque. Observed MSP braking torques already place meaningful constraints on the allowed gravitational wave amplitudes and dissipation mechanisms.

1 R-modes, the CFS Instability, and Viscous Damping

The Chandrasekhar-Friedman-Schutz instability[1] can make retrograde r-modes (inertial modes due to the Coriolis force) on rapidly rotating neutron stars grow and emit gravitational waves at the expense of the stellar rotational energy and angular momentum[2]. It is choked[3] at very high temperatures by bulk viscosity due to non-equilibrium weak interactions such as the Urca processes ($n \to p + e + \bar{\nu}_e$ and $p + e \to n + \nu_e$, where n, p, e, $\bar{\nu}_e$, and ν_e denote neutrons, protons, electrons, electron antineutrinos, and electron neutrinos, respectively, or analogous processes involving hyperons), and at very low temperatures by standard shear viscosity. Additional damping could be provided at intermediate temperatures by hyperon bulk viscosity[4], caused by the processes $\Lambda^0 + n \leftrightarrow n + n$ and $\Sigma^- + p \leftrightarrow n + n$. If strong enough, this can split the instability region on the temperature-frequency plane into two

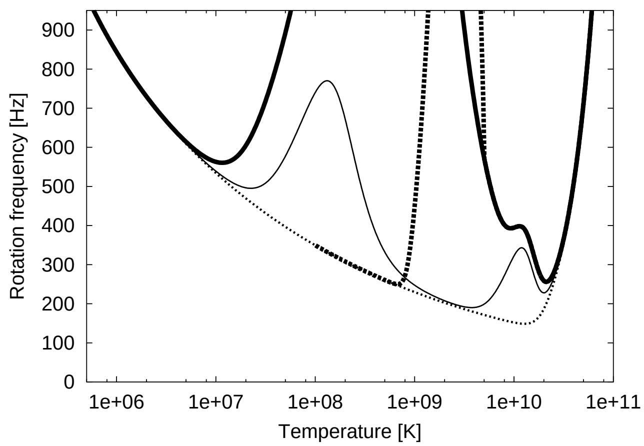

Figure 1. Instability boundaries for the $l = m = 2$ r-mode. The curves all use the same shear viscosity, determined by Levin & Ushomirsky [9] to be active at the boundary between the liquid core and a thin, elastic crust, but different assumptions about the bulk viscosity in the core. The thin dotted curve considers only modified Urca processes, whereas the other three also include direct Urca processes and the hyperon bulk viscosities proposed by Jones [4] (thin solid line) and by Lindblom & Owen [5] (thick solid line), in both cases ignoring superfluid effects, and by Lindblom & Owen affected by hyperon superfluidity with a uniform, high critical temperature, $T_c \sim 5 \times 10^9$ K (thick dotted line). In each case, the unstable region lies above the curve.

separate windows, as illustrated in Fig. 1.

2 Neutron Star "Recycling"

In the standard "recycling" model, described in somewhat more detail in A. Reisenegger's other contribution to these Proceedings, millisecond pulsars (MSPs) reach their high rotation rates due to the accretion torque in a low-mass X-ray binary system (LMXB)[6]. For this to happen, some mechanism must suppress the CFS instability enough to prevent the gravitational wave torque from being larger than the accretion torque. The simplest scenario[7] requires a shear viscosity substantially higher than that used in Fig. 1, so that the instability boundary at the equilibrium temperature T_{acc} reached during the accretion process[8] lies higher than the fastest rotation frequencies observed in MSPs, ~ 650 Hz. This is not unlikely, since a stronger effective shear viscosity might be obtained with a thicker crust[9], superfluids [10], or a magnetic field[11]. An instability boundary just above the frequency of the fastest pulsars might even explain why there are no MSPs close to the break-up frequency, but it would tend to predict an accumulation of MSPs toward the instability boundary, rather than the observed decline in their numbers towards short periods[12].

3 Steady-state Gravitational Radiation in LMXBs

Wagoner[13] has shown that fast rotation periods can be reached even if there is a substantial unstable region at T_{acc}. In his scenario, the low-temperature instability window is reached from below as the neutron star is spun up by accretion (see Fig. 2(a)). A stable balance between cooling through neutrino emission and heating through nuclear reactions undergone by the accreted matter in the deep crust (which release ~ 1 MeV per baryon[8]) keeps the stellar interior warm, at a temperature $T_{acc} \sim 10^{7.5-8.8}$ K that depends on the allowed neutrino emission processes (e.g., direct vs. modified Urca). As the star moves into the unstable region, its most unstable r-mode [3] (with spherical-harmonic indices $l = m = 2$) is excited, producing viscous dissipation. The resulting heat release moves the star to the high-temperature boundary of the instability window,[a] set by

$$0.051\nu_{\rm kHz}^6 \ {\rm s}^{-1} = 0.13 f_h \nu_{\rm kHz}^2 T_8^2 \ {\rm s}^{-1}. \tag{1}$$

The left-hand side gives the r-mode driving rate through gravitational radiation reaction[3] for a "fiducial" neutron star with a polytropic ($n = 1$) equation of state, mass $M = 1.4\,M_\odot$, and radius $R = 12.5$ km, in terms of the rotation frequency in kHz, $\nu_{\rm kHz}$. The right-hand side is the damping rate, dominated by hyperonic processes, whose large uncertainty (including the reduction due to superfluid effects) is parameterized by the dimensionless parameter f_h. This parameter is unity for the preferred bulk viscosity of Lindblom and Owen[5] with no superfluid effects, acting in a hyperonic core of radius $r = R/2$, whereas $f_h = 0.16$ corresponds to the bulk viscosity previously proposed by Jones [4]. The resulting interior temperature, in units of 10^8 K, turns out to be $T_8 = 0.63 f_h^{-1/2} \nu_{\rm kHz}^2$.

[a] Our numerical calculations show that, even if the instability boundary is reached on its low-temperature side (where $d\nu/dT < 0$), the heating due to shear viscosity and hyperon bulk viscosity combined is strong enough to move the star across the window to the high-temperature side, so the present analysis still applies.

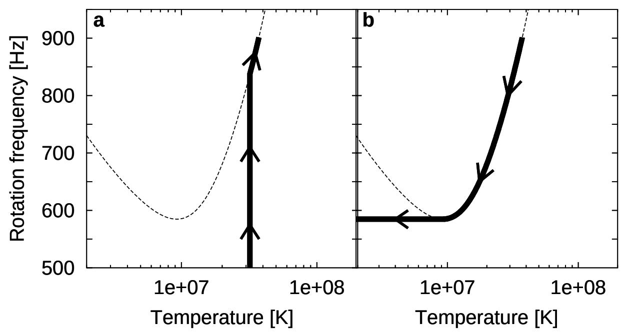

Figure 2. Evolutionary tracks of LMXBs and MSPs. Panel (a) shows the evolution of a neutron star in a LMXB being spun up by accretion[13], and panel (b) that of a MSP being spun down by gravitational radiation and/or a magnetic torque (solid lines). The instability boundary (dashed) considers the same shear viscosity as in Fig. 1, and the hyperon bulk viscosity normalized to fit the parameters of PSR B1957+20 (see text).

Once the temperature is high enough for accretion heating to be negligible, the dimensionless mode amplitude[3] α is set by the balance of the hyperon bulk viscosity heating and the neutrino emission (the latter assumed to be dominated by direct Urca processes[14]),

$$3.6 \times 10^{50} f_h T_8^2 \nu_{\rm kHz}^4 \alpha^2 {\rm erg\ s^{-1}} = 1.2 \times 10^{39} f_U T_8^6 {\rm erg\ s^{-1}}, \tag{2}$$

where we again parameterize our ignorance by a dimensionless factor, f_U, which is unity when the threshold for nucleonic direct Urca processes ($n \leftrightarrow p + e^-$) is reached at half the stellar radius. It can take different (possibly temperature-dependent) values if the threshold radius is different, if only hyperonic direct Urca processes are allowed [15], or if superfluidity reduces the reaction rates [16]. The resulting amplitude is quite small, $\alpha = 7.5 \times 10^{-7} (f_U/f_h^3)^{1/2} \nu_{\rm kHz}^2$, so the linear approximation used in the expressions above is amply justified, and it appears unlikely that the mode could be saturated at lower amplitudes by any other processes such as turbulent viscosity[17] or mode coupling[18].

With the temperature and mode amplitude at any given rotation rate set by Eqs. (1) and (2), the accretion torque,

$$\dot{M}\sqrt{GMR} = 1.5 \times 10^{34} (\dot{M}/\dot{M}_{\rm Edd})\ {\rm dyne\ cm}$$

(where $\dot{M}$ is the mass accretion rate and $\dot{M}_{\rm Edd} = 1.9 \times 10^{-8} M_\odot {\rm yr}^{-1}$ is the standard "Eddington limit"[19]), gradually drives the star "up" towards faster rotation. It may eventually settle in a quasi-steady state, in which the accretion torque is balanced by the angular momentum loss due to gravitational radiation [13], $6.8 \times 10^{46} \nu_{\rm kHz}^7 \alpha^2$ dyne cm, at an equilibrium period

$$P_{\rm eq} = \frac{1}{\nu_{\rm eq}} = 1.1 \left(\frac{f_U}{f_h^3} \frac{\dot{M}_{\rm Edd}}{\dot{M}} \right)^{1/11} \rm ms \,, \tag{3}$$

which is only very weakly dependent on the uncertain parameters. In this state, the neutron star is a source of low-amplitude gravitational waves, whose steadiness depends on that of the accretion rate.

It is worth noting that Eq. (3) and the expression above for the accretion torque are only valid if the star's rotation is much slower than the mass-shedding limit and the accretion disk extends down to the stellar surface, not being constrained by the star's magnetic field. We also point out that the possibility of balancing accretion torques in LMXBs by gravitational wave emission had previously been suggested[20] in somewhat different scenarios.

4 Evolution of Millisecond Pulsars and Constraints from Timing

In what follows, we analyze what happens in Wagoner's scenario once the accretion stops and the star turns on as a (millisecond) radio pulsar. It slowly slides down the instability boundary as it loses angular momentum through the combined effects of gravitational radiation and standard magnetic braking (Fig. 2(b)). Its heating and cooling time scales, as well as the growth and damping times for the r-modes, are much shorter than the spin-down time, and therefore the star will remain in the quasi-steady state described by Eqs. (1) and (2). Since no external perturbations are present, the spin-down is extremely smooth and predictable, as observed in MSPs[21]. For a given neutron star model (which sets the "fudge factors" f_U and f_h), the rotation period, P (observed as the periodicity of the radio pulses), uniquely determines the gravitational-wave contribution to the spin-down rate,

$$\dot{P}_{GW} = 5.3 \times 10^{-18} \frac{f_U}{f_h^3} \left(\frac{1 \rm ms}{P} \right)^9 . \tag{4}$$

Two representative lines for this relation are shown on Fig. 3. We note that, if the particles in the neutron star core are superfluid, the factors f_U and

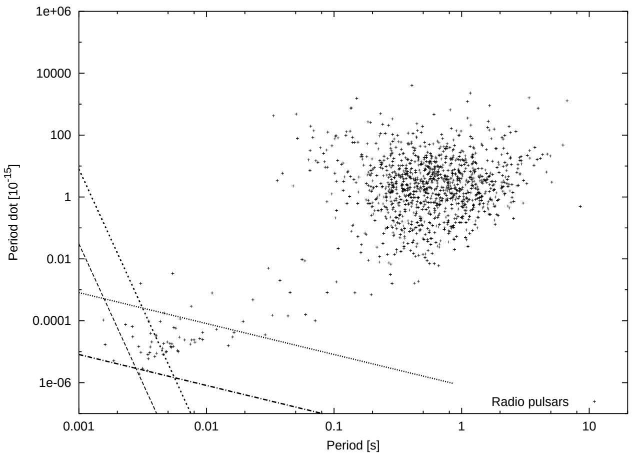

Figure 3. $P - \dot{P}$ diagram for known radio pulsars (data taken from the ATNF Pulsar Catalogue, http://www.atnf.csiro.au/research/pulsar/catalogue/). The strongly sloped lines represent the evolution through gravitational radiation torques from r-modes stabilized by the hyperon bulk viscosity proposed by Jones[4] ($f_h/f_U^{1/3} = 0.16$; short-dashed) and by Lindlom & Owen[5] ($f_h/f_U^{1/3} = 1$; long-dashed), in both cases disregarding superfluid effects. The gently sloped lines represent magnetic dipole torquing with $B = 10^8$ (dash-dotted) and 10^9 G (dotted). If both processes are present, pulsars will move down one of the former lines at small periods, and then gradually turn over to follow one of the latter.

f_h will depend on temperature, and therefore implicitly on P, which would modify the dependence of $\dot{P}_{GW}$ from the proportionality to P^{-9} implied by Eq. (4). However, for typical cases that we evaluated numerically, a strongly decreasing function $\dot{P}_{GW}(P)$ is still obtained.

Of course, for periods P of actually observed MSPs in the range of applicability of this model, $\dot{P}_{GW}(P)$ can be at most as high as the observed $\dot{P}$. For the "standard" values of our "fudge factors", $f_h \lesssim 1 = f_U$, this is not always the case, as some pulsars on Fig. 3 lie below the lines. The most constraining case is the so-called "black widow"[22], PSR B1957+20, because of its short period, $P = 1.60$ ms, and its small intrinsic period derivative[23], $\dot{P} = 1.2 \times 10^{-20}$. For this pulsar, we require $f_h^3/f_U \geq 6.4$, i.e., a somewhat larger bulk viscosity and/or a substantially lower neutrino emissivity than in our favored model. Neither of these would be too surprising, in view of their strong dependence on the uncertain state of very dense matter. The presence of superfluid energy gaps for the particles in the neutron star core could substantially reduce the phase space and therefore the rates of all reactions, thereby decreasing both the Urca cooling rate [16] and the hyperon bulk viscosity [5], potentially down to $f_U, f_h \ll 1$. Since the reactions determining f_h involve more potentially superfluid particles (both hyperons and nucleons) than the Urca processes (just nucleons), it is likely that $f_h < f_U$, making it difficult to satisfy the "black widow" constraint. However, f_U may be reduced if, instead of nucleonic direct Urca processes, only hyperonic direct Urca reactions are allowed, which would set f_U (without superfluid reduction) at ~ 0.2 [15]. The latter processes are almost certainly allowed as long as hyperons exist in the star, but again may be reduced (perhaps more strongly than the neutrino-less processes contributing to f_h) if the hyperons become superfluid at high temperatures. Therefore, instead of ruling out the model, the observations may be used to constrain the internal properties of neutron stars once the model is validated in other ways.

5 No Limit on Accretion Spin-up?

Of course, the values of f_h and f_U are most likely not identical for all MSPs, as these probably have different masses, and therefore different mean densities and different fractions of their interior in which the relevant processes take place. Assuming, for simplicity, that the condition $f_h^3/f_U \geq 6.4$ does apply to all MSPs and LMXBs, we can in principle constrain the LMXB equilibrium period to $P_{\rm eq} \leq 0.9(\dot{M}_{\rm Edd}/\dot{M})^{1/11}$ ms. Strictly speaking, this period is beyond the mass-shedding limit for our adopted neutron star model, and should therefore not be taken literally. It shows that, in order to be consistent with MSP observations, the r-mode instability cannot limit the rotation of LMXBs to much below the maximum allowed equilibrium rotation for any neutron star, unless magnetic stresses prevent the accretion disk from reaching the surface, reducing the accretion torque and increasing the equilibrium period. A similar conclusion has been reached before[24], based on the low luminosity of LMXBs in quiescence, compared to the expectation from internal dissipation.

6 Few Very Fast Millisecond Pulsars

The spin-down of a newly born MSP, initially spinning at a period P_0 near the mass-shedding limit, is likely to be dominated by the gravitational radiation torque. The time it takes to spin down to a slower period P is $t \approx 3.9 \times 10^6 (f_h^3/6.4 f_U)[(P/{\rm ms})^{10} - (P_0/{\rm ms})^{10}]$ yr, substantially shorter (and much more period-dependent) than for magnetic braking at the inferred magnetic field strengths, $\sim 10^{8-9}$ G. This makes it much less likely to find such extremely rapidly spinning MSPs, perhaps in this way explaining the observed absence of these objects[12]. An independent test of this scenario would be a measurement of the braking index[19], $n \equiv \nu\ddot{\nu}/\dot{\nu}^2$, which reaches a huge value ($n = 11$, if f_h^3/f_U is constant) in the regime where the torque is dominated by gravitational-wave emission (compared to $n = 3$ for pure magnetic dipole braking). Unfortunately, measurements of braking indices in MSPs are out of reach.

7 Thermal X-ray emission

In this state, the effective surface temperature of the MSP can be inferred from its interior temperature[25] to be $T_s \approx 8 \times 10^5 f_h^{-0.28}(P/\mathrm{ms})^{-1.1}$ K, not far below the upper limits obtained from ROSAT observations of MSPs [26], and perhaps within the reach of more careful determinations based on XMM-Newton spectra.

8 Gravitational Radiation

Perhaps most interestingly, the MSP will radiate gravitational waves at the r-mode frequency. In the limit of slow rotation and weak gravity, this frequency is 4/3 times the rotation frequency[27], or ≈ 850 Hz for the fastest pulsars. A more precise determination, including centrifugal and relativistic corrections, should be possible. The angle-averaged amplitude at a distance D is[28]

$$h \approx 7 \times 10^{-28} \left(\frac{6.4 f_U}{f_h^3}\right)^{1/2} \left(\frac{1.6\mathrm{ms}}{P}\right)^5 \frac{1.5\mathrm{kpc}}{D}, \tag{5}$$

where the reference numbers correspond to the "black widow" pulsar, the most favorable known so far. According to recent sensitivity curves[29], this signal is almost within reach of the advanced LIGO with signal recycling, tuned at the appropriate frequency and integrating for 1/3 yr. Therefore, future gravitational-wave observatories may well give us information about weak interaction processes in superdense matter.

9 "Strange" Millisecond Pulsars

A recent study[30] has found that "strange stars", composed of "deconfined" u, d, and s quarks, may have a time evolution qualitatively similar to that later found by Wagoner[13] for neutron stars in LMXBs and here for neutron stars as MSPs. In the corresponding strange stars, the bulk viscosity is dominated by the process $u + d \leftrightarrow s + u$, the quark analog for the hyperonic processes considered in the present discussion. In that study, the reheating due to bulk viscous dissipation is ignored, and therefore the evolution time scale is set by passive cooling, which is much faster and thus more violent than in the more realistic case discussed here. In view of the present theoretical uncertainties and lack of observational constraints, LMXBs and MSPs may well be strange stars rather than neutron stars. Although the numerical details are different (and even more uncertain), "strange MSPs" may follow a similar evolutionary path and be subject to observational constraints analogous to those discussed here for neutron stars.

10 Conclusions

We conclude that, if there is a substantial instability window at low temperatures, neutron stars (or strange stars) in LMXBs can generally spin up to MSP rotation rates by the mechanism suggested by Wagoner[13]. The gravitational wave torque will still be active in their MSP phase, until they reach the bottom of the instability window, which some of the observed MSPs may not yet have done. Thus, these MSPs will be sources of gravitational waves and thermal X-rays. Even if born rotating near break-up, MSPs will spin down to near the observed periods much more quickly than in the magnetic braking model, perhaps explaining the scarcity of very fast MSPs.

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