Momentum space saturation model for deep inelastic scattering and inclusive hadron production

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In collaboration with M. B. Gay Ducati and E. G. de Oliveira

Outline

- Introduction

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  - ⇒ Saturation and dipole frame
  - ⇒ Geometric scaling

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  - ⇒ Traveling waves and the AGBS model

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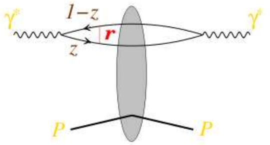
- Discussion

Saturation and Dipole Frame

- e-P scattering at HERA: strong rise of the gluon dist. function for small-x
  - Untamed rising should violate unitarity
  - For $Q^2 \leq Q_s^2(x)$, parton recombination should happen
  - Semihard scale from pQCD $\mathbf{Q_s(x)}$: Saturation scale
- **Dipole frame** is convenient to investigations on small-x



z: longitudinal photon momentum fraction carried by the quark
$\mathbf{r}$: transverse size of the pair $q\bar{q}$

- γ^* splits into the $q\bar{q}$ pair and the cross section factorizes as [Nikolaev & Zakharov '90; Mueller '94]

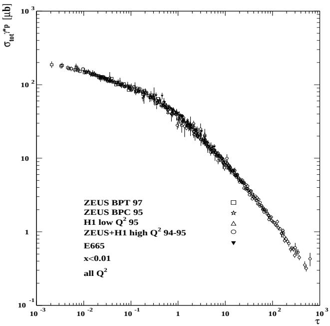
$$\sigma_{L,T}^{\gamma^* P}(x, Q^2) = \int_0^1 dz \int d^2\mathbf{r} |\Psi_{L,T}(z, r; Q^2)|^2 \sigma_{dip}(r = |\mathbf{r}|, x) \qquad (1)$$

- $\Psi_{L,T}(z, r; Q^2)$ (Describes the γ^* splitting into the $q\bar{q}$): Computed from pQED

σ_{dip} and geometric scaling

- σ_{dip} includes the target non-perturbative terms (proton) and satisfies the pQCD properties
  - Color transparency: $\sigma_{dip}(r, x) \propto r^2$
  - Black disc limit: $r \leq 1/Q_s(x)$, $\sigma_{dip}(r, x) \sim \pi R_h^2$
- σ_{dip} models describe the geometric scaling observed in the DIS at HERA

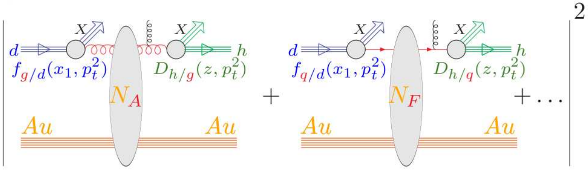
[Stasto, Golec-Biernat and Kwiecinski '2001]



- $\tau = Q^2/Q_s^2(x)$
- $\sigma_{dip}(x, Q^2) \Rightarrow \sigma_{dip}(\tau)$
- Scaling behavior is quite model independent
- Holds outside the saturation region (geometric scaling window)
- Stands as a strong evidence of the saturation phenomena

Hadron Production from the CGC

- d-Au scattering at LO accuracy ($d + Au \to h + X$)



- Amplitude: sums diagrams of parton-nucleus (CGC) interaction
- |Amplitude|^2: only 2-point functions (**dipoles**) $N_{A,F}$ enter the cross section [Dumitru, Hayashigaki and Jalilian-Marian '2006]

$$\frac{dN_h(d\,Au \to h(p_t, y_h)\,X)}{dy_h d^2 p_t} = \frac{K(y_h)}{(2\pi)^2} \int_{x_F}^{1} dx_1 \, \frac{x_1}{x_F} \Big[f_{q/p}(x_1, p_t^2)\, N_F\,(q_t, x_2)\; D_{h/q}\,(x_F/x_1, p_t^2) + f_{g/p}(x_1, p_t^2)\, N_A\,(q_t, x_2)\; D_{h/g}\,(x_F/x_1, p_t^2) \Big]$$

Hadron Production from the CGC

- d-Au scattering at LO accuracy ($d + Au \to h + X$)
  - ($x_F = \frac{p_t}{\sqrt{s}} \exp{(y_h)}$): Feynman x of the produced hadron
  - $q_t = \frac{x_1}{x_F} p_t$ transv. momentum of the dipole probing the target nucleus (CGC)
  - $x_2 = x_1 \exp{(-2y_h)}$: momentum fraction of the target partons
  - Loop effects absorbed in DGLAP evolution of $f_{q/p}(x_1, p_t^2)$ and $D_{h/q}\left(x_F/x_1, p_t^2\right)$
  - $K(y_h)$: Accounts from NLO uncertainties
  - $N_{A,F}$ are the scattering amplitudes in the adjoint (gluons) and fundamental (quarks) representations
  - $N_{A,F}\left(q_t, x_2\right)$ are obtained through the Hankel transform

$$N_{F(A)}(k, Y) = \int d^2\mathbf{r}\, e^{-\imath \mathbf{k}\cdot\mathbf{r}}\, \mathcal{N}_{F(A)}(r, Y) = 2\pi \int dr\, r\, J_0(kr) \mathcal{N}_{F(A)}(r, Y) \quad (2)$$

  - N_F obtained from N_A by the replacement $Q_s^2 \to (C_F/C_A) Q_s^2$, with $C_F/C_A = 4/9$

DHJ and BUW amplitude models

- Glauber like amplitudes

$$N_A(r_t, x_2) = 1 - \exp\left[-\frac{1}{4}(r_t^2 Q_s^2(x_2))^{\gamma(y_h, r_t)}\right] \tag{3}$$

- Saturation scales:

$$Q_s^2(x_2) = Q_0^2\, A_{\rm eff}^{1/3}\,(x_0/x_2)^{\lambda}, \qquad \lambda = 0.3, \qquad x_0 = 3\cdot 10^{-4} \tag{4}$$

- DHJ anomalous dimension [Dumitru, Hayashigaki and Jalilian-Marian '2006]

$$\gamma(q_t, x_2) = \gamma_s + (1-\gamma_s)\,\frac{\log(q_t^2/Q_s^2(x_2))}{\lambda y + d\sqrt{y} + \log(q_t^2/Q_s^2(x_2))} \qquad y = \log 1/x(x_2)$$

  - Violation of geometric scaling
- BUW anomalous dimension [Boer, Utermann and Wessels '2008]

$$\gamma(w) = \gamma_1 + (1-\gamma_1)\frac{(w^a-1)}{(w^a-1)+b} \qquad w = q_t^2/Q_s^2(x_2) \tag{5}$$

  - Geometric scaling preserved

Modeling amplitudes: Traveling Wave method

- The HEQCD amplitudes evolution are described by nonlinear equations (parton recombination)
  - The simplest equation is the **BK** equation for the dipole-target amplitude Balitski '96 and Kovchegov '00

$$\partial_Y \mathcal{N}_Y(\mathbf{x},\mathbf{y}) = \bar{\alpha} \int d^2z \, \frac{|xy|^2}{|xz|^2|zy|^2} \left[\mathcal{N}_Y(\mathbf{x},\mathbf{z}) + \mathcal{N}_Y(\mathbf{z},\mathbf{y}) - \mathcal{N}_Y(\mathbf{x},\mathbf{y}) - \mathcal{N}_Y(\mathbf{x},\mathbf{z})\mathcal{N}_Y(\mathbf{z},\mathbf{y})\right],$$

where $|xy|^2 = (\mathbf{x} - \mathbf{y})^2$ is the dipole size, $Y = \ln 1/x$ the rapidity variable and $\bar{\alpha} = \alpha_s N_c/\pi$.

  - The b-independent form ($\mathcal{N}_Y(\mathbf{x},\mathbf{y}) \equiv \mathcal{N}_Y(r)$, with $r = |xy|$) could be Fourier transformed through

$$N_Y(k) = \frac{1}{2\pi} \int \frac{d^2 r}{r^2} \, e^{i\mathbf{k}\cdot\mathbf{r}} \, \mathcal{N}_Y(r) = \int_0^\infty \frac{dr}{r} J_0(kr) \mathcal{N}_Y(r), \qquad (6)$$

so that the **BK** equation in momentum space reads

$$\partial_Y N_Y = \bar{\alpha}\chi(-\partial_L) N_Y - \bar{\alpha} {N_Y}^2,$$

where

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$$

is the **BFKL** kernel and $L = \log(k^2/k_0^2)$, with k_0 some fixed soft scale.

Modeling amplitudes: Traveling Wave method

- There are no analytical solution to BK equation
  Asymptotic forms obtained through QCD $\Rightarrow$ Reaction-diffusion processes
  - Diffusive approximation for $\chi(\gamma) \Rightarrow$ **BK** $\equiv$ **FKPP** equation
  - **FKPP** admits *traveling waves as solutions*
  - *For large L, the BK solutions take the form $N(L - v_g\bar{\alpha}Y)$*
    - $\rightarrow$ *Conditions: $N(L, Y_0)$ decreases faster than $\exp(-\gamma_0 Y)$ for large L, with $\gamma_0 > \gamma_c$.*
  - *In the QCD variables, traveling waves translates into the geometric scaling form of amplitudes [Munier and Peschanski '2004]*

$$N_Y(k) \overset{k \gg Q_s}{\approx} \left(\frac{k^2}{Q_s^2(Y)}\right)^{-\gamma_c} \log\left(\frac{k^2}{Q_s^2(Y)}\right) \exp\left[-\frac{\log^2\left(k^2/Q_s^2(Y)\right)}{2\bar{\alpha}\chi''(\gamma_c)Y}\right],$$

with

$$Q_s^2(Y) = Q_0^2 \exp\left(\lambda Y - \frac{3}{2\gamma_c}\log Y\right),$$

where $\lambda = \bar{\alpha}v_g = \bar{\alpha}\chi(\gamma_c)/\gamma_c$.

AGBS model for σ_{dip}

- Parametrization in momentum space for the dipole-proton scattering amplitude [Amaral, Ducati, Betemps and Soyez '2007]
  - The model uses the traveling wave BK solutions for large the L (dilute) region
  - A Fourier transform of a Theta function models the saturated region

$$N(k) \overset{k \ll Q_s}{=} c - \log\left(\frac{k}{Q_s(Y)}\right)$$

  - The AGBS model interpolates between the two behaviors through ($\rho \equiv \ln(k^2/k_0^2)$ and $\rho_s \equiv \ln(k_0^2/Q_s^2)$):

$$N^{\mathrm{AGBS}}(\rho, Y) = L_F \left(1 - e^{-N_{\mathrm{dil}}}\right),$$

where

$$N_{\mathrm{dil}} = \exp\left[-\gamma_c(\rho - \rho_s) - \frac{\mathcal{L}^2 - \log^2(2)}{2\bar{\alpha}\chi''(\gamma_c)Y}\right],$$

$$\mathcal{L} = \ln\left[1 + e^{(\rho-\rho_s)}\right] \qquad \text{with} \quad Q_s^2(Y) = k_0^2\, e^{\lambda Y},$$

and

$$L_F = 1 + \ln\left[e^{\frac{1}{2}(\rho-\rho_s)} + e^{-\frac{1}{2}(\rho-\rho_s)}\right]$$

Simultaneous fit to HERA and RHIC

[EB, Gay Ducati and Oliveria hep-ph/1103.2145 '2011]

- The AGBS model was simultaneously fitted to the last HERA (combined H1 and ZEUS) and RHIC minimum-bias (BRAHMS and STAR) data.
  - DIS was investigated through the proton structure function in momentum space

$$F_2(x, Q^2) = \frac{Q^2 R_p^2 N_c}{4\pi^2} \int_0^\infty \frac{dk}{k} \int_0^1 dz\, |\tilde{\Psi}_{L,T}(z, k; Q^2)|^2 N(k, Y) \qquad (7)$$

    where the photon wave function is now expressed in momentum space
  - Hadron collisions were described by AGBS through the inclusive hadron yield

$$\frac{dN_h(d\,Au \to h(p_t, y_h)\,X)}{dy_h d^2 p_t} = \frac{K(y_h)}{(2\pi)^2} \int_{x_F}^1 dx_1\, \frac{x_1}{x_F} \Big[f_{q/p}(x_1, p_t^2)\, N_F\,(q_t, x_2)\, D_{h/q}\,(x_F/x_1, p_t^2) + f_{g/p}(x_1, p_t^2)\, N_A\,(q_t, x_2)\, D_{h/g}\,(x_F/x_1, p_t^2) \Big] \qquad (8)$$

Simultaneous fit to HERA and RHIC

- The AGBS model was simultaneously fitted to the last HERA (combined H1 and ZEUS) and RHIC minimum-bias (BRAHMS and STAR) data.
  - $N_{A,F}$ and the $N^{\mathrm{AGBS}}(\rho, Y)$ amplitudes were derived in distinct Fourier spaces (see (2) and (6))

$$N(k,Y) = \frac{1}{2\pi} H_0(r^2 N^{\mathrm{AGBS}}(r,Y))$$

  - Using the property

$$H_0(r^2 T(r)) = -\frac{d^2 T_0(k)}{dk^2} - \frac{1}{k}\frac{dT_0(k)}{dk}$$

    one get the AGBS amplitude in the appropriate Fourier space of (8), which reads

$$N_{A,F}(k,Y) = 2\pi\left[-\frac{d^2 N^{\mathrm{AGBS}}(k,Y)}{dk^2} - \frac{1}{k}\frac{dN^{\mathrm{AGBS}}(k,Y)}{dk}\right] \qquad (9)$$

Results: fit to HERA

- Before proceed with the simultaneous fit, the AGBS model was fitted to the last H1 and ZEUS combined data [JHEP **0110** 109 (2010)]
  - Fixed parameters: $\gamma_c = 0.6285$ from the LO BFKL and $\bar{\alpha} = 0.2$
  - Free parameters: k_0^2, $\chi''(\gamma_c)$, λ and R_p
  - Kinematic range:

$$\begin{cases} x \leq 0.01, \ \text{small-}x \\ 0.1 \leq Q^2 \leq 150\,\text{GeV}^2 \end{cases}$$

  - Only light quarks were considered, with mass $m_{u,d,s} = 140$ MeV

χ^2/d.o.f	k_0^2 ($\times 10^{-3}$)	λ	$\chi''(\gamma_c)$	$R(\text{GeV}^{-1})$
0.903	1.129 ± 0.024	0.165 ± 0.002	7.488 ± 0.081	5.491 ± 0.039

Results: simultaneous fit to HERA and RHIC

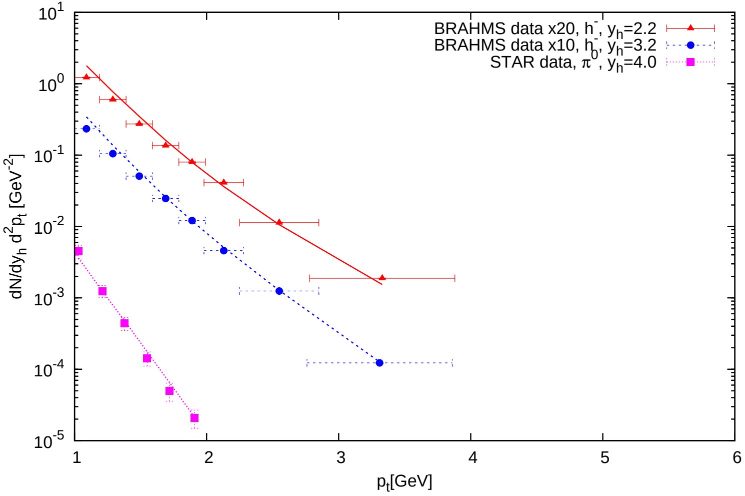
- Simultaneous fit: In addition to the H1 and ZEUS combined data, the model was fitted simultaneously to the BRAHMS [Phys. Rev. Lett. **93**, 242303 (2004)] and STAR [Phys. Rev. Lett. **97**, 152302 (2006)] data on inclusive hadron production
  - Fixed parameters: $\gamma_c = 0.6285$ from the LO BFKL and $\bar{\alpha} = 0.2$
  - Also fixed: $K_{y_h=4} = 0.7$ from DHJ and BUW LO models
  - Free parameters: k_0^2, $\chi''(\gamma_c)$, λ, R_p and $K(y_h)$
  - **CTEQ06** PDF and **KKP** fragmentation functions at scale of $P_t \geq 1$ GeV
  - Kinematic range:

$$HERA \begin{cases} x \leq 0.01, \ (\text{small-}x) \\ 0.1 \leq Q^2 \leq 150\,\text{GeV}^2 \end{cases} \qquad RHIC \begin{cases} P_t \geq 1\,\text{GeV}, \\ 2.2 \leq y_h \leq 4.0 \ (\text{small-}x) \\ 1.0 \leq y_h \leq 4.0 \ (\text{mid-rapidity test}) \end{cases}$$

  - Only light quarks were considered, with masses $m_{u,d,s} = 140$ MeV
  - $A_{eff} = 18.5$ for d-Au collisions

χ^2/d.o.f.	k_0^2 ($\times 10^{-3}$)	λ	$\chi''(\gamma_c)$	R(GeV^{-1})	$K(y_h = 1.0)$	$K(y_h = 2.2)$	$K(y_h = 3.2)$
0.799	2.760 ± 0.130	0.190 ± 0.003	5.285 ± 0.123	4.174 ± 0.053	–	2.816 ± 0.110	2.390 ± 0.098
1.056	1.660 ± 0.137	0.186 ± 0.003	6.698 ± 0.223	4.695 ± 0.112	6.172 ± 0.379	3.783 ± 0.259	3.256 ± 0.226

Results: simultaneous fit to HERA and RHIC ($d + Au$)



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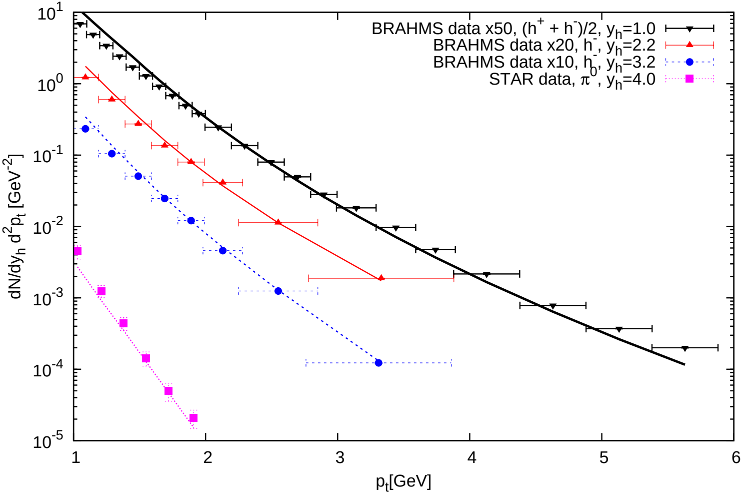
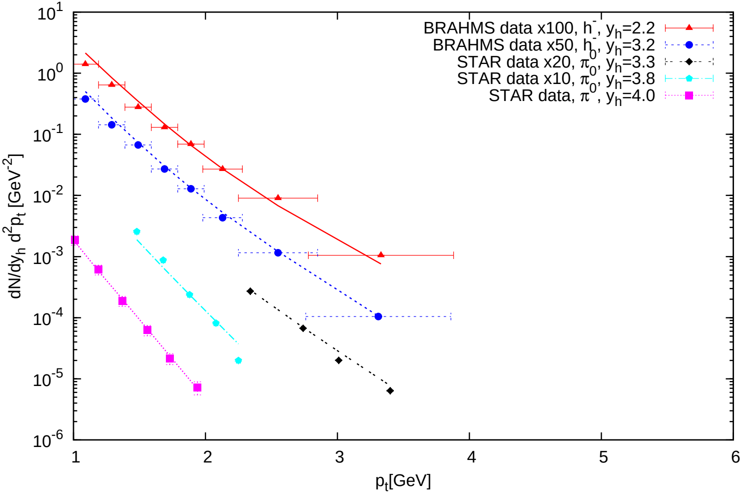


Results: simultaneous fit to HERA and RHIC ($p + p$)



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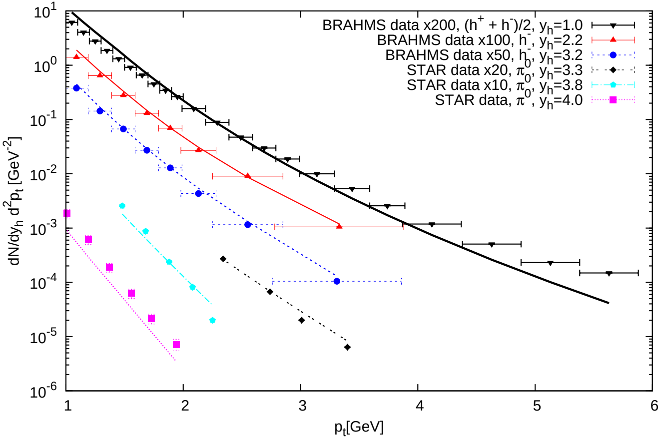
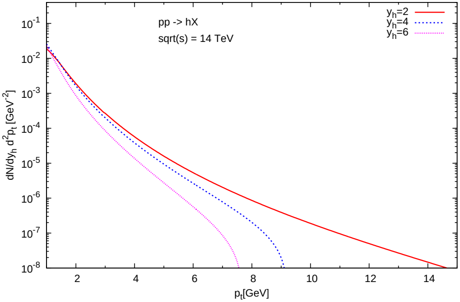
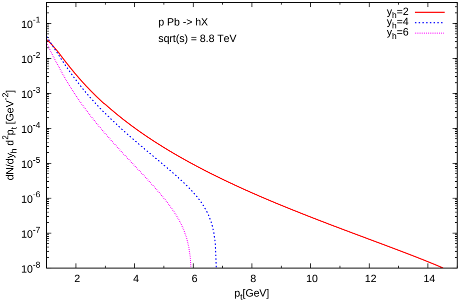


- Model does not fit the mid-rapidity data

Results: Predictions to LHC

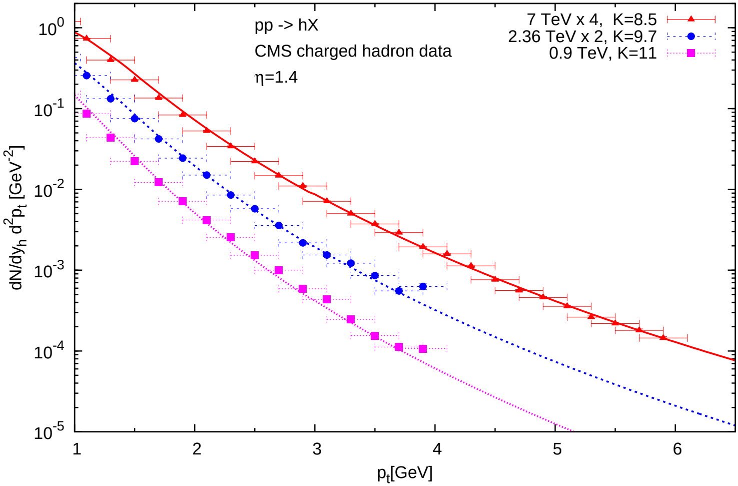
- Parameters extracted from the fit to the forward RHIC data ($y_h \geq 2.2$)





Results: Predictions to LHC (CMS $p + p$)

$$y(\eta, p_t, m) = \frac{1}{2} \ln \left[\frac{\sqrt{m^2 + p_t^2 \cosh^2 \eta} + p_t \sinh \eta}{\sqrt{m^2 + p_t^2 \cosh^2 \eta} - p_t \sinh \eta} \right]$$



Summary and outlook

- We have shown how to use the traveling wave method (AGBS model) to describe both DIS and inclusive hadron production
- The simultaneous AGBS fit to HERA and RHIC data agree with the one performed to HERA data alone
  - $\Rightarrow$ The AGBS change to the correct Fourier space of (8) works
- Model works in the forward region
  - $\Rightarrow$ The CGC formulation of (8) is not supposed to work in this region too ($x_1 \sim x_2$)
- Saturation scale for $d + Au$ is small compared to the HERA fit and another predictions
  - $\Rightarrow$ Minimum bias data implies in an impact-parameter averaged value for $Q_s(x)$
- Pretty good description of the LHC CMS data for $p + p$ collisions
  - $\Rightarrow$ Large K due to the averaged CMS data over the region $|\eta| < 2.4$

Summary and outlook

- Inclusion of parameter impact dependence is important
- Other observables
  - $\Rightarrow$ Prompt photon production
- Other parametrizations between dilute (traveling wave) and saturated regions should be tested
- Modeling of the NLO traveling wave method