Running coupling BK evolution and heavy quarks at HERA

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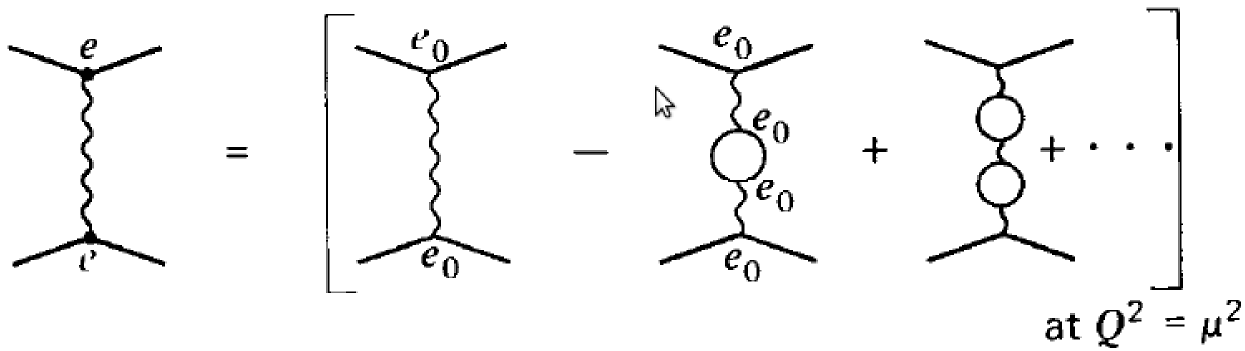
Outline

- Introduction
- BK Evolution
- Running Coupling BK evolution
- Parametrization of dipole-proton scattering amplitude
- Describing F_2^c with RC corrections
- Results

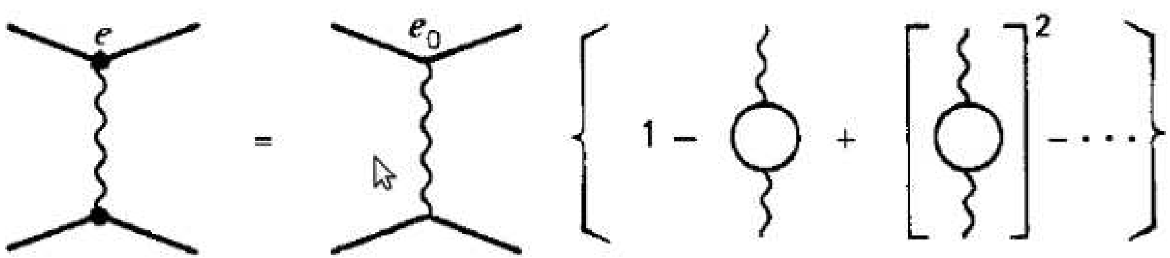
Introduction

Why (and what mean) Running Coupling?

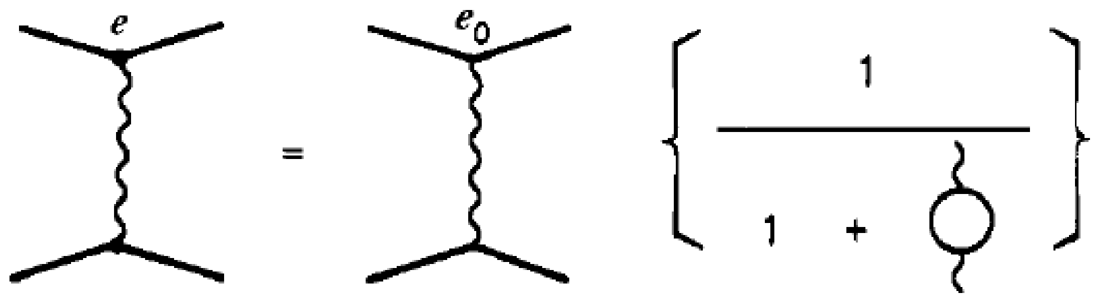
- QED example: Higher order corrections $\Rightarrow$ infinties quantities, that are absorbed in the eletric charge.
- Ward identities: Only vacuum polarization graphs modifies the charge



$$e^2 = e_0^2 \left[1 - I(q^2 = -\mu^2) + \mathcal{O}(e_0^4)\right] \qquad (1)$$



Introduction



$$e^2(Q^2) = e_0^2 \left(\frac{1}{1 + I(q^2)} \right) \tag{2}$$

- The running coupling constant $\alpha(Q^2) \equiv e^2(Q^2)/4\pi$ is given, for large Q^2:

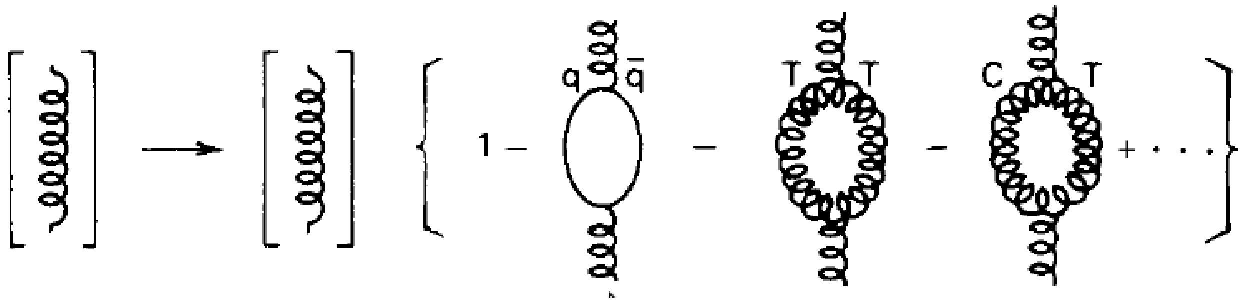
$$\alpha(Q^2) = \frac{\alpha_0}{1 - \frac{\alpha_0}{3\pi} \log\left(\frac{Q^2}{M^2}\right)}, \tag{3}$$

or

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \log\left(\frac{Q^2}{\mu^2}\right)}. \tag{4}$$

Introduction

QCD case



$$\frac{\alpha_s(\mu^2)}{4\pi}\left(-\frac{2}{3}N_f - 5 + 16\right) \quad (5)$$

- The QCD running coupling constant is given by

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{12\pi}(33 - 2N_f)\log\left(Q^2/\mu^2\right)}. \quad (6)$$

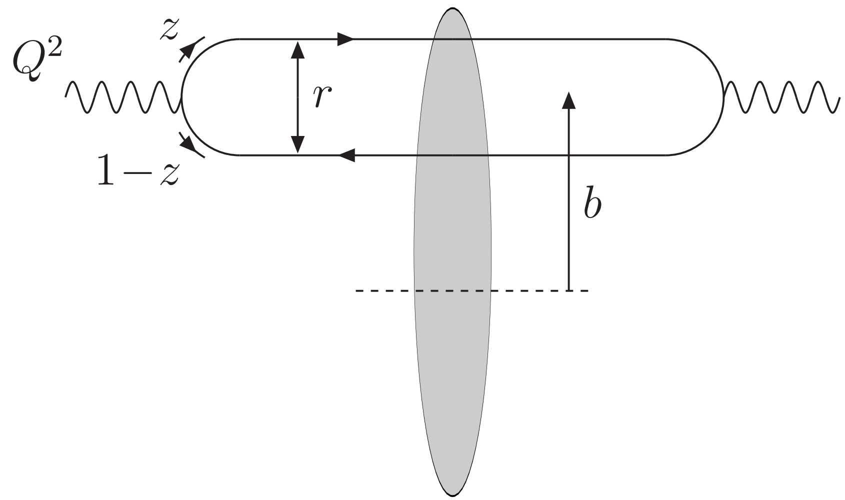
- With the introduction

$$\Lambda^2 = \mu^2 \exp\left[\frac{-12\pi}{(11N_c - 2N_f)\alpha_s(\mu^2)}\right], \quad (7)$$

we have

$$\alpha_s(Q^2) = \frac{12\pi}{(11N_c - 2N_f)\log\left(Q^2/\Lambda^2\right)}. \quad (8)$$

Dipole frame



- For small x, the inclusive structure function of DIS can be expressed as

$$F_2(x,Q^2) = \frac{Q^2}{4\,\pi^2 \alpha_{em}} \left(\sigma_T + \sigma_L\right) \,, \tag{9}$$

- The cross section is factorized like

$$\sigma_{T,L}(x,Q^2) = \sigma_0 \int_0^1 dz \int d\mathbf{r}\, |\Psi_{T,L}(z,Q^2,\mathbf{r})|^2 \,\mathcal{N}(r,Y)\,, \tag{10}$$

where $\sigma_{dip}^{\gamma^* p}(Y,r) \equiv \sigma_0 \mathcal{N}(r,Y)$ is the dipole-proton cross section.

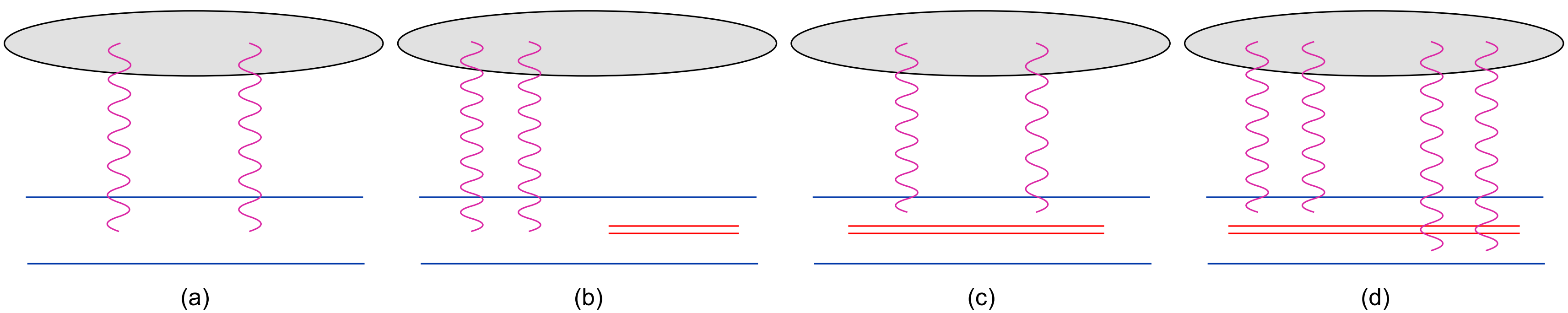
BK evolution

- The LO BK equation reads

$$\frac{\partial S(\underline{x}_0,\underline{x}_1;Y)}{\partial Y} = \int d^2z\, K(\underline{x}_0,\underline{x}_1,\underline{z})\,[S(\underline{x}_0,\underline{z};Y)\,S(\underline{z},\underline{x}_1;Y) - S(\underline{x}_0,\underline{x}_1;Y)]\;, \quad (11)$$

where

$$K(\underline{x}_0,\underline{x}_1,\underline{z}) = \frac{\alpha_s\,N_c}{2\pi^2}\,\frac{r^2}{r_1^2\,r_2^2} \quad (12)$$

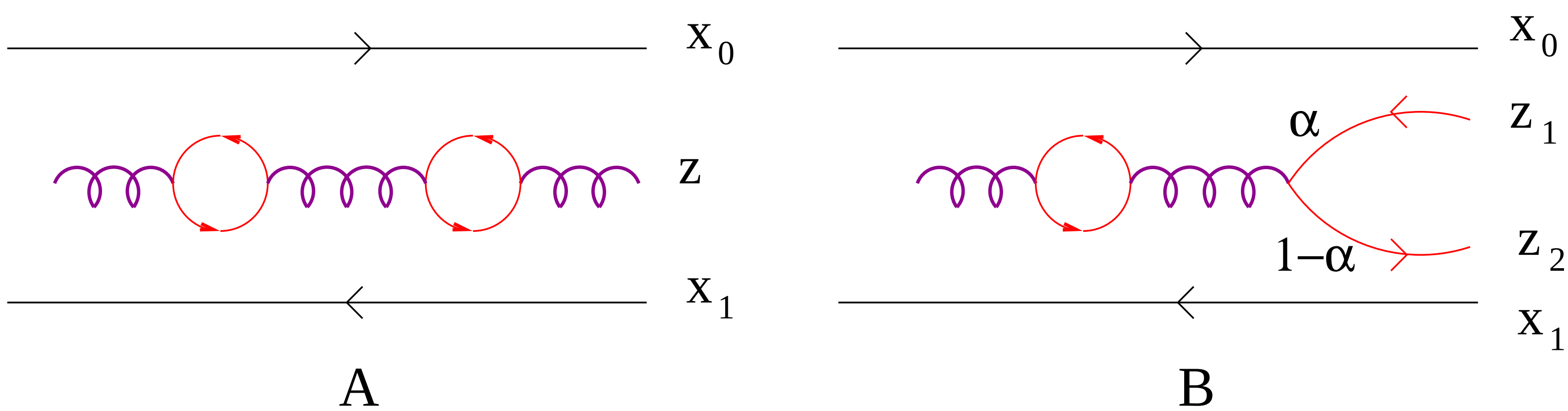


- LO equation has a three point structure

$$S(\underline{x}_0,\underline{x}_1;Y) = \frac{1}{N_c}\langle \mathrm{tr}\left\{V(\underline{x}_0)V^\dagger(\underline{x}_1)\right\}\rangle\;. \quad (13)$$

Running Coupling BK evolution

- Higher order corrections via ressumation of $\alpha_s N_f$ contributions



- Evolution $\sim S(\underline{x}_0, \underline{z}_1) S(\underline{z}_2, \underline{x}_1)$.
- New interaction picture: Four point structure
- To investigate higher order corrections, the evolution is writed as [Albacete and Kovchegov]

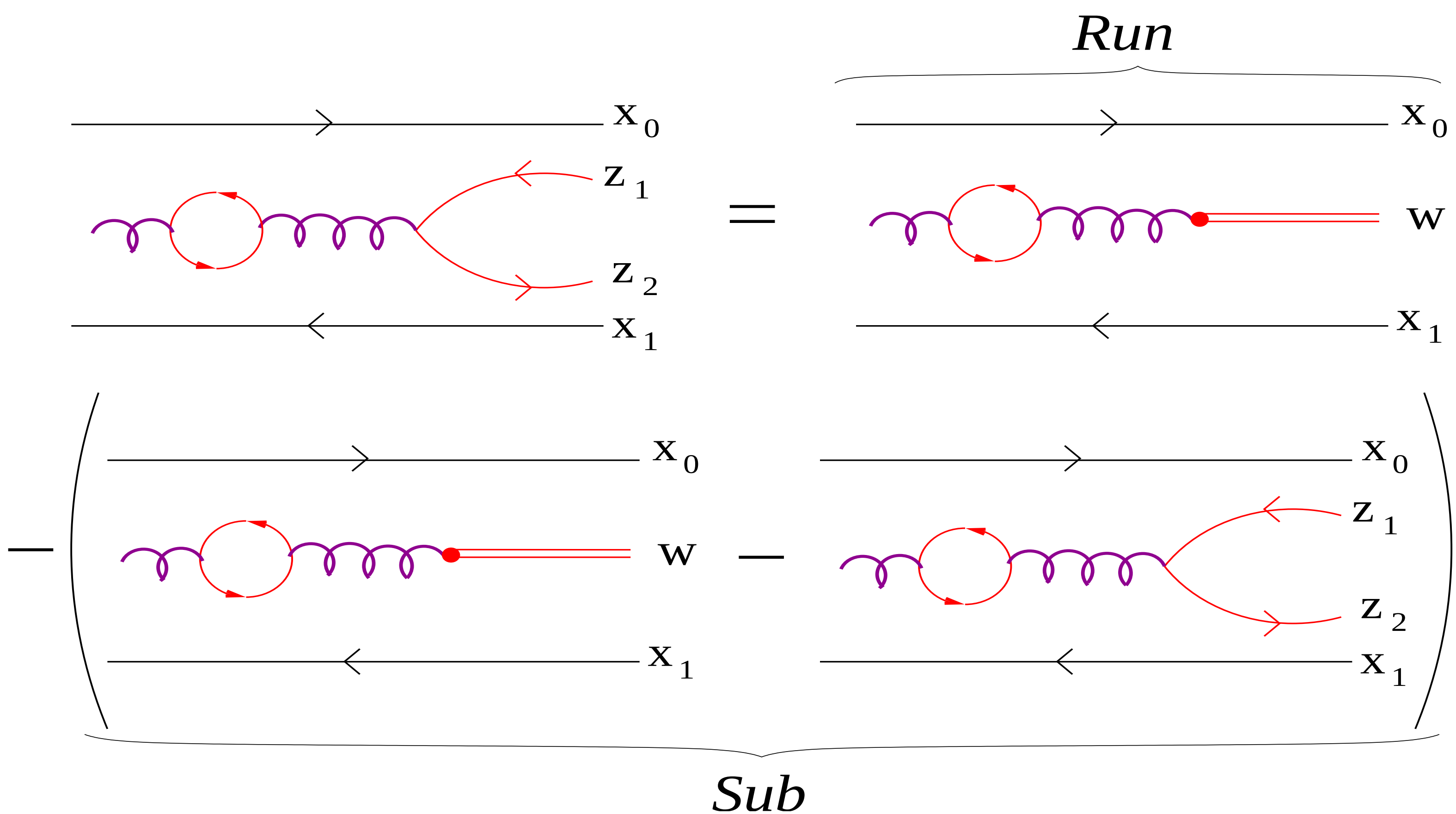
$$\frac{\partial S(\underline{x}_0, \underline{x}_1; Y)}{\partial Y} = \mathcal{F}\left[S(\underline{x}_0, \underline{x}_1; Y)\right] \tag{14}$$

where

$$\mathcal{F}[S] = \mathcal{R}[S] - \mathcal{S}[S] \ . \tag{15}$$

RC BK evolution

- The decomposition of $\mathcal{F}$ is not unique.
- Two different separation schemes have been proposed by Balitsky and Kovchegov & Weigert
- Both based on the same idea: The subtraction point, that recover the three point structure of the interaction



RC BK evolution

- The two approaches have different choices for the subtraction point ($\underline{w}$)

  Bal: $\underline{w}$ chosen to be the transverse coordinate of the quark $\underline{z}_2$ (or antiquark $\underline{z}_1$)

  KW: $\underline{w}$ fixed at the transverse coordinate of the gluon $z = \alpha z_1 + (1-\alpha) z_2$

- The Kernels for the $\mathcal{R}\,[S]$ contribution are

$$\tilde{K}^{\text{Bal}}(\underline{r}, \underline{r}_1, \underline{r}_2) = \frac{N_c\, \alpha_s(r^2)}{2\pi^2} \left[\frac{r^2}{r_1^2\, r_2^2} + \frac{1}{r_1^2} \left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1 \right) + \frac{1}{r_2^2} \left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1 \right) \right] . \tag{16}$$

$$\tilde{K}^{\text{KW}}(\underline{r}, \underline{r}_1, \underline{r}_2) = \frac{N_c}{2\pi^2} \left[\alpha_s(r_1^2) \frac{1}{r_1^2} - 2\, \frac{\alpha_s(r_1^2)\, \alpha_s(r_2^2)}{\alpha_s(R^2)}\, \frac{\underline{r}_1 \cdot \underline{r}_2}{r_1^2\, r_2^2} + \alpha_s(r_2^2) \frac{1}{r_2^2} \right] , \tag{17}$$

where

$$R^2(\underline{r}, \underline{r}_1, \underline{r}_2) = r_1\, r_2 \left(\frac{r_2}{r_1} \right)^{\frac{r_1^2 + r_2^2}{r_1^2 - r_2^2} - 2\, \frac{r_1^2\, r_2^2}{\underline{r}_1 \cdot \underline{r}_2}\, \frac{1}{r_1^2 - r_2^2}} . \tag{18}$$

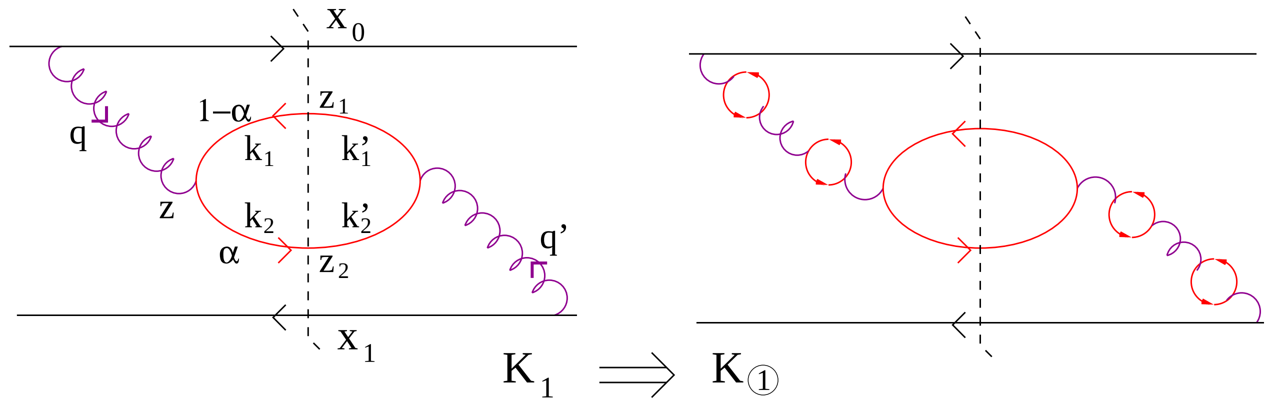
- Bal and KW have not calculated the $\mathcal{S}\,[S]$ contribution.

- Albacete & Kovchegov - AK have calculated the $\mathcal{F}\,[S]$ functional with the schemes: Bal and KW and another one where

$$\tilde{K}^{pd}(\underline{r}, \underline{r}_1, \underline{r}_2) = \frac{N_c\, \alpha_s(r^2)}{2\pi^2}\, \frac{r^2}{r_1^2\, r_2^2} . \tag{19}$$

RC BK evolution

The subtraction term



- The NLO subtraction term reads

$$\mathcal{S}_{\text{NLO}}[S] = \alpha_\mu^2 \int d^2z_1\, d^2z_2\, K_1^{\text{NLO}}(\underline{x}_0, \underline{x}_1; \underline{z}_1, \underline{z}_2) \times [S(\underline{x}_0, \underline{w}, Y)\, S(\underline{w}, \underline{x}_1, Y) - S(\underline{x}_0, \underline{z}_1, Y)\, S(\underline{z}_2, \underline{x}_1, Y)], \quad (20)$$

where α_μ is the bare coupling.

- Including the infinite chain of quark bubbles we have

$$K_1^{\text{NLO}} \Rightarrow K_{①}(\underline{x}_0, \underline{x}_1; \underline{z}_1, \underline{z}_2) = C_F \sum_{m,n=0}^{1} (-1)^{m+n}\, \mathcal{K}_{①}(\underline{x}_m, \underline{x}_n; \underline{z}_1, \underline{z}_2). \quad (21)$$

RC BK evolution

The subtraction term

- In the Bal scheme, $\underline{w} = \underline{z}_1$ or $\underline{w} = \underline{z}_2$ and

$$\mathcal{S}^{\text{Bal}}[S] = \alpha_\mu^2 \int d^2 z_1 \, d^2 z_2 \, K_{①}(\underline{x}_0, \underline{x}_1; \underline{z}_1, \underline{z}_2) \times [S(\underline{x}_0, \underline{z}_1, Y)\, S(\underline{z}_1, \underline{x}_1, Y) - S(\underline{x}_0, \underline{z}_1, Y)\, S(\underline{z}_2, \underline{x}_1, Y)] \tag{22}$$

- In the KW scheme, $\underline{w} = \underline{z} = \alpha\, \underline{z}_1 + (1-\alpha)\, \underline{z}_2$ and

$$\mathcal{S}^{\text{KW}}[S] = \alpha_\mu^2 \int d^2 z_1 \, d^2 z_2 \, K_{①}(\underline{x}_0, \underline{x}_1; \underline{z}_1, \underline{z}_2) \times [S(\underline{x}_0, \underline{z}, Y)\, S(\underline{z}, \underline{x}_1, Y) - S(\underline{x}_0, \underline{z}_1, Y)\, S(\underline{z}_2, \underline{x}_1, Y)] \tag{23}$$

- The kernel is scheme independent. AK have found it using the $\overline{\text{MS}}$ renormalization scheme so that the physical running coupling is given by

$$\alpha_s(1/R^2) = \frac{\alpha_\mu}{1 + \alpha_\mu \beta_2 \, \ln\left(\frac{1}{R^2\, \mu_{\overline{\text{MS}}}^2}\right)}. \quad \beta_2 = \frac{11 N_c - 2 N_f}{12\, \pi}. \tag{24}$$

RC BK evolution

AK numerical implementation

- $\underline{b}$ independent approximation.
- IC's for the dipole scattering amplitude $N(r,Y) = 1 - S(r,Y)$:

$$N^{MV}(r, Y=0) = 1 - \exp\left[-\frac{r^2 Q_s'^2}{4} \ln\left(\frac{1}{r^2\Lambda^2} + e\right)\right]. \quad (25)$$

$$N^{AN}(r) = 1 - \exp\left[-\frac{(r\, Q_s')^{2\gamma}}{4}\right], \quad (26)$$

  with $\gamma = 0.6$ and $\gamma = 0.8$.
- They have set $Q_s' = 1$ GeV at Y=0 and put $\Lambda = 0.2$ GeV.
- Saturation scale given by the condition $N(r = 1/Q_s(Y), Y) = \kappa = 0.5$.
- To regularize the running coupling at large r they have used:

$$\alpha_s(1/r^2) = \frac{1}{\beta_2 \ln\left(\frac{1}{r^2\,\Lambda^2}\right)} \quad \text{for } r < r_{fr} \text{ small}$$

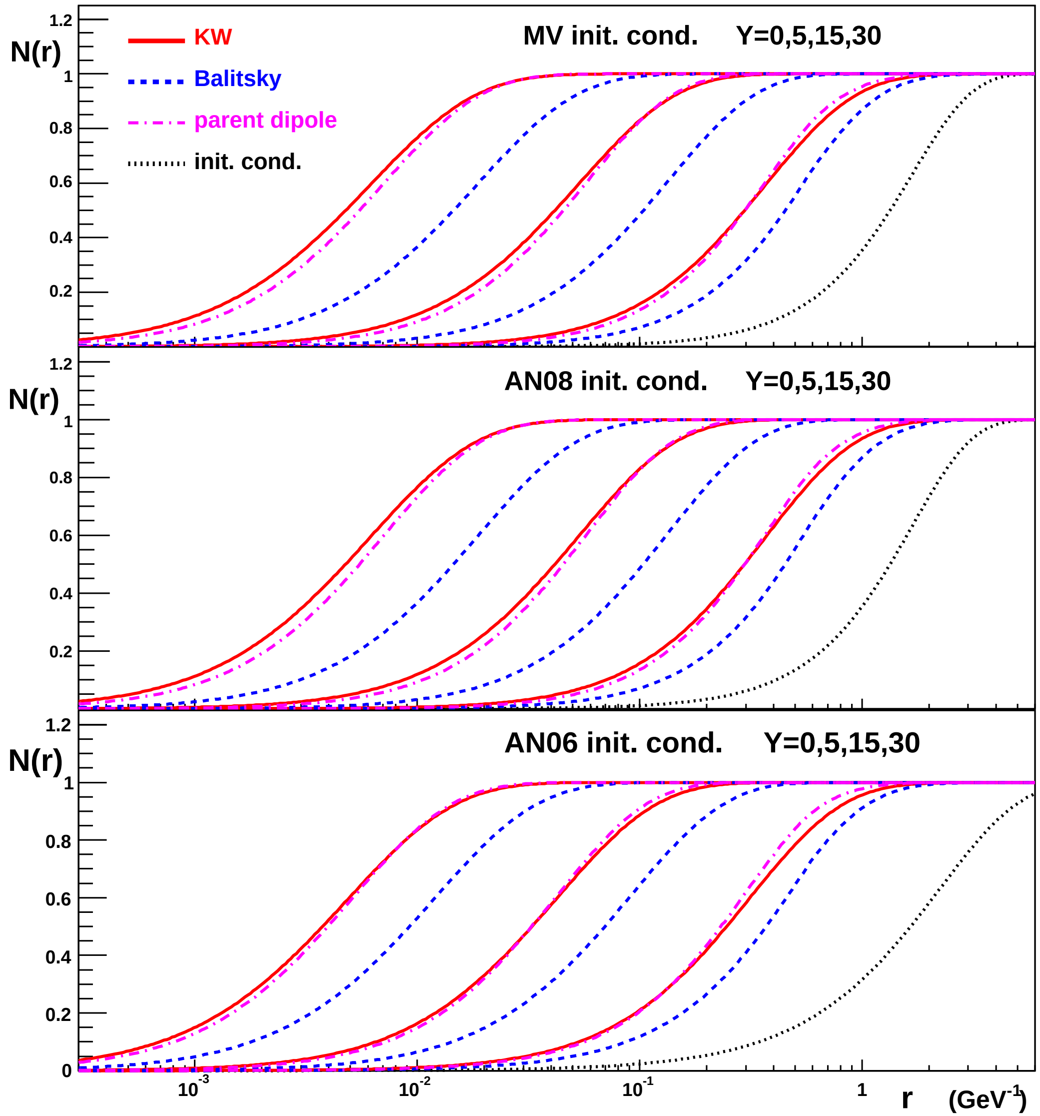
$$\alpha_s = 0.5 \quad \text{for } r > r_{fr}, \quad (27)$$

  with r_{fr} defined by $\alpha_s(1/r_{fr}^2) = 0.5$.
- N_f completed to the QCD beta function: $N_f \rightarrow -6\pi\beta_2$.

RC BK evolution

AK results - Phys. Rev. D75, 125021 (2007)

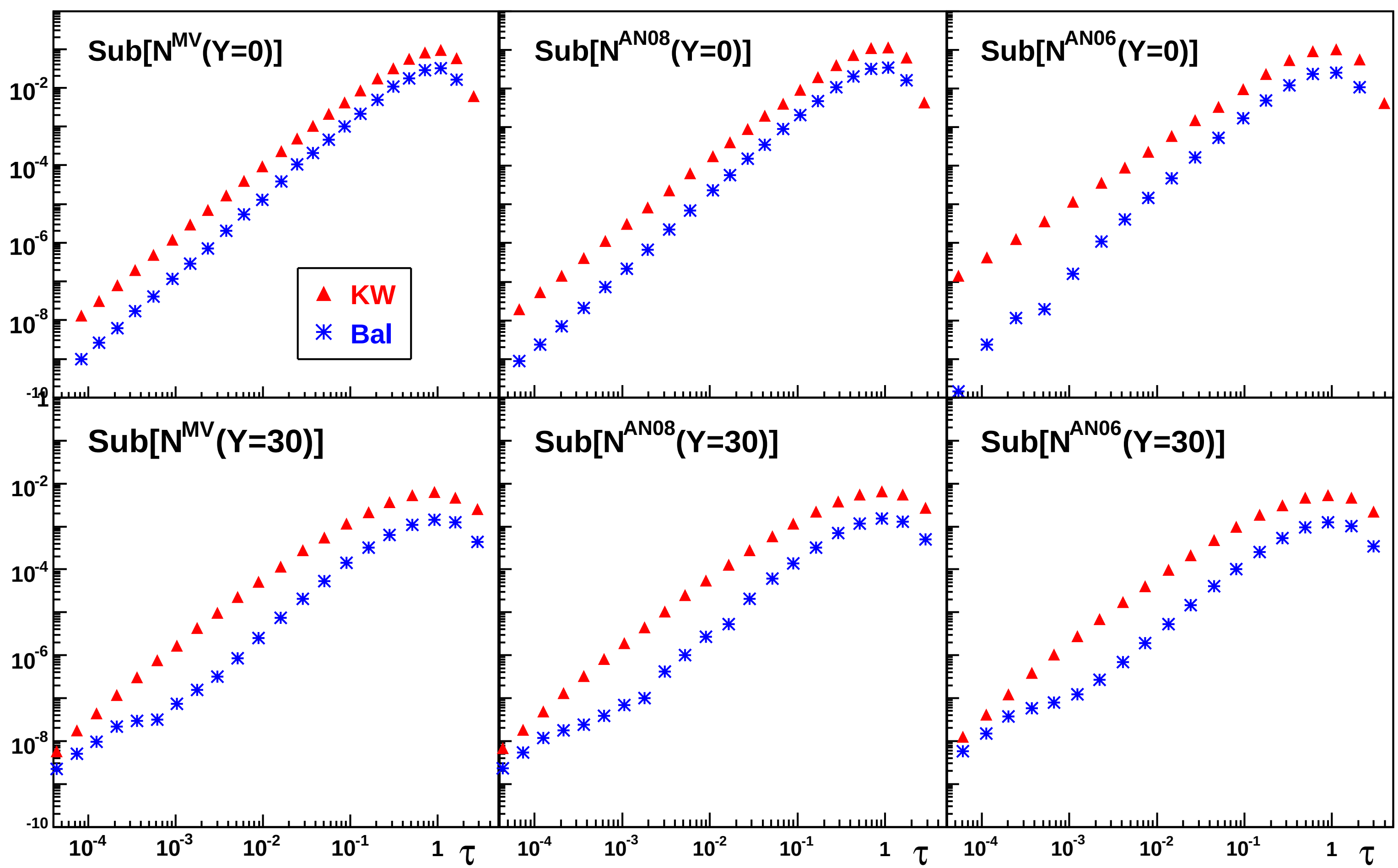
- Running coupling contribution



RC BK evolution

AK results - Phys. Rev. D75, 125021 (2007)

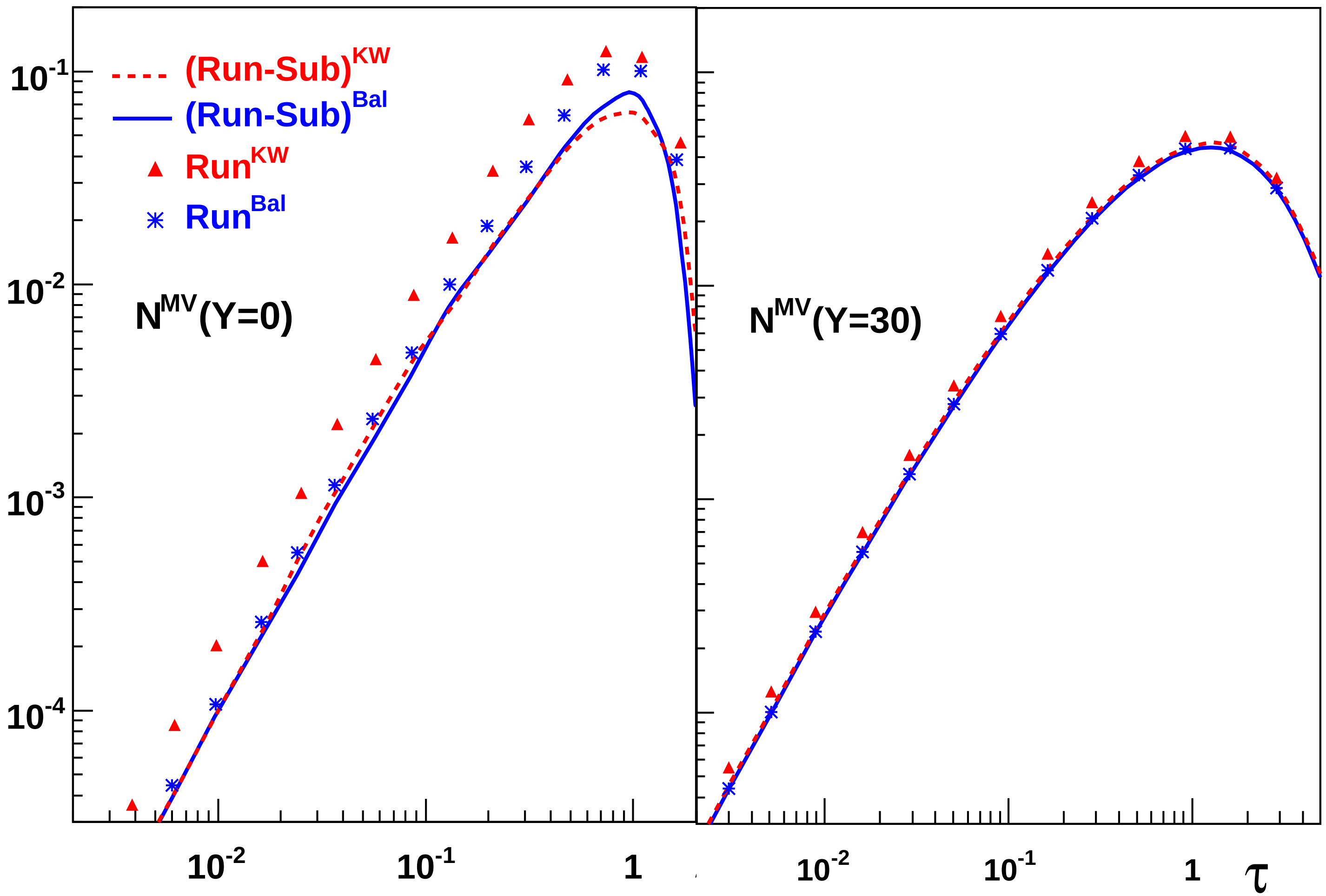
- Subtraction term contribution



RC BK evolution

AK results - Phys. Rev. D75, 125021 (2007)

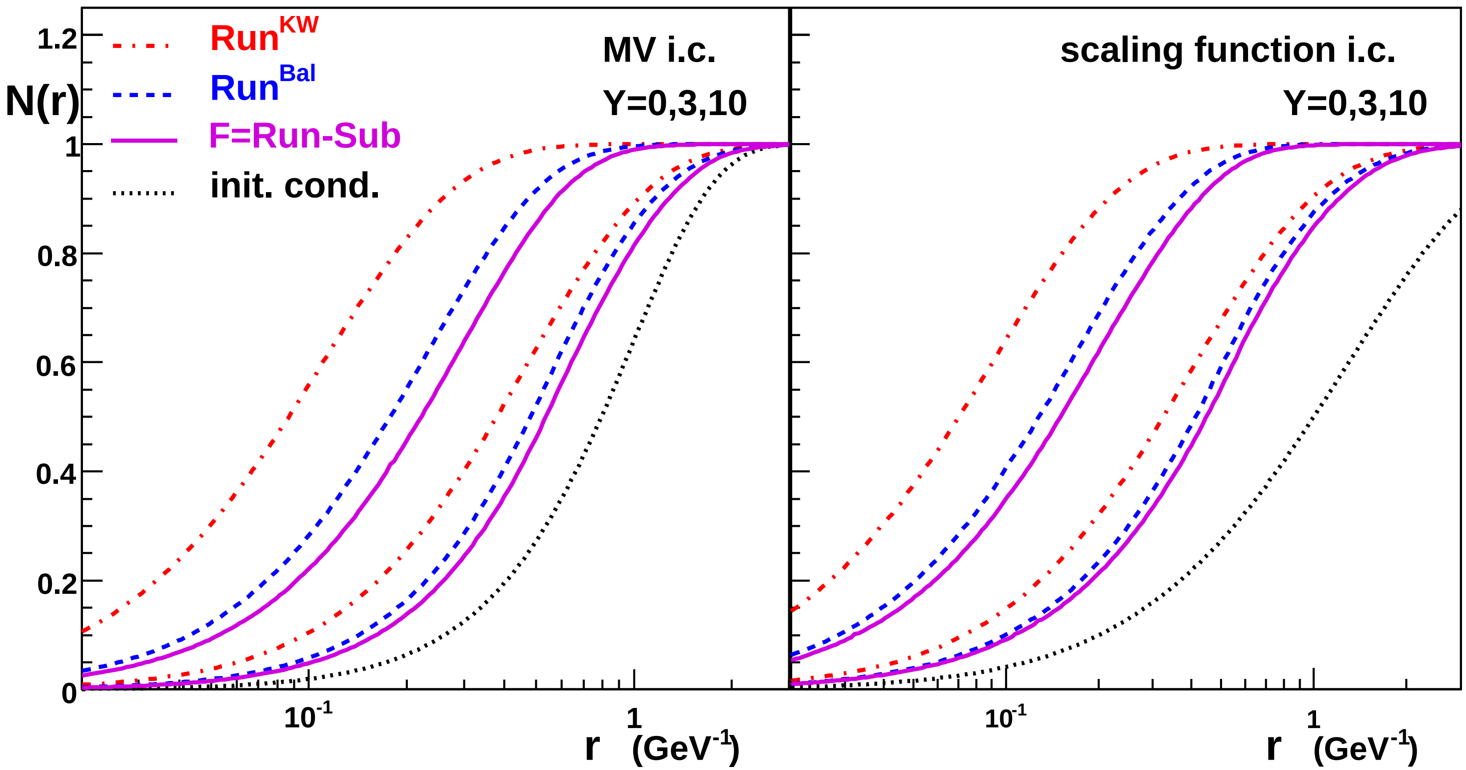
- Total kernel $\mathcal{F} = \mathcal{R} - \mathcal{S}$



RC BK evolution

AK results - Phys. Rev. D75, 125021 (2007)

- Complete RC BK equation



RC BK Fit to F_2

- Albacete et. al. - AAMS have used only the running copuling corrections

$$\frac{\partial \mathcal{N}(r,Y)}{\partial Y} = \mathcal{R}[\mathcal{N}]\,, \tag{28}$$

within the Balitsky's prescription.

- The IF regularization is made taking

$$\alpha_s(r^2) = \frac{12\pi}{\left(11N_c - 2N_f\right)\,\ln\left(\frac{4\,C^2}{r^2\Lambda^2_{QCD}}\right)} \quad \text{for } r < r_{fr} \text{ small}$$
$$\alpha_{fr} = 0.7 \quad \text{for } r < r_{fr}, \tag{29}$$

with $N_f = 3$.

- $\Lambda_{QCD} = 0.241$ GeV, such that $\alpha_s(M_Z) = 0.1176$.
- The factor C^2 has been left free.
- The σ_0 constant resulting for the $\underline{b}$ integration has been left free too.

RC BK Fit to F_2

- IC's for the evolution:

$$\mathcal{N}^{GBW}(r, Y=0) = 1 - \exp\left[-\left(\frac{r^2\, Q_{s\,0}^2}{4}\right)^{\gamma}\right]. \tag{30}$$

$$\mathcal{N}^{MV}(r, Y=0) = 1 - \exp\left[-\left(\frac{r^2 Q_{s\,0}^2}{4}\right)^{\gamma} \ln\left(\frac{1}{r\,\Lambda_{QCD}} + e\right)\right], \tag{31}$$

- $Q_{s\,0}^2 \equiv Q_s^2(x = 0.01)$ and γ has been left free.
- Kinematical region:

$$\begin{aligned} & x \leq 10^{-2} \\ & 0.045\ \mathrm{GeV}^2 \leq Q^2 \leq 800\ \mathrm{GeV}^2 \end{aligned} \tag{32}$$

- They have used only light quarks, with $m_f = 0.14$ GeV, and

$$\tilde{x} = x\left(1 + \frac{4m_f^2}{Q^2}\right). \tag{33}$$

RC BK Fit to F_2

AAMS results

Initial condition	σ_0 (mb)	Q^2_{s0} (GeV^2)	C^2	γ	$\chi^2/\mathrm{d.o.f.}$
GBW	31.59	0.24	5.3	1 (fixed)	916.3/844=1.086
MV	32.77	0.15	6.5	1.13	906.0/843=1.075

- We have used the AAMS results, and the GBW dipole cross section, to describe the HERA data for F_2 and F_2^c ($m_c = 1.3$ GeV and $m_l = 0.14$ GeV)
- GBW dipole model:

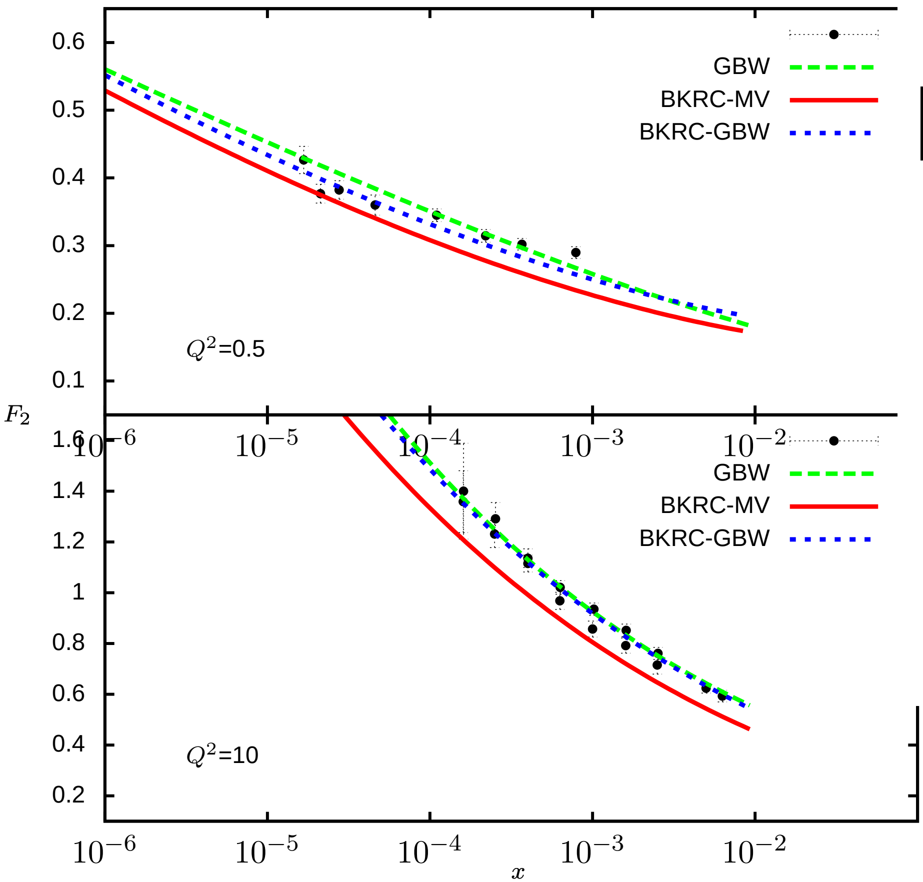
$$\sigma_{dip}^{\text{GBW}}(r,x) = 1 - exp\left[-\frac{r^2}{2R_0(x)}\right], \quad (34)$$

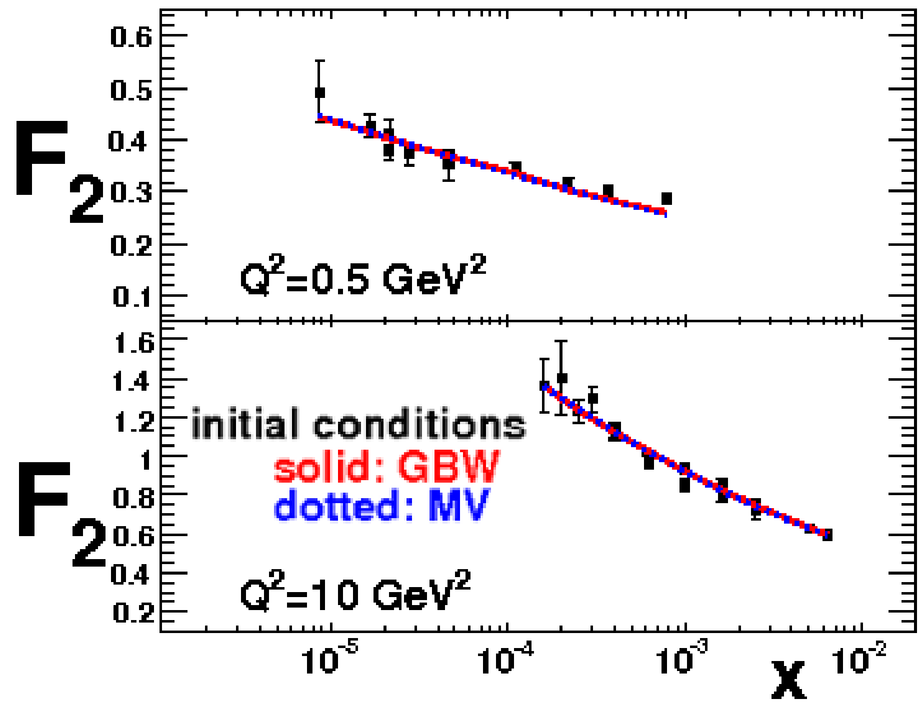
$$R_0(x) = \frac{1}{Q_0}\left(\frac{x}{x_0}\right)^{\lambda/2} \quad (35)$$

- With charm: $\sigma_0 = 74.78\ \lambda = 0.277\ x_0 = 0.41e-4$
- Without charm: $\sigma_0 = 59.141\ \lambda = 0.288\ x_0 = 3.04e-4$

F_2 Results

only light quarks





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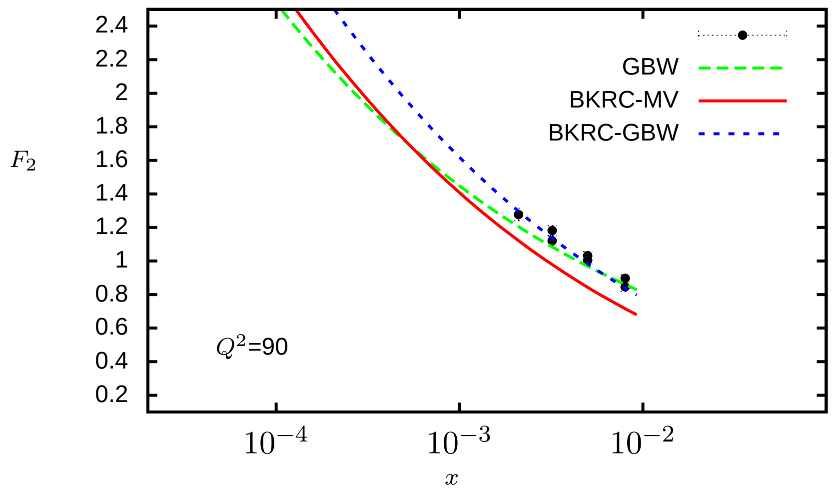
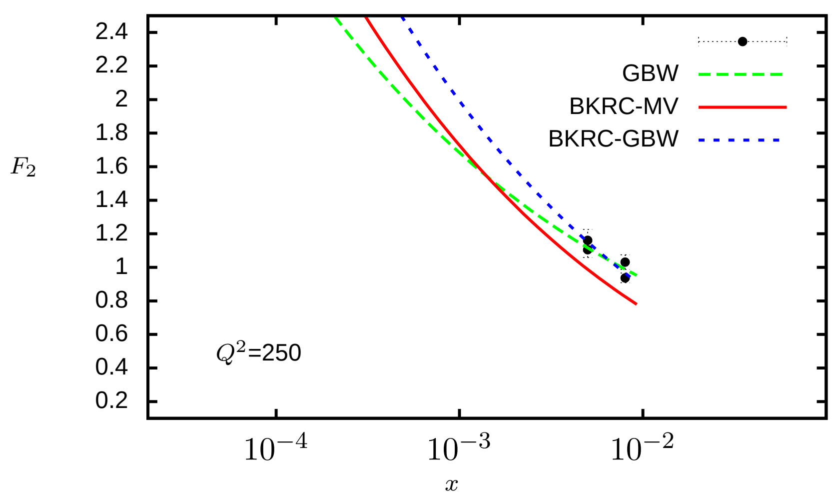
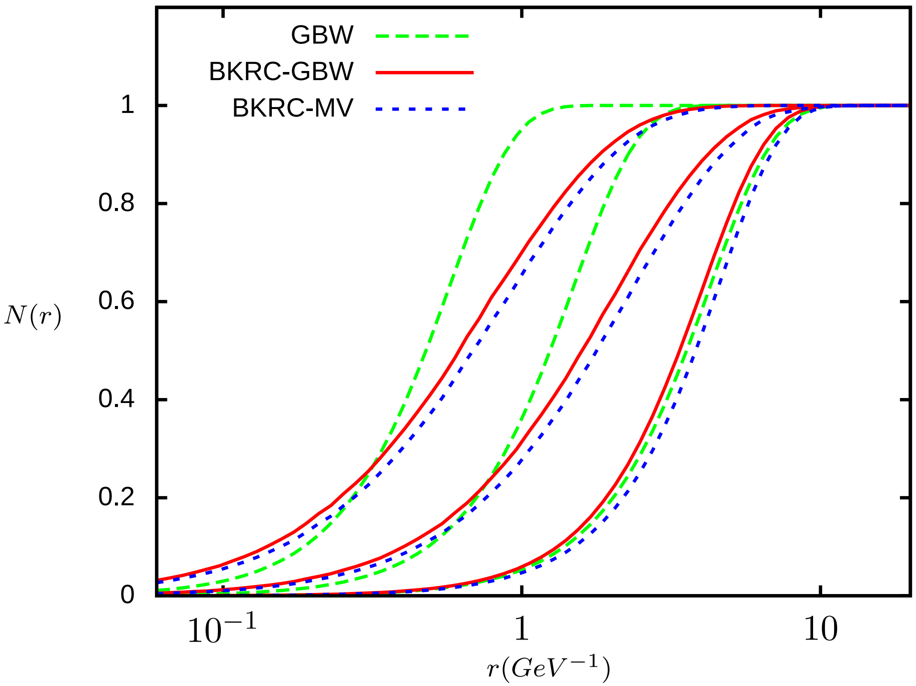
F_2 Results

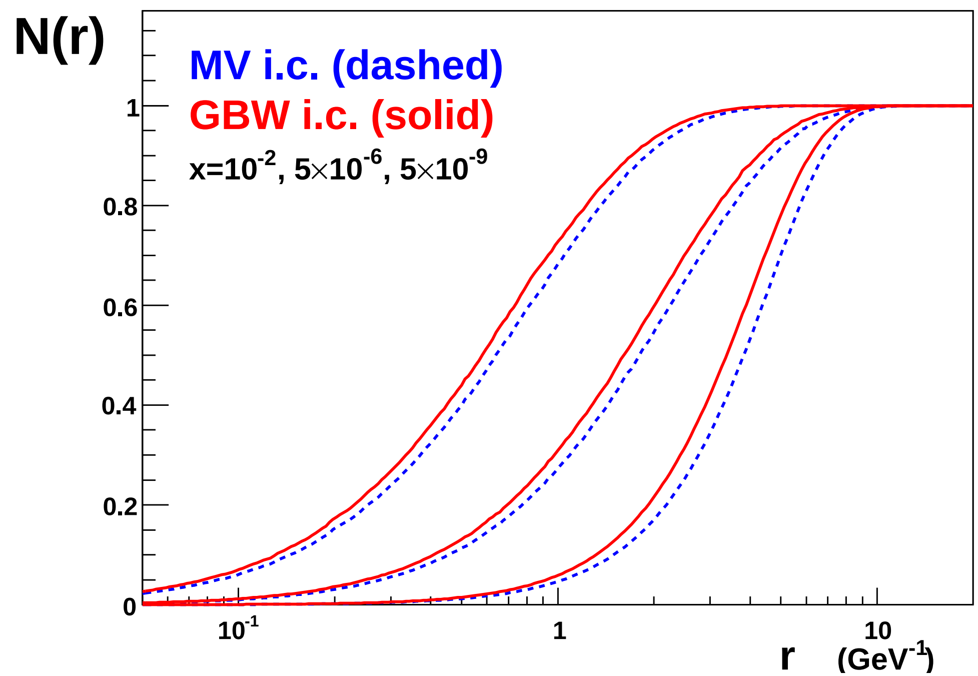
only light quarks





$N(r)$ Results





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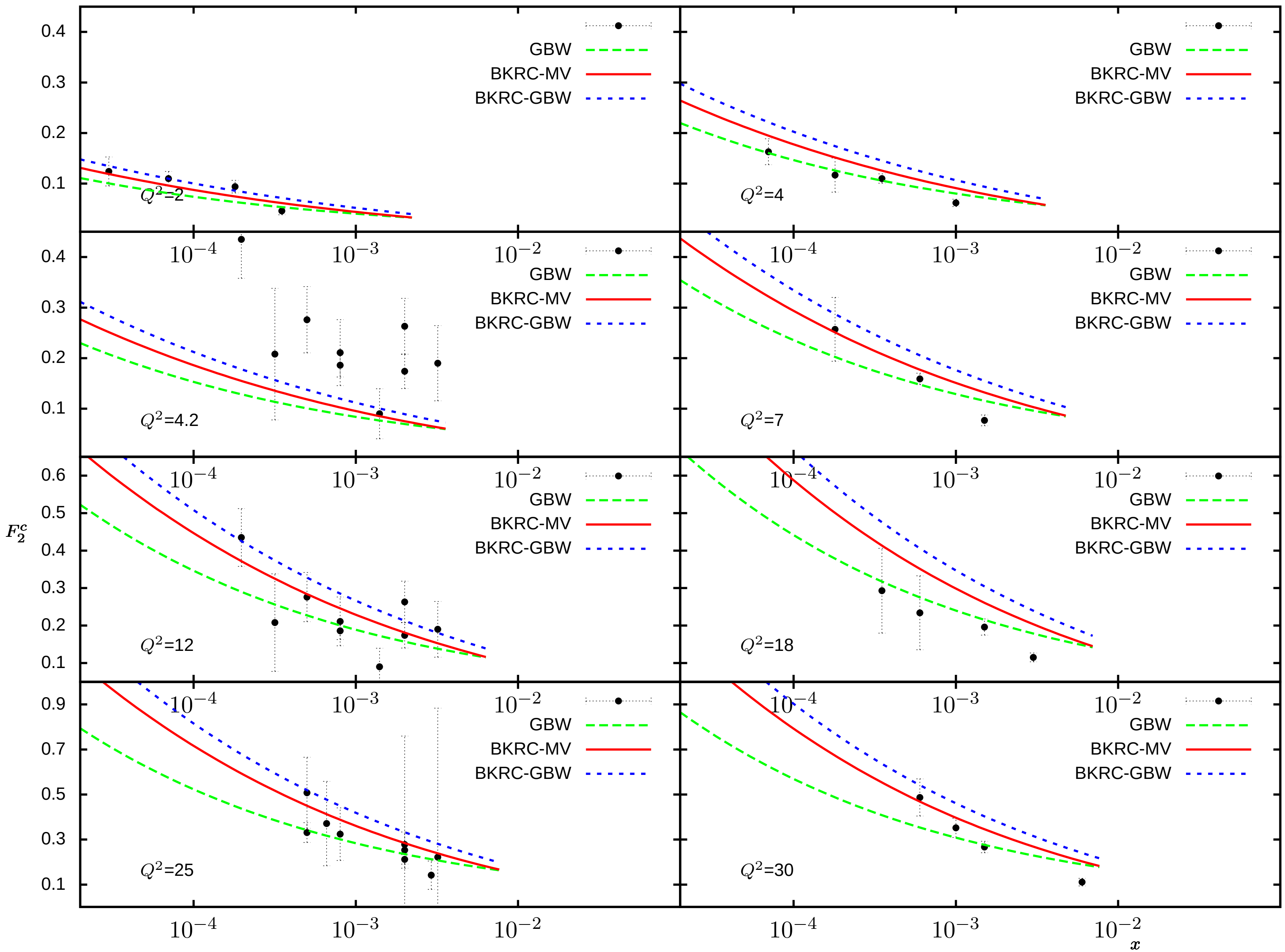
F_2^c Results



GBW
BKRC-MV
BKRC-GBW
Q^2=2
Q^2=4
Q^2=4.2
Q^2=7
Q^2=12
Q^2=18
Q^2=25
Q^2=30
F_2^c
x
10^{-4}
10^{-3}
10^{-2}