Drell-Yan process at next-to-leading order

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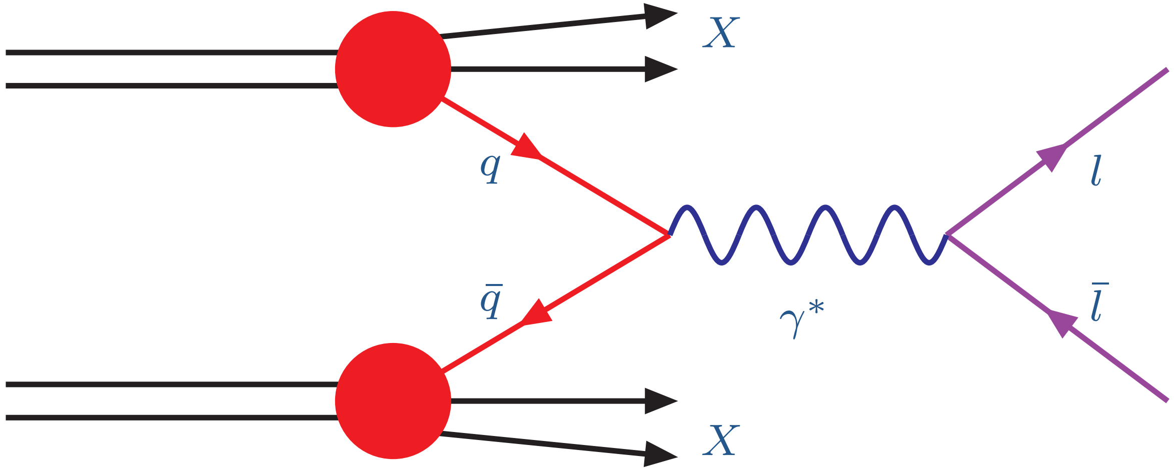
Summary

- High energy hadron collisions
- Drell-Yan process
- Parton model
- Intrinsic transverse momentum
- Next-to-leading order corrections
- Conclusions and perspectives
- References

High energy hadron collisions

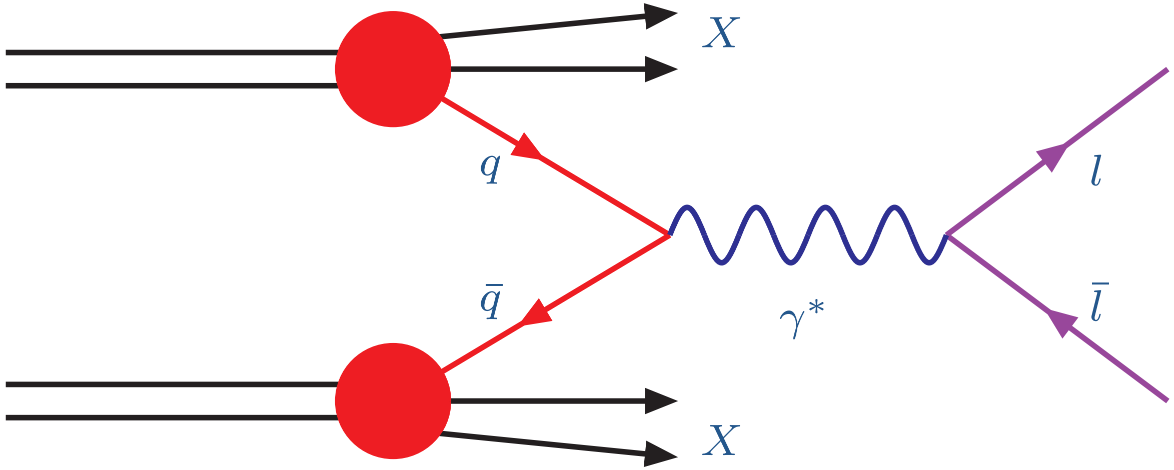
- Why high energy hadron collisions?
- Although knowing that hadrons are composed by partons, we do not know the details in this process.
- Partons have color charge, while hadrons do not have, in a way that partons are confined inside hadrons.
- The only way to study (“collide”) partons is studying (“colliding”) hadrons. (In other words, it is no possible to create a quark or gluon beam.)
- In high energy, perturbative techniques and the parton model are expected to be applicable.
- In the framework presented, the Drell-Yan process is relevant.

Drell-Yan process



- Drell-Yan process is the production of dileptons (leptons-antilepton pairs) following the combination of partons in a hadron collision.
- It was proposed in S.D. Drell, T.M. Yan, Phys. Rev. Lett. 25, 316 (1970).
- At leading order, it is the annihilation of a quark-antiquark pair into a virtual boson that splits into the dilepton.
- At leading order, only QED vertices and the parton model are needed (no QCD vertices).

Drell-Yan process



- The lepton can be an electron (0.51 MeV), muon (105 MeV) or tauon (1777 MeV).
- In addition to the lepton-antilepton pair, there is a remainder X, product of the other partons.
- The dilepton does not strongly interact, i. e., it is not affected by the remainder X.
- If the dilepton mass M is much smaller than the boson Z mass (91 GeV), the virtual boson that dominates in the Drell-Yan process is the photon (zero mass).

Drell-Yan process variables at LO

- Natural units ($c = 1, \hbar = 1$).
- Hadron momenta: P_A e P_B.
- Infinite momentum frame: $P_A^2 = 0 = P_B^2$
- Parton momenta: $p_A = x_A P_A$ e $p_B = x_B P_B$ (collinear).
- Lepton momenta: p_1 e p_2.
- Hadron center of mass energy squared:

$$s = (P_A + P_B)^2 = P_A^2 + P_B^2 + 2P_A \cdot P_B \approx 2P_A \cdot P_B \tag{1}$$

- Dilepton mass:

$$M^2 = (p_1 + p_2)^2 = (p_A + p_B)^2 = p_A^2 + p_B^2 + 2p_A \cdot p_B \approx 2p_A \cdot p_B = 2x_A x_B P_A \cdot P_B = x_A x_B s \tag{2}$$

Parton Model

- To determine the parton distribution of a hadron is a problem the requires a solution from the nonperturbative QCD.
- That solution is not available.
- What can be done is to use a parametrization obtained from experiments.
- In Drell-Yan process, parton distribution momentum functions are used: $f_q(x_A)$ is the probability of finding parton q with momentum between x_A and $x_A + dx_A$ times the A hadron momentum.
- The differential cross-section for the Drell-Yan process is (at LO):

$$d\sigma = \sum_q \left[f_q(x_A) f_{\bar{q}}(x_B) + f_{\bar{q}}(x_A) f_q(x_B) \right] dx_A dx_B \hat{\sigma}_q \tag{3}$$

- $\hat{\sigma}_q$ is the subprocess ($q + \bar{q} \rightarrow \gamma^* \rightarrow l + \bar{l}$) cross-section.

Subprocess $q + \bar{q} \to \gamma^* \to l + \bar{l}$

At leading order, the Drell-Yan cross section is an application of QED.

$$\imath\mathcal{M} = \bar{v}^{s'}(p_B)(\imath e_q e\gamma^\mu)u^s(p_A)\left(-\frac{\imath g_{\mu\nu}}{M^2}\right)\bar{u}^r(p_1)(\imath e\gamma^\nu)v^{r'}(p_2) \tag{4}$$

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2 = \frac{e_q^2 e^4}{4M^4}\text{tr}\left[(p_A)_\sigma\gamma^\sigma\gamma^\mu(p_B)_{\sigma'}\gamma^{\sigma'}\gamma^\nu\right]\text{tr}\left[(p_1)_\rho\gamma^\rho\gamma_\mu(p_2)_{\rho'}\gamma^{\rho'}\gamma_\nu\right] \tag{5}$$

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2 = \frac{8e_q^2 e^4}{M^4}\left[(p_A\cdot p_1)(p_B\cdot p_2) + (p_A\cdot p_2)(p_B\cdot p_1)\right] \tag{6}$$

In the center of mass frame (also $e^2 = 4\pi\alpha$):

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2 = \frac{16e_q^2(4\pi)^2\alpha^2}{M^4}\frac{M^4}{16}\left[1+\cos^2\theta\right] = e_q^2(4\pi)^2\alpha^2\left[1+\cos^2\theta\right] \tag{7}$$

$$\frac{d\hat{\sigma}}{d\Omega} = \frac{1}{64\pi^2 M^2}\sum_{\text{spins}}|\mathcal{M}|^2 = \frac{e_q^2\alpha^2}{4M^2}\left[1+\cos^2\theta\right] \to \hat{\sigma} = \frac{4\pi e_q^2\alpha^2}{3M^2} \tag{8}$$

LO Cross section

- It is usual to change the variables x_A and x_B to variables better suited to be measured in the laboratory:

$$\tau = x_A x_B = \frac{M^2}{s} \qquad\qquad y = \frac{1}{2}\ln\frac{x_A}{x_B} \tag{9}$$

- Therefore:

$$\frac{d\sigma}{d\tau dy} = \frac{4\pi\alpha^2}{9M^2}\sum_q e_q\left[f_q(x_A)f_{\bar{q}}(x_B) + f_{\bar{q}}(x_A)f_q(x_B)\right] \tag{10}$$

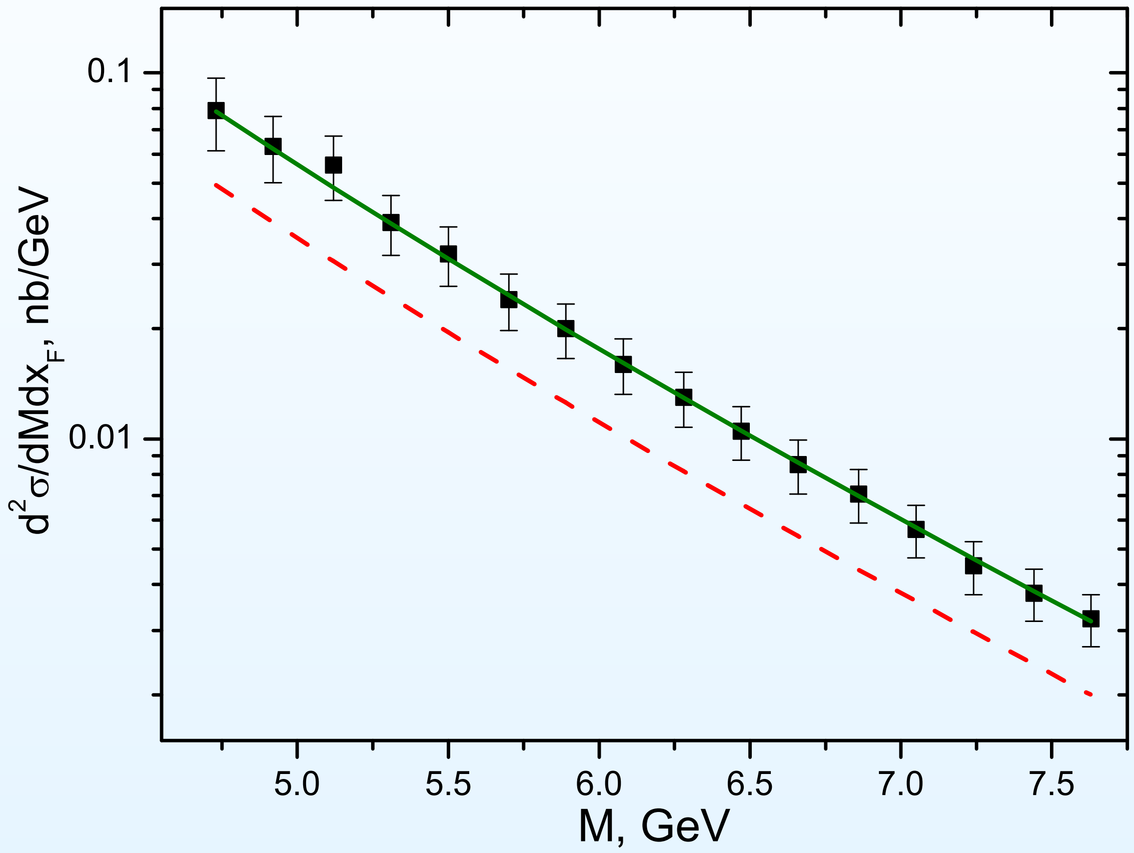
- One can integrate in the variable y ($dy = \frac{dx_A}{2x_A}$ and, for fixed τ, x_A is minimal when $x_B = 1$ and as a result $x_{A\,\mathrm{min}} = \tau$):

$$\frac{d\sigma}{d\tau} = \int_\tau^1 \frac{dx_A}{2x_A}\frac{4\pi\alpha^2}{9M^2}\sum_q e_q\left[f_q(x_A)f_{\bar{q}}(\tau/x_A) + f_{\bar{q}}(x_A)f_q(\tau/x_A)\right] \tag{11}$$

$$M^4\frac{d\sigma}{dM^2} = \tau\int_\tau^1 \frac{dx_A}{2x_A}\frac{4\pi\alpha^2}{9}\sum_q e_q\left[f_q(x_A)f_{\bar{q}}(\tau/x_A) + f_{\bar{q}}(x_A)f_q(\tau/x_A)\right] \tag{12}$$

Experiment: leading order

- Data from Fermilab E439 experiment ($\sqrt{s} = 20$GeV and $x_F = 0.1$).
- The leading order calculation is wrong by a factor of 1.6 (K factor, does not depend in x).



Intrinsic transverse momentum

- Experimentally, one observes dilepton transverse momentum ($\vec{p_T}$, bidimensional), not described in the model so far.
- One source for this momentum can be the intrinsic transverse momentum of the partons.
- The parton distribution function changes by:

$$f(x)dx \longrightarrow f(x)h(\vec{k_T})dxd^2k_T \tag{13}$$

with $\int h(\vec{k_T})d^2k_T = 1$. The p_T-dependent cross section is given by:

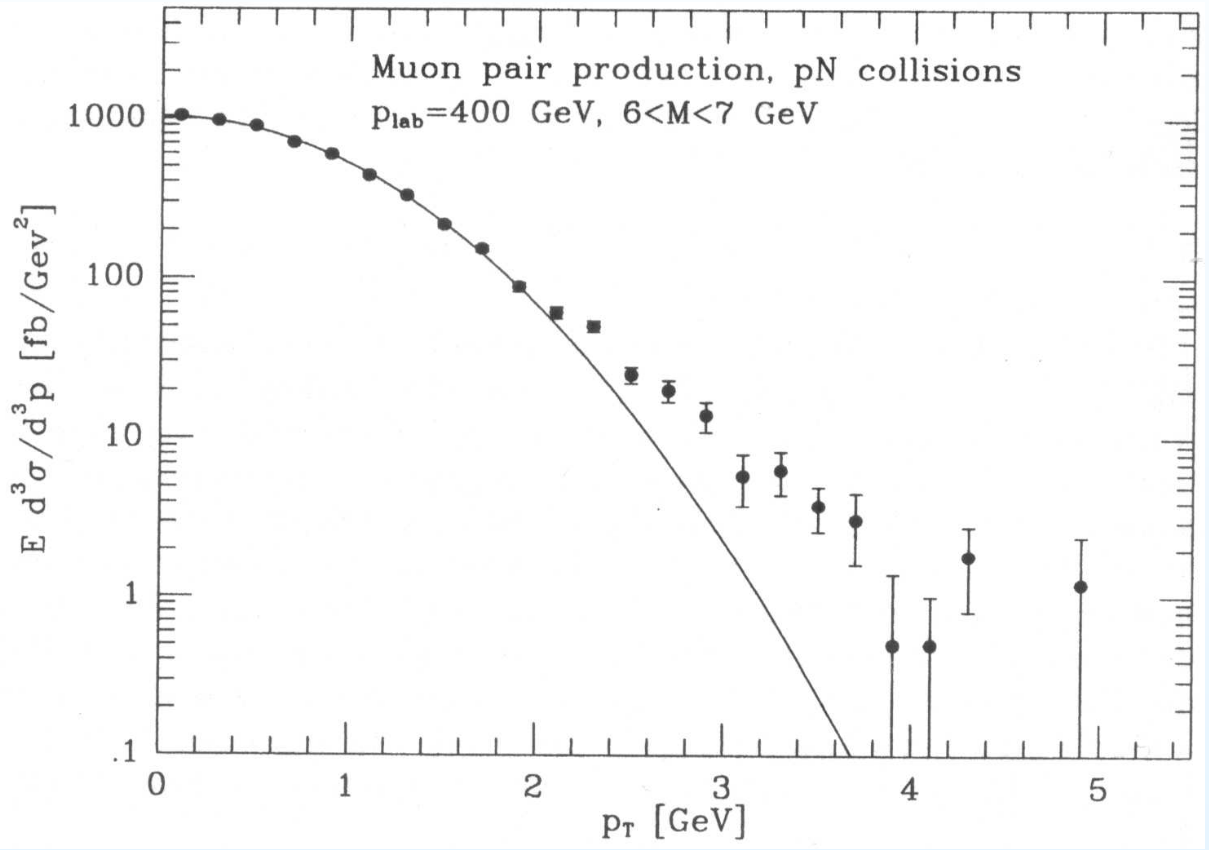
$$\frac{1}{\sigma}\frac{d\sigma}{d^2p_T} = \int d^2k_{T1}d^2k_{T2}\delta^{(2)}(\vec{k_{T1}} + \vec{k_{T2}} - \vec{p_T})h(\vec{k_{T1}})h(\vec{k_{T2}}). \tag{14}$$

- If the distribution is a Gaussian ($h(\vec{k_{T1}}) = \frac{1}{2\pi b^2}\exp(\frac{k_{T1}^2}{2b^2})$), the result is simply:

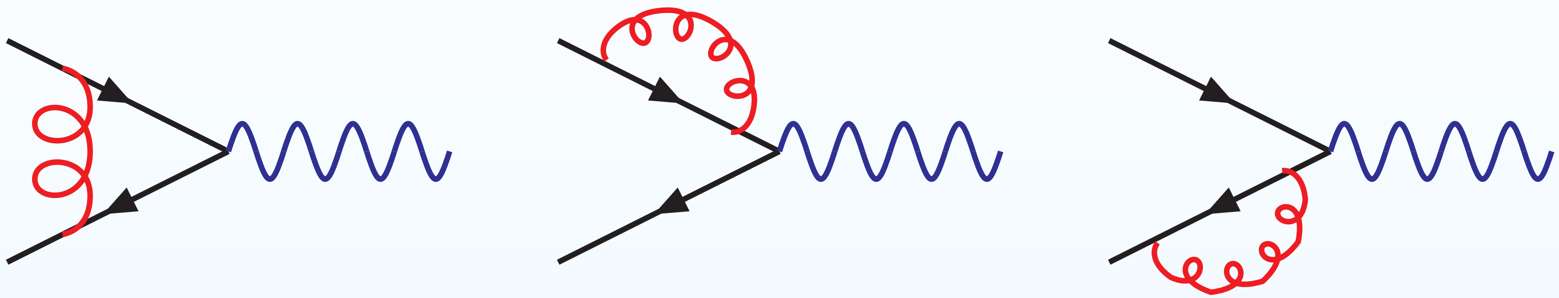
$$\frac{1}{\sigma}\frac{d\sigma}{d^2p_T} = \frac{1}{4\pi b^2}\exp\left(\frac{p_T^2}{4b^2}\right). \tag{15}$$

Experiment: intrinsic transverse momentum

- Data from Fermilab E288 experiment ($\sqrt{s} = 20$GeV, 6 GeV $< M <$ 7GeV and $x_F = 0.1$).
- For small p_T, a good description of the cross section is achieved.
- For large p_T, the cross section follows a power law.

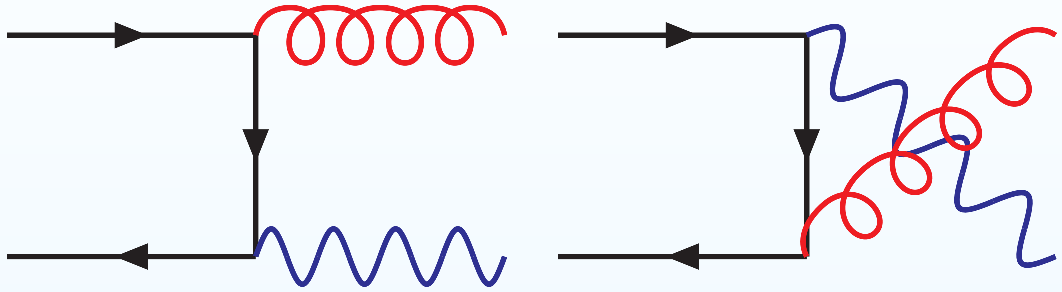


NLO – Virtual corrections



- At next-to-leading order, more diagrams are involved.
- Now, QCD is applicable in the quark-gluon vertices.
- The diagrams above interfere with the Born diagram.
- The first diagram is a vertex correction, that contributes to the α_s renormalization.
- The other two contribute to the self-energy of the quarks.
- No transverse momentum generated.

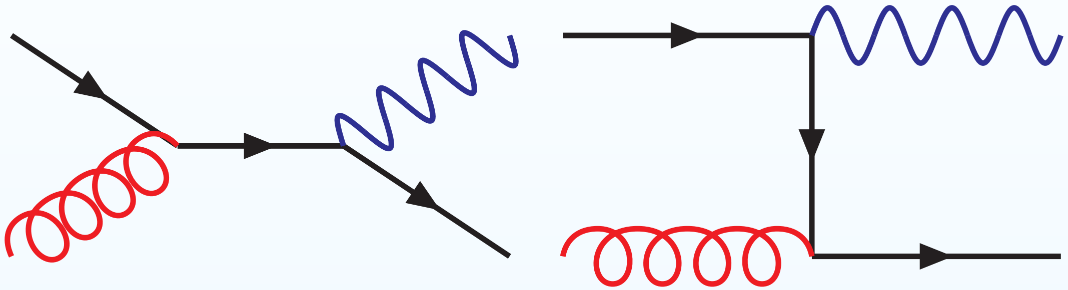
NLO – Annihilation



- Annihilation diagrams, in which a quark and an antiquark combine to form a photon and a gluon.
- The photon can have transverse momentum, opposite to the gluon transverse momentum.
- The partonic cross section is given by:

$$\frac{d\hat{\sigma}_{\text{annih}}}{dM^2 d\hat{t}}(\hat{s},\hat{t}) = \frac{8}{27}\frac{\alpha^2\alpha_{\text{s}}e_q^2}{M^2\hat{s}^2}\frac{2M^2\hat{s}+\hat{u}^2+\hat{t}^2}{\hat{t}\hat{u}} \tag{16}$$

NLO – Compton



- Compton diagrams, in which a quark (or antiquark) absorbs a gluon and emits a photon.
- Again, there is photon transverse momentum.

$$\frac{d\hat{\sigma}_{\mathrm{Compt}}}{dM^2 d\hat{t}}(\hat{s},\hat{t}) = \frac{1}{9}\frac{\alpha^2 \alpha_{\mathrm{s}} e_q^2}{M^2 \hat{s}^2}\frac{2M^2\hat{u} + \hat{s}^2 + \hat{t}^2}{-\hat{s}\hat{t}} \qquad (17)$$

NLO cross section

- The double differential Drell-Yan cross section at next-to-leading order is given by:

$$\frac{d\sigma}{dM^2dy} = \frac{\hat{\sigma}_0}{s}\int_0^1 dx_A dx_B dz \delta(x_A x_B z - \tau)\delta\left(y - \frac{1}{2}\ln\frac{x_A}{x_B}\right) \left\{\left[\sum_q e_q^2 (f_q(x_A) f_{\bar{q}}(x_B) + x_A \leftrightarrow x_B)\right]\left[\delta(1-z) + \frac{\alpha_s(M^2)}{2\pi} D_q(z)\right] \left[\sum_q e_q^2 \left[f_g(x_A)(f_q(x_B) + f_{\bar{q}}(x_B)\right] + x_A \leftrightarrow x_B\right]\left[\frac{\alpha_s(M^2)}{2\pi} D_g(z)\right]\right\}. \quad (18)$$

- At next-to-leading order, $f_q(x) = f_q(x, M)$.
- Also, $\alpha_s = \alpha_s(M^2)$.
- The functions $D_q(z)$ and $D_g(z)$ depend on the renormalization scheme used.

NLO cross section

- In the $\overline{\mathrm{MS}}$ scheme they are given by (with $C_F = 4/3$, $T_R = 1/2$):

$$D_q(z) = C_F\left[\delta(1-z)\left(\frac{2\pi^2}{3}-8\right) - 2\frac{1+z^2}{1-z}\ln z + 4(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+\right] \quad (19)$$

$$D_g(z) = T_R\left[(z^2+(1-z)^2)\ln\frac{(1-z)^2}{z} + \frac{1}{2} + 3z - \frac{7}{2}z^2\right]. \quad (20)$$

- In the DIS scheme they are given by:

$$D_q^{\mathrm{DIS}}(z) = C_F\left[\delta(1-z)\left(1+\frac{4\pi^2}{3}\right) - 6 - 4z + \left(\frac{3}{1-z}\right)_+ + 2(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+\right] \quad (21)$$

$$D_g^{\mathrm{DIS}}(z) = T_R\left[(z^2+(1-z)^2)\ln(1-z) + \frac{3}{2} - 5z + \frac{9}{2}z^2\right]. \quad (22)$$

+ functions

- The + functions are defined by
- They are defined as:

$$(F(x))_+ = \lim_{\beta \to 0} \{F(x)\theta(1-x-\beta) + \log(\beta)\delta(1-x-\beta)\} \quad (23)$$

- For $x < 1-\beta$, $(F(x))_+ = F(x)$, but the integral over x vanishes.

$$\int_0^1 (F(x))_+ dx = 0$$

- They a way to mathematically account virtual corrections in the expressions.
- For practical purposes:

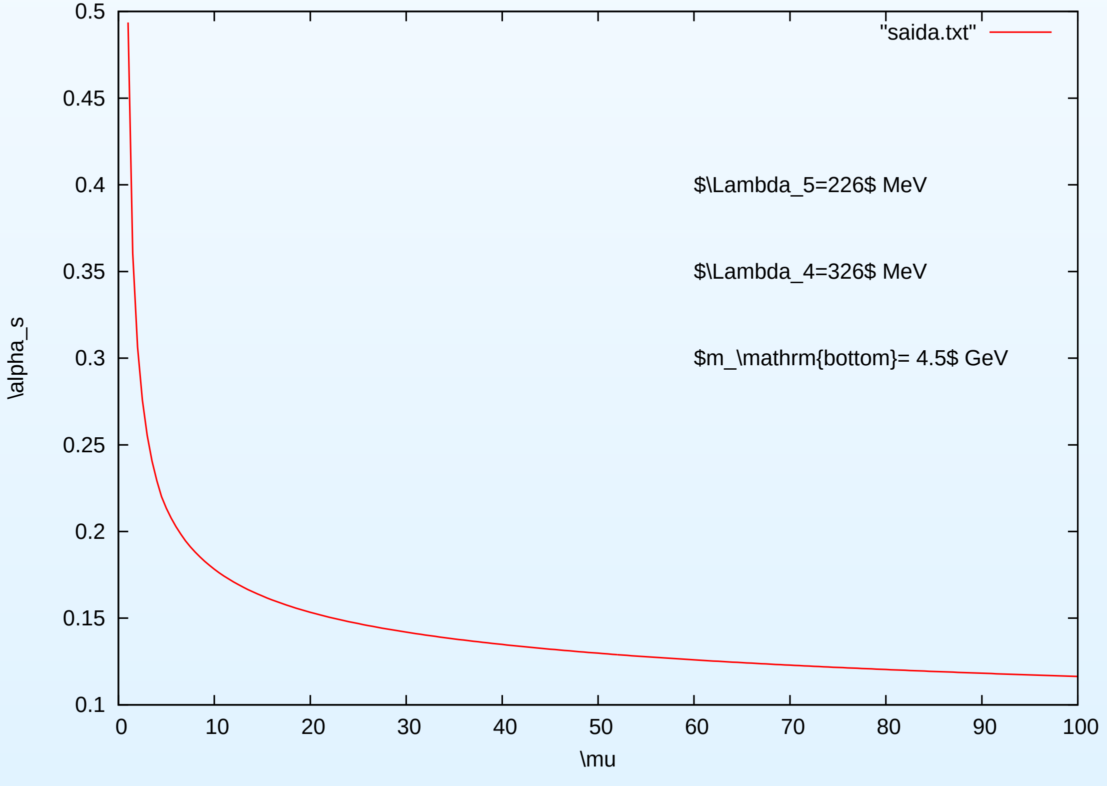
$$\int_0^1 dx g(x)(F(x))_+ = \int_0^1 dx(g(x) - g(1))F(x) \quad (24)$$

NLO α_s

- At NLO, α_s is given by:

$$\beta_0 = 11 - \frac{2}{3} n_f \qquad \beta_1 = 51 - \frac{19}{3} n_f$$

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2)} \left[1 - \frac{2\beta_1}{\beta_0^2} \frac{\ln[\ln(\mu^2/\Lambda^2)]}{\ln(\mu^2/\Lambda^2)} \right] \tag{25}$$



NLO cross section

- In the $\overline{\mathrm{MS}}$ scheme, the cross section can be rewritten as ($x_{A,B} = \sqrt{\tau/z}\exp \pm y$):

$$\frac{d\sigma}{dM^2dy} = \frac{\hat{\sigma}_0}{s}\int_\tau^1 \frac{dz}{z} \tag{26}$$

$$\left\{\left[\sum_q e_q^2(f_q(x_A)f_{\bar{q}}(x_B) + x_A \leftrightarrow x_B)\right]\left[\delta(1-z) + \frac{\alpha_s(M^2)}{2\pi}D_q(z)\right]\right.$$

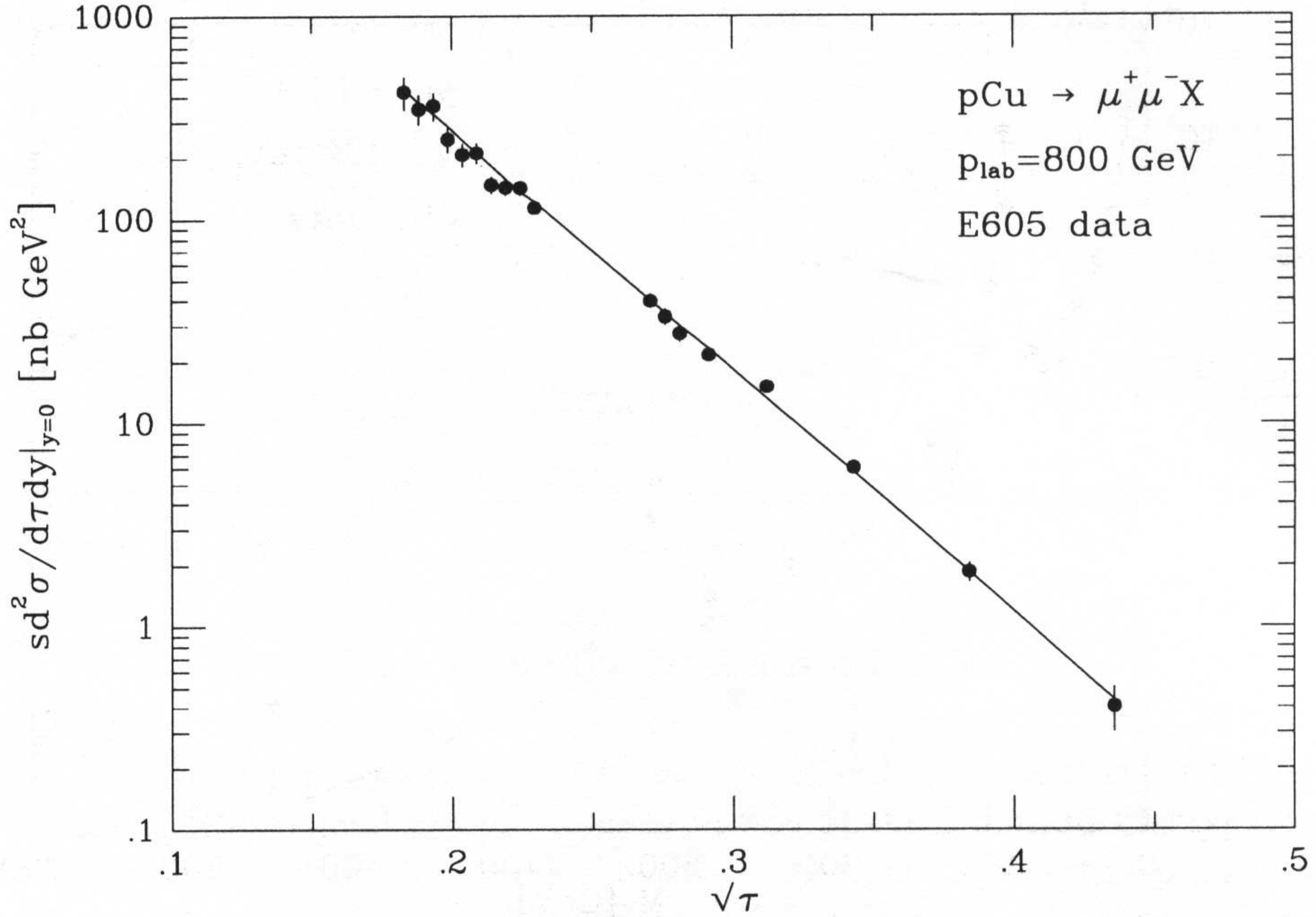
$$\left.\left[\sum_q e_q^2\left[f_g(x_A)(f_q(x_B) + f_{\bar{q}}(x_B)\right] + x_A \leftrightarrow x_B\right]\left[\frac{\alpha_s(M^2)}{2\pi}D_g(z)\right]\right\}$$

$$\frac{d\sigma}{dM^2dy} = \frac{\hat{\sigma}_0}{s}\left(1 + \frac{2\alpha_s(M^2)}{3\pi}\left(\frac{2\pi^2}{3} - 8\right)\right)\left[\sum_q e_q^2(f_q(x_A)f_{\bar{q}}(x_B) + x_A \leftrightarrow x_B)\right]$$

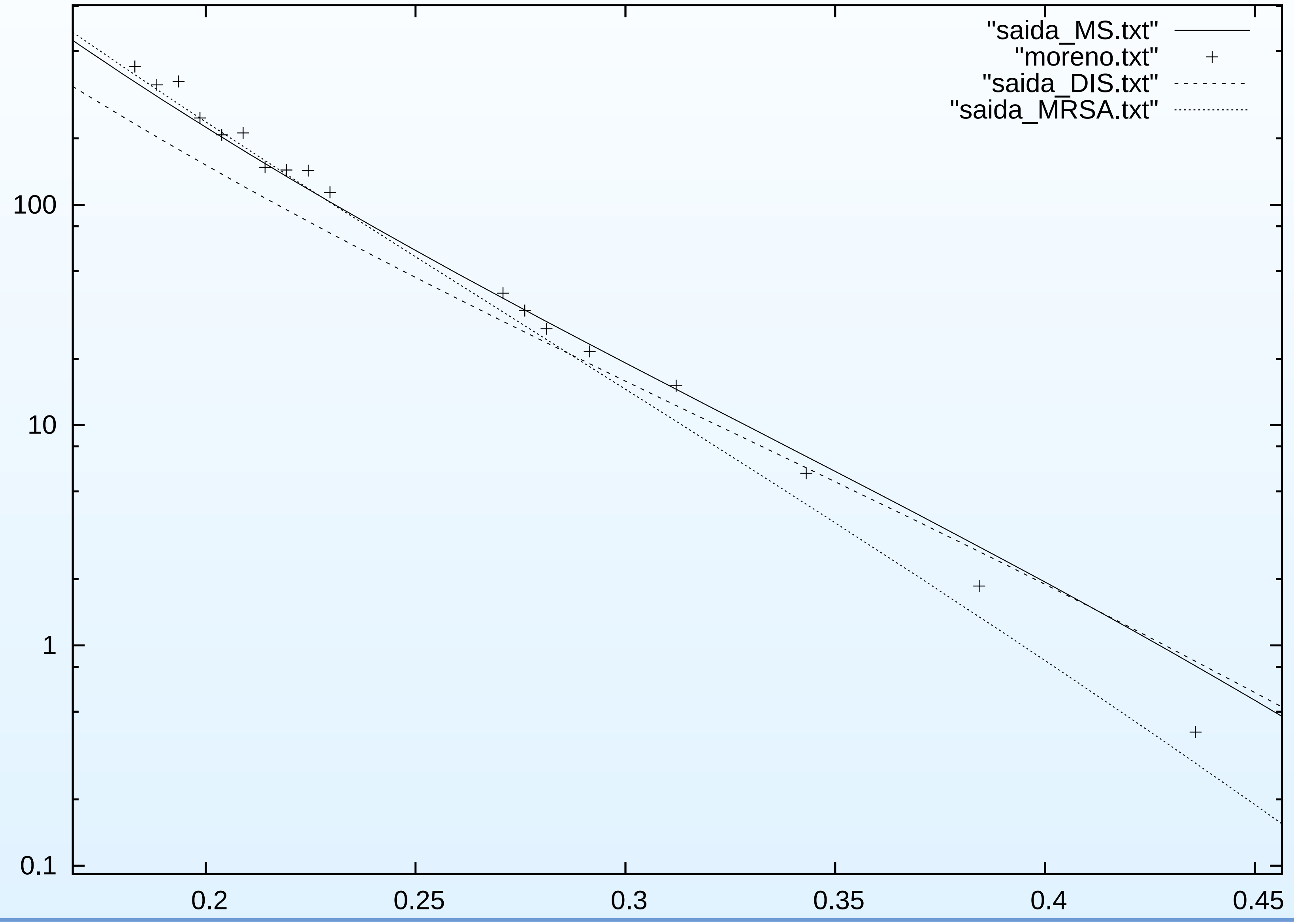
$$+\frac{\hat{\sigma}_0}{s}\frac{\alpha_s(M^2)}{2\pi}\int_\tau^1 \frac{dz}{z}\left\{\left[\sum_q e_q^2(f_q(x_A)f_{\bar{q}}(x_B) + x_A \leftrightarrow x_B)\right]D_q(z)\right.$$

$$\left.\left[\sum_q e_q^2\left[f_g(x_A)(f_q(x_B) + f_{\bar{q}}(x_B)\right] + x_A \leftrightarrow x_B\right]D_g(z)\right\} \tag{27}$$

Experiment: NLO



Experiment: NLO



Nuclear parton distributions

- Nuclear structure function $f_q^A(x, M^2)$ are needed.
- Two different approaches used:

  Eskola, Kolhinen and Salgado (EKS parametrization) *Eur. Phys. J. C* **9**, *61 (1999); J. High Energy Phys.* **05**, *002 (2007)* and

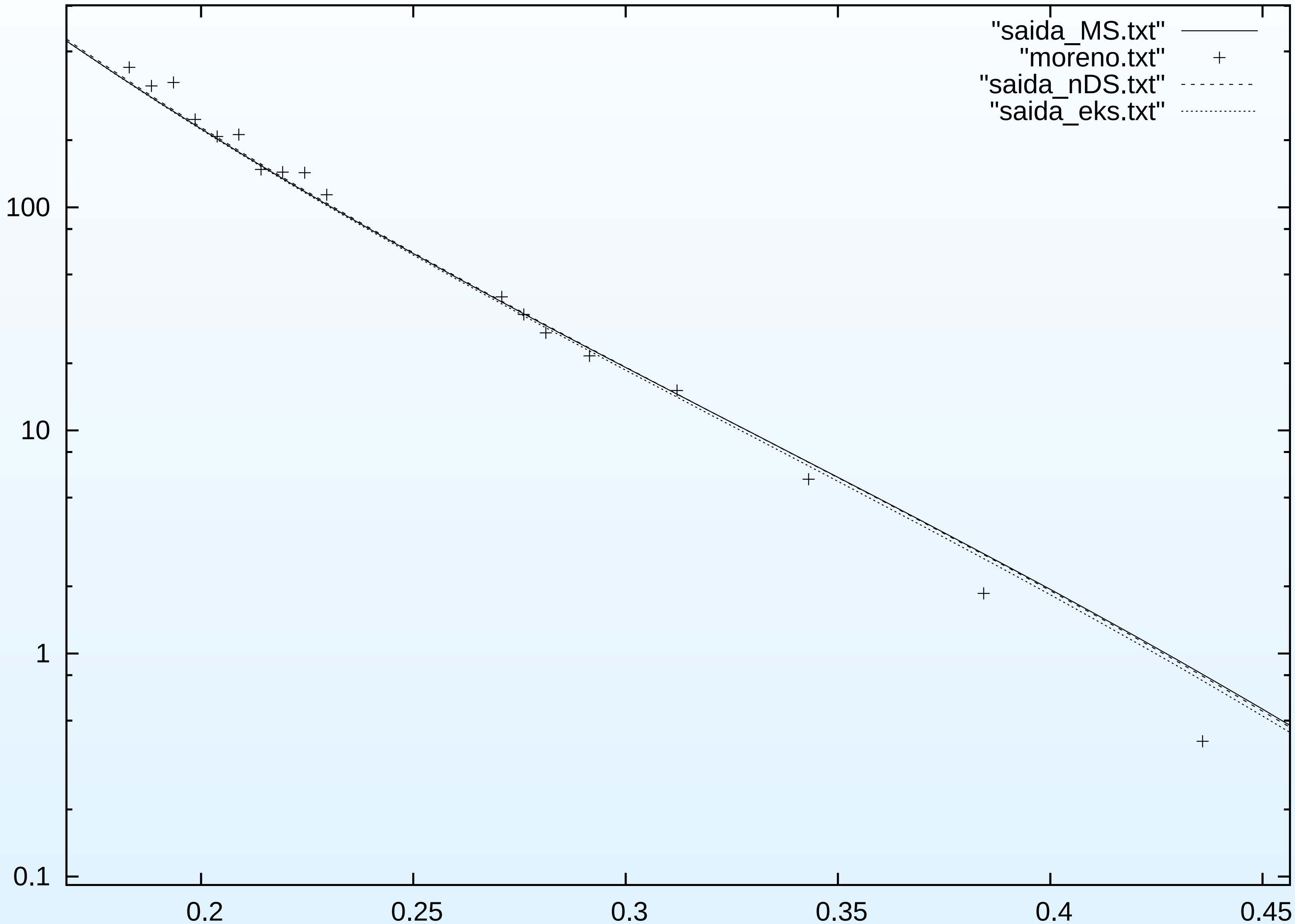
  D. de Florian and R. Sassot (nDS parametrization) *Phys. Rev. D* **69**, *074028 (2004)*
- EKS parametrization gives NPDFs simply by multiplying proton PDFs by a factor:

$$f_q^A(x, Q_0^2) = R_q^A(x, Q_0^2) f_q^p(x, Q_0^2)$$

- nDS parametrization obtains NPDFs from proton PDFs by a convolution:

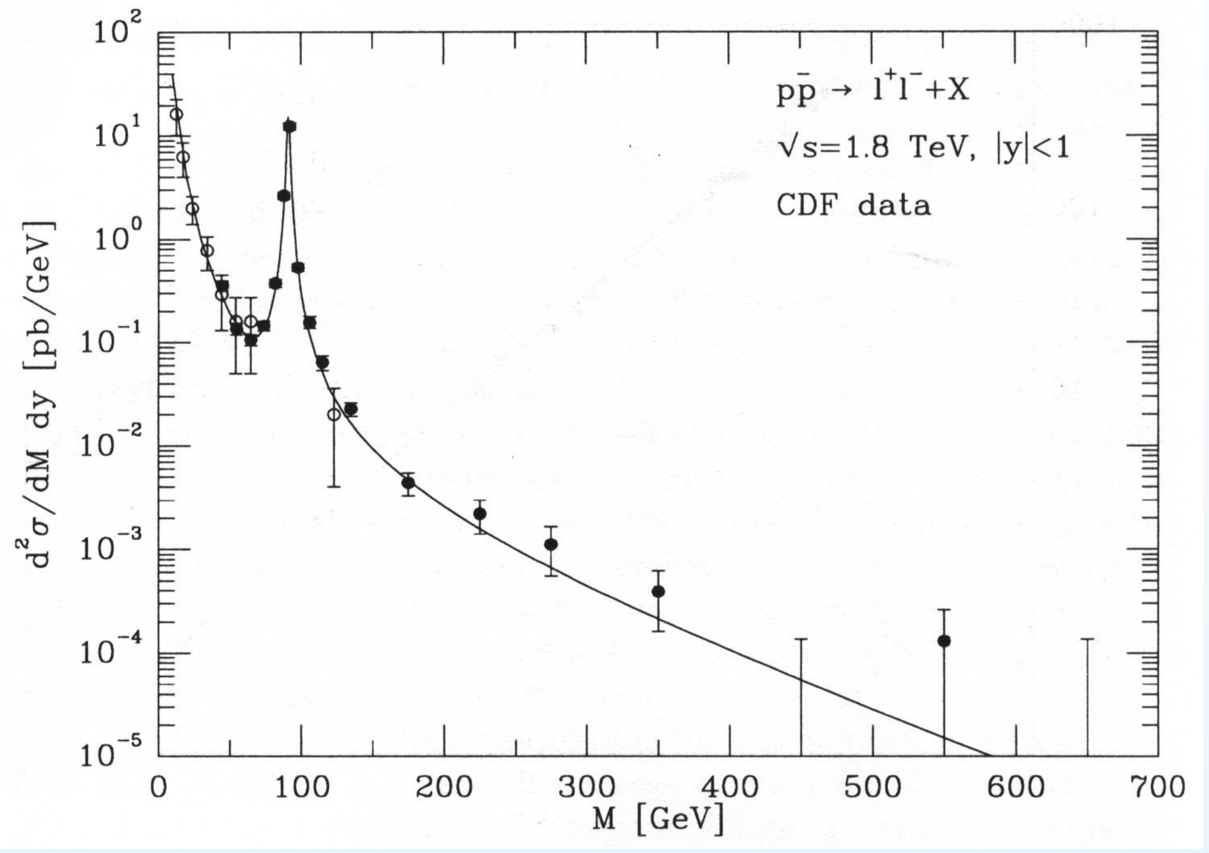
$$f_q^A(x, Q_0^2) = \int_x^A \frac{dy}{y} W_q(y, A) f_q^p\left(\frac{x}{y}, Q_0^2\right)$$

Experiment: NLO



Experiment: NLO

- Data from Fermilab E741 ($\sqrt{s} = 1.8$TeV and $|y| < 1$).
- $p\bar{p}$: do we needed antiproton parton distributions?
- Z Boson is considered.



NLO transverse momentum

- One can combine intrinsic transverse momentum with partonic CS transverse momentum with the following integral:

$$\sigma_S = \int d^2k_T f(k_T^2)\left[\sigma_P(s,M^2,(\vec{p}_T-\vec{k}_T)^2)+\sigma_V(s,M^2)\delta(\vec{p}_T-\vec{k}_T)^2)\right]. \quad (28)$$

- Adding and subtracting the same term:

$$\begin{aligned}\sigma_S &= \int d^2k_T\sigma_P(s,M^2,(\vec{p}_T-\vec{k}_T)^2)[f(k_T^2)-f(p_T^2)]\\ &+f(p_T^2)\int d^2k_T\left[\sigma_P(s,M^2,(\vec{p}_T-\vec{k}_T)^2)+\sigma_V(s,M^2)\delta(\vec{p}_T-\vec{k}_T)^2)\right]\end{aligned}$$

and exchange of integration variable $\vec{q}_T=\vec{p}_T-\vec{k}_T$ gives:

$$\begin{aligned}\sigma_S &= \int d^2q_T\sigma_P(s,M^2,q_T^2)[f((\vec{p}_T-\vec{q}_T)^2)-f(p_T^2)]\\ &+f(p_T^2)\sigma_{\rm tot}(s,M,y)\end{aligned}$$

NLO transverse momentum

$$\sigma_P(s, M^2, q_T^2) = \sigma_{\text{annih}}(s, M^2, y, p_T) + \sigma_{\text{Compt}}(s, M^2, y, p_T) \tag{29}$$

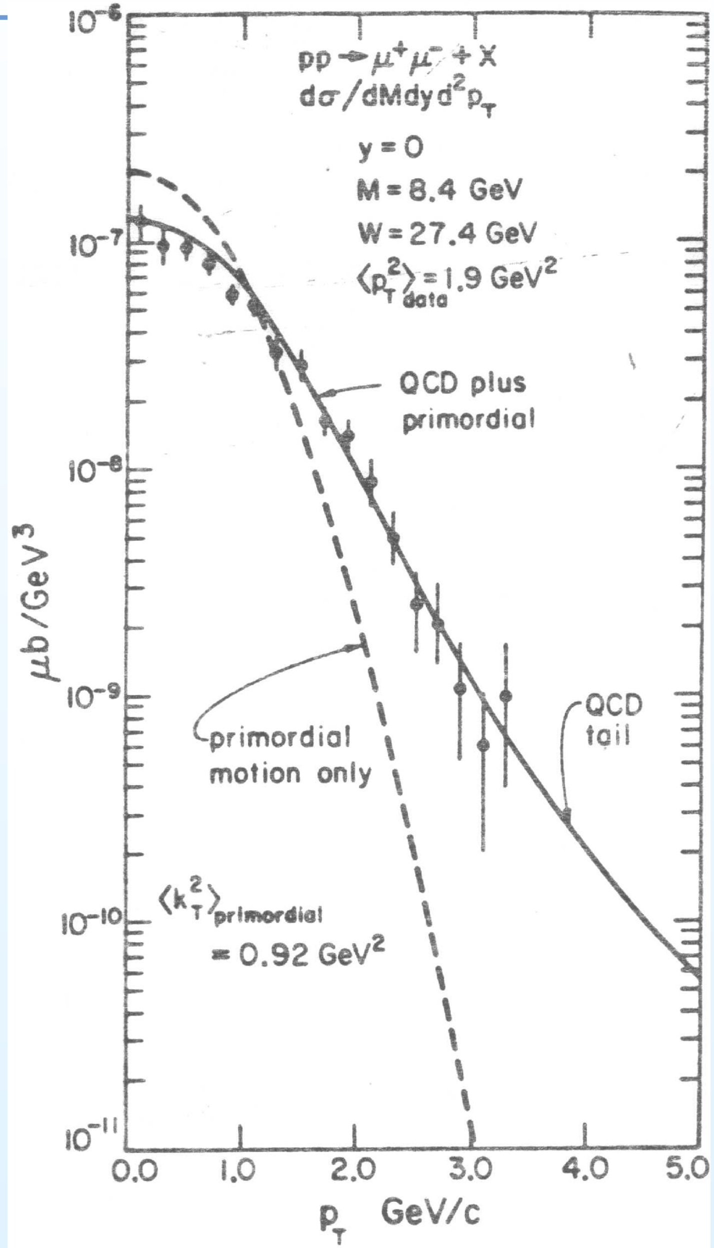
$$\sigma_{\text{annih}}(s, M^2, y, p_T) = \frac{d\sigma_{\text{annih}}}{dM^2, dy, dp_T^2} \tag{30}$$

$$= \int_{x_{\text{a}\,\text{min}}}^{1} dx_{\text{a}} \frac{x_{\text{b}} x_{\text{a}}}{x_{\text{a}} - x_1} \sum_q P_{q\bar{q}}(x_{\text{a}}, x_{\text{b}}, M^2) \frac{1}{\pi} \frac{8}{27} \frac{\alpha^2 \alpha_{\text{s}} e_q^2}{M^2 \hat{s}^2} \frac{2M^2 \hat{s} + \hat{u}^2 + \hat{t}^2}{\hat{t}\hat{u}}$$

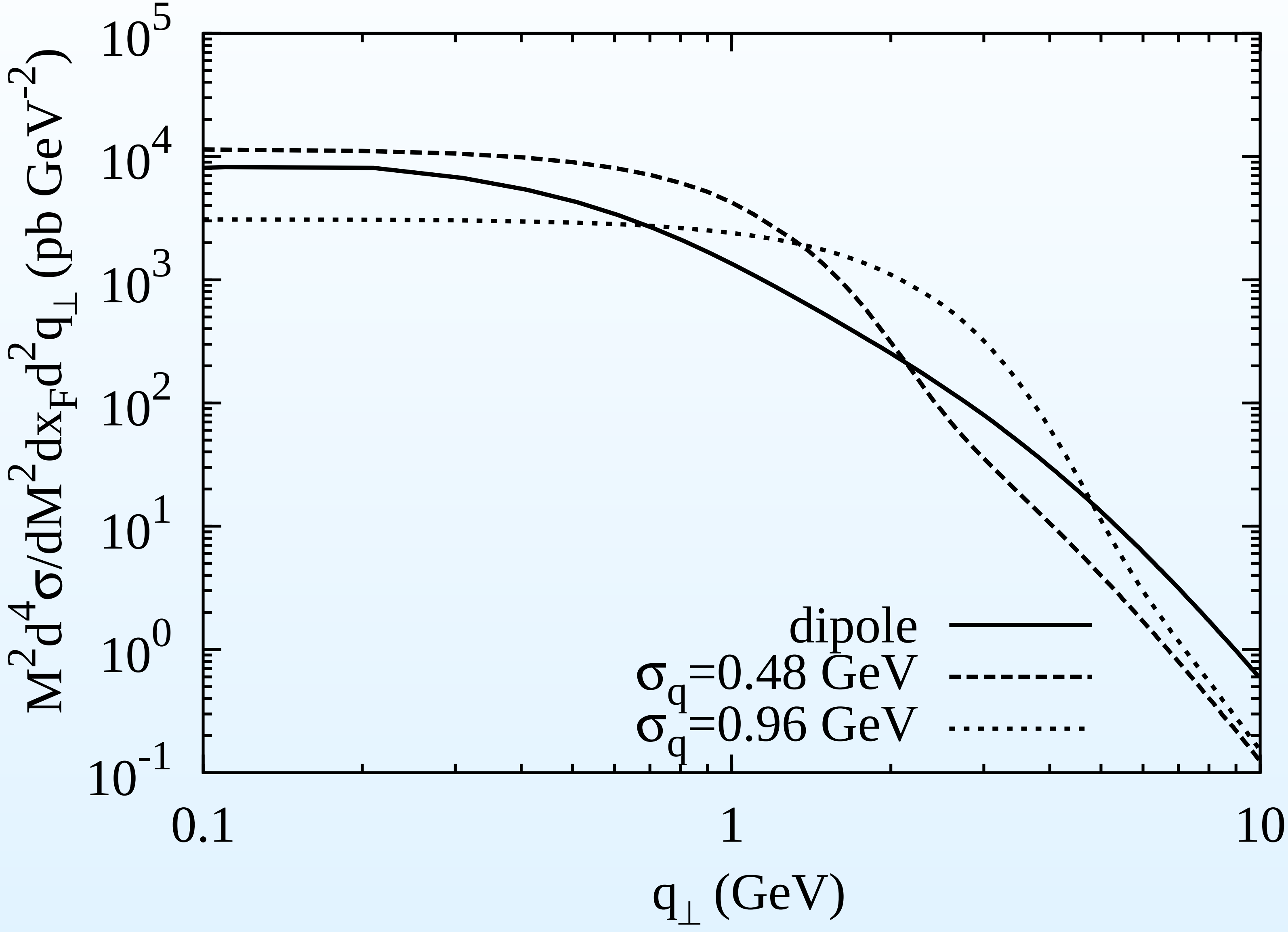
$$\sigma_{\text{Compt}}(s, M^2, y, p_T) = \int_{x_{\text{a}\,\text{min}}}^{1} dx_{\text{a}} \frac{x_{\text{b}} x_{\text{a}}}{x_{\text{a}} - x_1} \sum_q [P_{gq}(x_{\text{a}}, x_{\text{b}}, M^2) + P_{g\bar{q}}(x_{\text{a}}, x_{\text{b}}, M^2)]$$

$$\times \frac{1}{\pi} \frac{1}{9} \frac{\alpha^2 \alpha_{\text{s}} e_q^2}{M^2 \hat{s}^2} \frac{2M^2 \hat{u} + \hat{s}^2 + \hat{t}^2}{-\hat{s}\hat{t}} \tag{31}$$

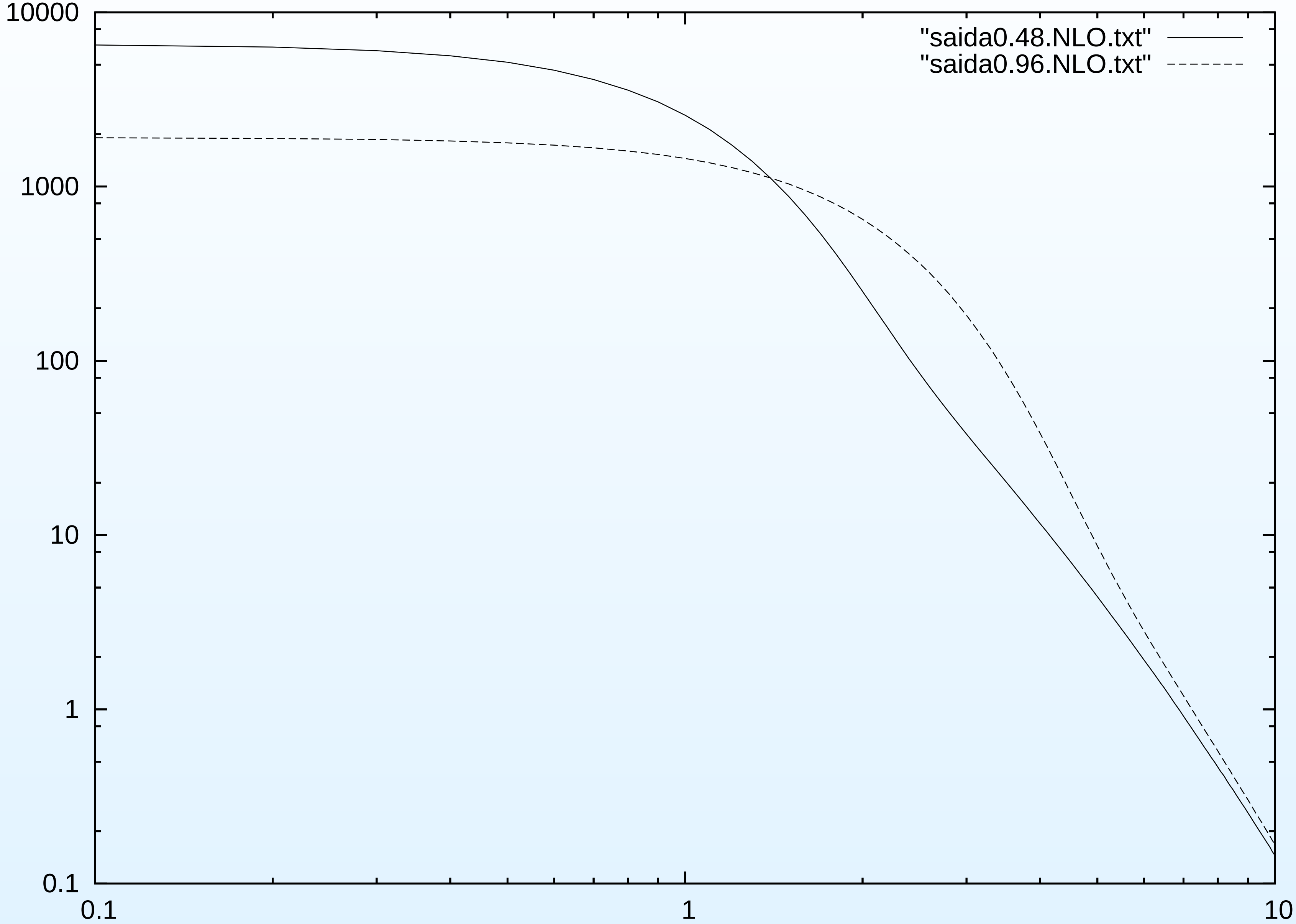
Experiment: NLO transverse momentum



Experiment: NLO transverse momentum



Experiment: NLO transverse momentum



Conclusions

- Work in progress.
- Match programs to results available.
- Calculate the nuclear modification ratio.
- Compare to dipole formalism.

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