

Threshold Resummation for single-inclusive Hadron production

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In collaboration with Werner Vogelsang

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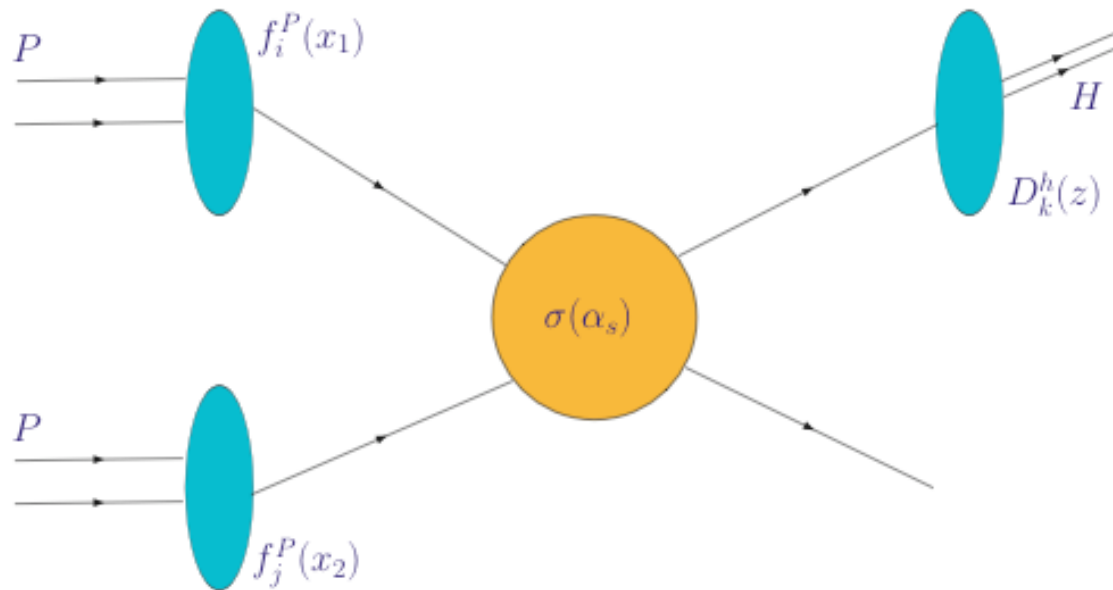
Phys. Rev.D72:014014,2005

and hep-ph/**0704.1677**

Outline:

- Introduction
- Large Logarithms
- Resummation
- Hadron production
- Prompt photon production
- Jet production
- Conclusions

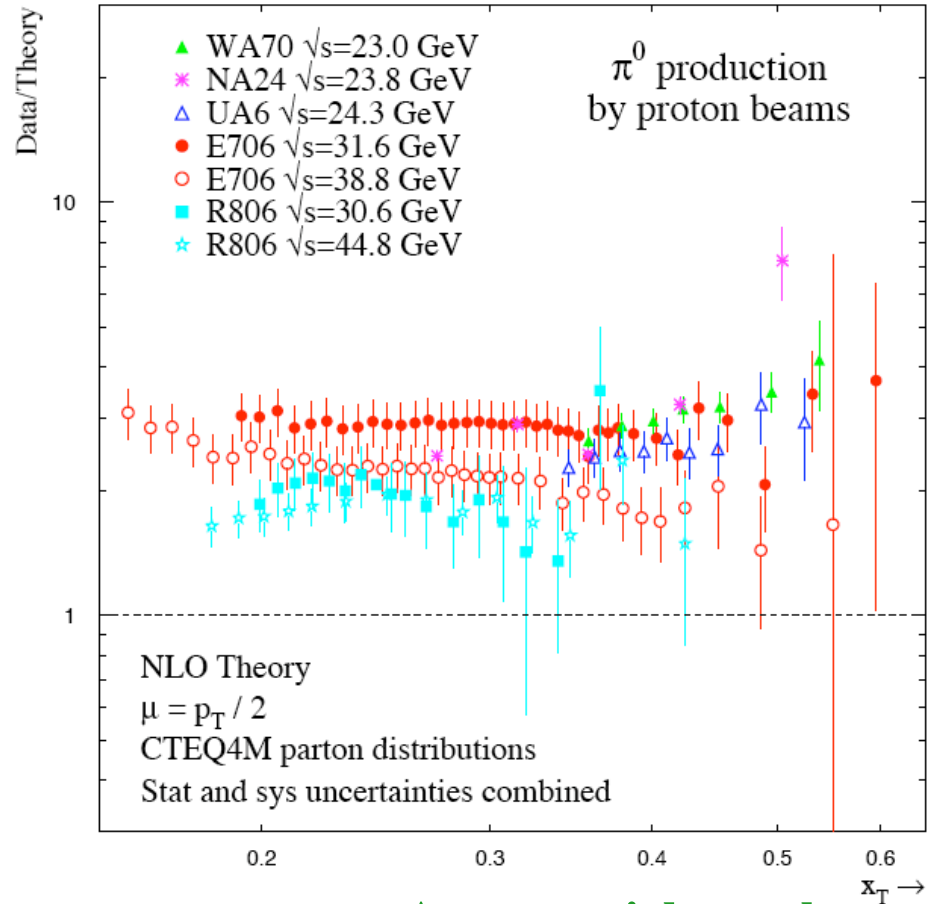
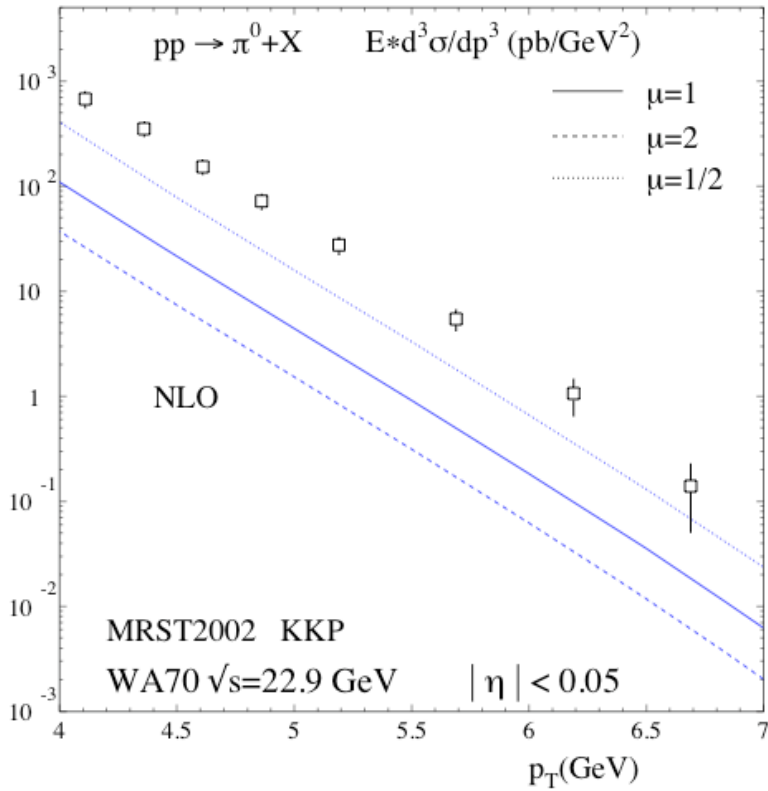
Relevance of hadron production in hadronic collisions



Involves almost all ingredients of QCD: coupling constant, hard cross-section, factorization, PDFs, FFs, non-pert. effects

$$d\sigma(pp \rightarrow h) = \sum_{i,j,k} \int_0^1 dx_1 f_i^P(x_1, \mu_{FI}^2) \int_0^1 dx_2 f_j^P(x_2, \mu_{FI}^2) \int_0^1 dz D_k^h(z, \mu_{FF}^2) d\hat{\sigma}(ij \rightarrow k)$$

Severe disagreement between data and pQCD (NLO) calculations for fixed target data



Very large
F&R scale
dependence

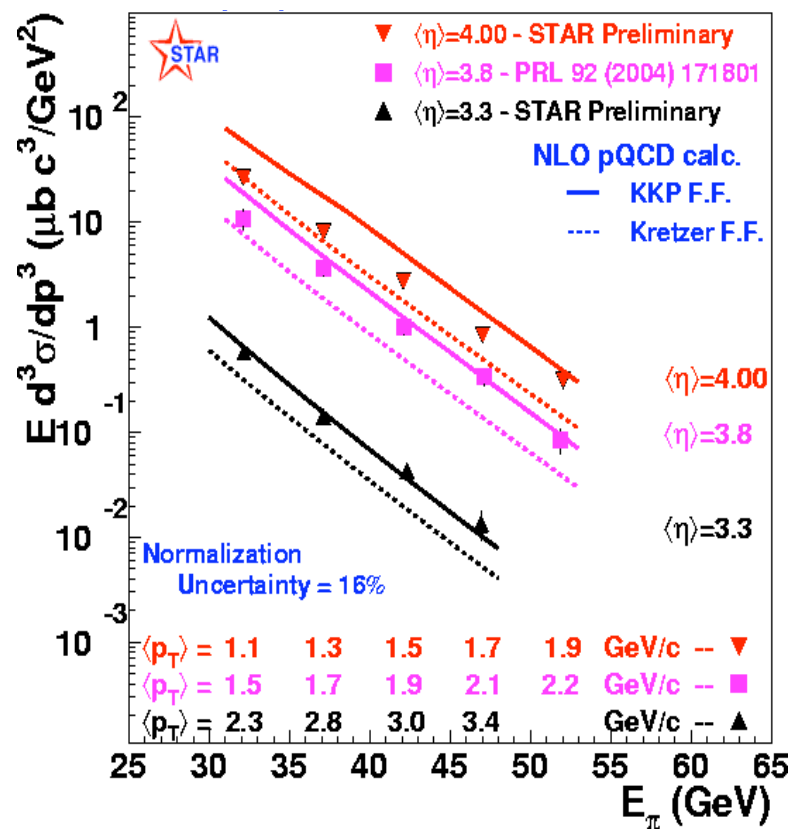
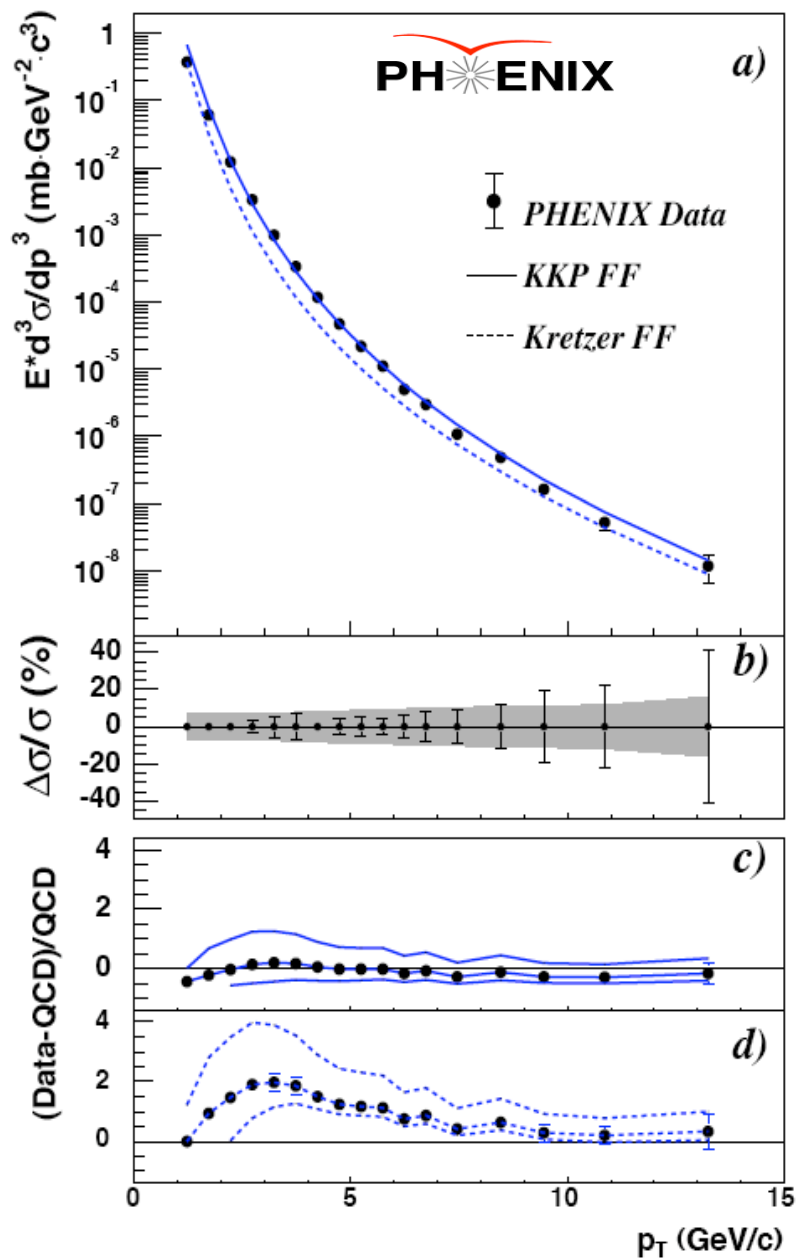
NLO already ~ 2.5 LO
Fixed order expansion valid?

Apanasevich et al.

$$x_T = \frac{4p_T^2}{S}$$

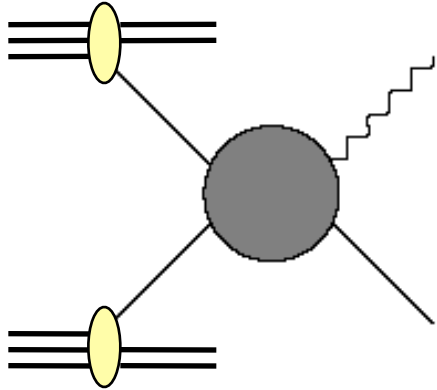
Situation is much better for collider experiments

$pp \rightarrow \pi^0 X$ at RHIC

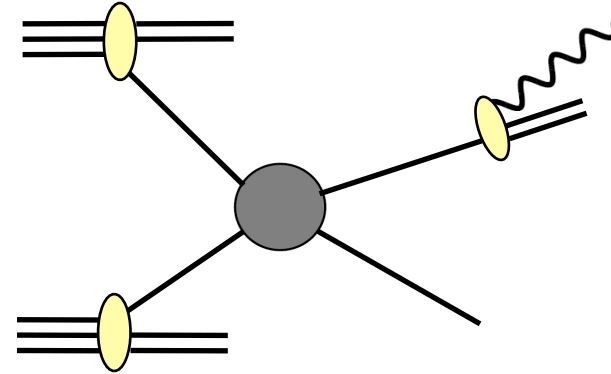


Hadron production relevant for prompt photons: $pp \rightarrow \gamma X$

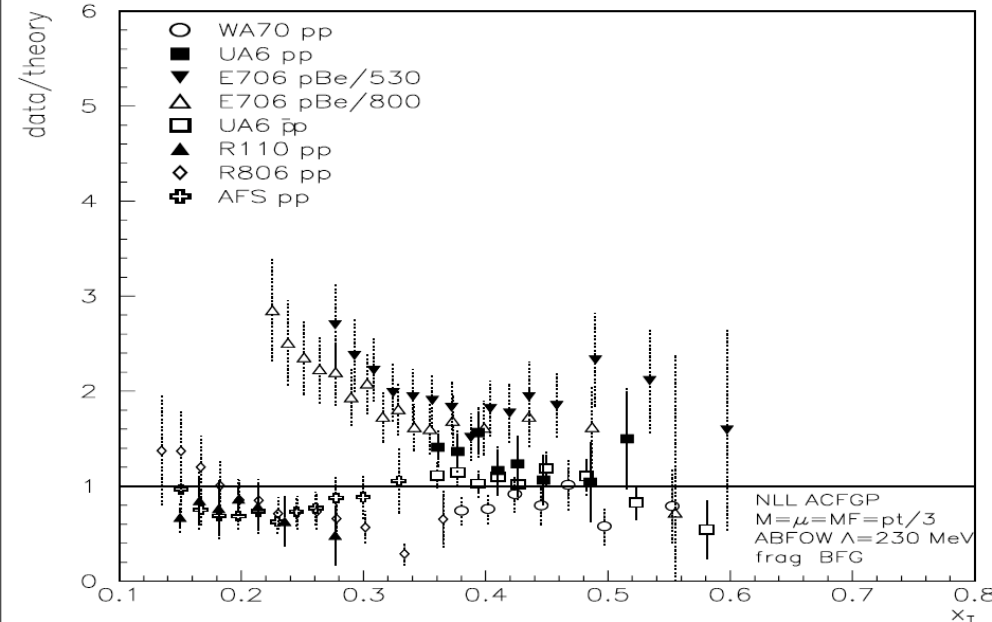
“direct” contribution



“fragmentation” contribution



like π^0 production, but different fragmentation function



• Problems with some fixed target data sets.

• Also some “extreme” choices of scales needed $\mu_{F,R} = P_T/3$

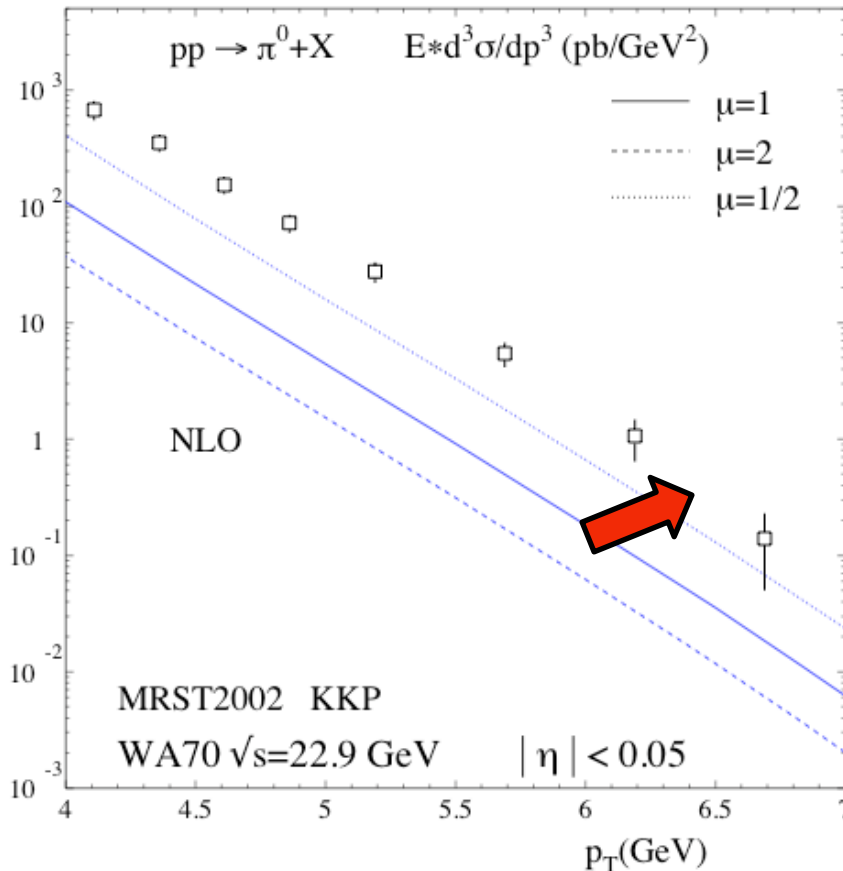
Aurenche et al.

One phenomenological approach:

- Ad-hoc intrinsic k_T needed to cover the gap

$$f(x, Q^2) \rightarrow \int dk_T f(x, Q^2, k_T)$$

$$f(x, Q^2, k_T) \sim f(x, Q^2) e^{-bk_T^2}$$



$k_T \sim \Lambda_{QCD}$ reasonable but
 $k_T > 1\text{ GeV}$ Needed!

Most of it could have
more “perturbative” origin:

Go beyond fixed order !

This talk: multiple soft gluon emission affects these observables
Resummation needed

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Resummation needed

- **Hadron production: corrections are large enough to bring agreement with fixed target data**
- **Photon production: resolved contribution increased.**
Reduction of Theoretical deficit at small transverse momentum

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Some phenomena usually explained by non-perturbative arguments can actually be (at least partially) understood in terms of (all orders) pQCD

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Some phenomena usually explained by non-perturbative arguments can actually be (at least partially) understood in terms of (all orders) pQCD

Why?



Large Logarithms in QCD

Any process involving two or more scales: Perturbative coefficients contain logs of scale ratios

Not a problem unless the scales are very different

Fixed order calculation $\sigma = C_0 + \alpha_s C_1 + \alpha_s^2 C_2 + \dots$

Logs appear in the coefficients as $C_n \sim \log^{2n-1} \frac{E_1}{E_2}$

when $E_1 \sim E_2 \gg \Lambda_{QCD} \rightarrow \alpha_s \ll 1$ and $C_n \sim \mathcal{O}(1)$

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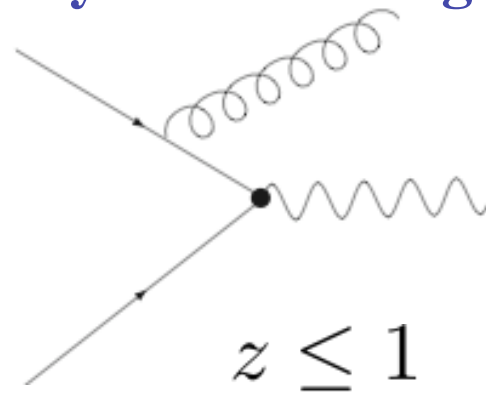
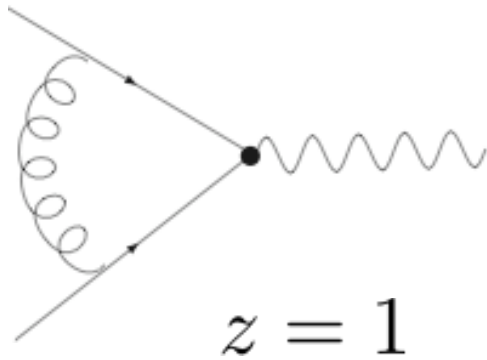
pQCD series converges well. Typically NLO calculations OK

But if $E_1 \gg E_2$ convergence is spoiled: even if coupling constant is small, power of Logs grows twice as fast! $\log(1 - 4p_T^2/\hat{s})$

Origin of the Logs

Unbalanced cancellation of real and virtual contributions at the boundaries of the phase space (soft gluon radiation)

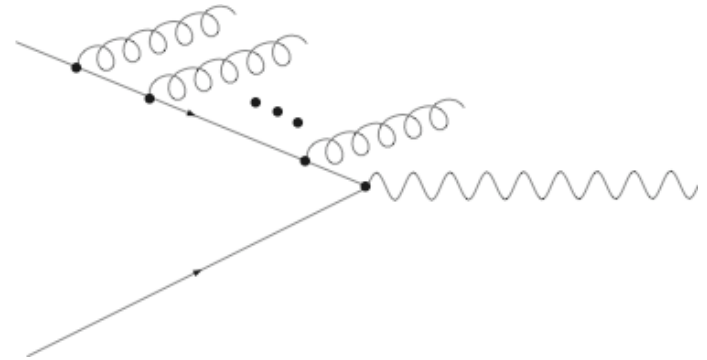
Example: Drell-Yan $z = \frac{Q^2}{\hat{s}}$
(1-z) Momentum fraction carried by the emitted gluon



Both contributions **infrared** divergent: cancellation at $z=1$
Full after $\int_0^1 dz$ (inclusive), otherwise some logs remain

In the elastic limit (threshold) $\left(\frac{\log^m(1-z)}{1-z} \right)_+$ can be very large

Each gluon contributes with a double log (soft-collinear) at most
 Typically one encounters corrections like $\alpha_s^n \log^{2n}$



•Where relevant?

$$\sigma(\tau) = \int_{\tau}^1 \frac{dz}{z} \sum_i \hat{\sigma}(z) q_i \left(\frac{\tau}{z}, Q^2 \right) \bar{q}_i \left(\frac{\tau}{z}, Q^2 \right)$$

For sure if $\tau = \frac{Q^2}{S_H}$ close to 1 (hadronic threshold)

- But even for $\tau \ll 1$ parton distributions prefer $z \rightarrow 1$
- Steeply falling densities leave partons with just enough energy for the process \rightarrow **partonic threshold (often) reached**
Even more when fragmentation functions involved!

Therefore large logs can be dominant even at collider energies

In order to be able to perform a quantitative analysis of the data

Logs have to be **resummed** to all orders in the coupling constant

- Restoration of perturbative series
- Precise predictions
- Structure of pQCD series at large orders

Technicalities

$$\sigma(\tau) = \sum_{i,j} \int f_i(x_i) \otimes f_j(x_j) \otimes \hat{\sigma}_{ij}(z)$$

$$\sigma(N) = \int_0^1 \tau^{N-1} \sigma(\tau)$$

Trade convolutions into products : Mellin

$$\sigma(N) = \sum_{i,j} f_i(N+1) \times f_j(N+1) \times \hat{\sigma}_{ij}(N)$$

$z \rightarrow 1$ ($\tau \rightarrow 1$) corresponds to $N \rightarrow \infty$

$$\left(\frac{\log^m(1-z)}{1-z} \right)_+ \downarrow \log^{m+1} N$$

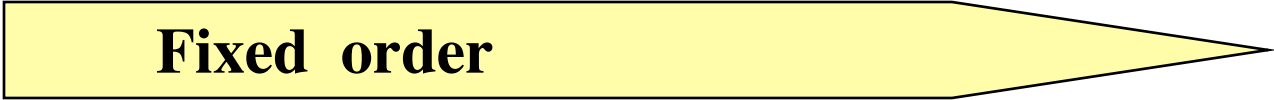
- **Resummation: reorganize perturbative structure**

$$\mathbf{L} \equiv \ln(\mathbf{N})$$

$$\alpha_s L \simeq 1$$

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Fixed order

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Fixed order

LO

1

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Fixed order

LO

1

NLO

$\alpha_s \mathbf{L}^2$

$\alpha_s \mathbf{L}$

α_s

+ ...

$$\alpha_s L \simeq 1$$

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Fixed order

LO	1				
NLO	$\alpha_s \mathbf{L}^2$	$\alpha_s \mathbf{L}$	α_s		+ ...
NNLO	$\alpha_s^2 \mathbf{L}^4$	$\alpha_s^2 \mathbf{L}^3$	$\alpha_s^2 \mathbf{L}^2$	$\alpha_s^2 \mathbf{L}$	+ ...

$$\alpha_s L \simeq 1$$

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	$\alpha_s^4 L^8$	$\alpha_s^4 L^7$	$\alpha_s^4 L^6$	$\alpha_s^4 L^5$ $+ \dots$
	\vdots	\vdots	\vdots	\vdots

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	\vdots	\vdots	\vdots	\vdots	
N^kLO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3}$	$+ \dots$

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N^kLO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3}$ + ...
	LL	NLL	NNLL	$\alpha_s L \simeq 1$

Resummation achieved by exponentiation of logarithmic terms: Sudakov form factor not trivial in QCD: color correlations

$$\sigma_{ij \rightarrow kl}^{\text{res}} \sim \sum_c e^{S_c} \sigma_{ij \rightarrow kl}^c$$

Catani, Trentadue

Sterman

Bonciani, Catani, Mangano, Nason

After rapidity integration

$$\int d\eta \sigma(\eta) \sim \sigma(\eta=0) + \mathcal{O}(1/N)$$

$$\sigma_{ij \rightarrow kl}^c \rightarrow G^c \sigma_{ij \rightarrow kl}^{\text{Born}}$$

- **How does resummed formula look ?**

usually,

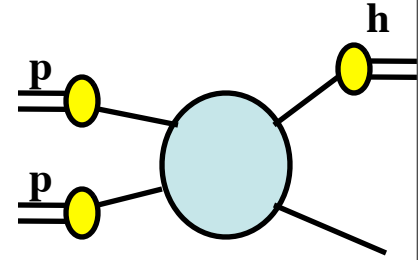
$$\exp \left[\underbrace{\mathbf{L} \mathbf{h}_1(\alpha_s \mathbf{L})}_{\alpha_s^k \mathbf{L}^{k+1}} + \underbrace{\mathbf{h}_2(\alpha_s \mathbf{L})}_{\alpha_s^k \mathbf{L}^k} + \frac{1}{\mathbf{L}} \mathbf{h}_3(\alpha_s \mathbf{L}) + \dots \right]$$

LL **NLL**

As a result of exponentiation: “simple” set of rules for each parton participating (in many cases universal): radiative factors

Initial state (pdf): soft gluon radiation collinear to parton

$$\Delta_N^a = \exp \left\{ \int_0^1 \frac{z^{N-1} - 1}{1-z} \int_{\mu_{FI}^2}^{(1-z)^2 Q^2} \frac{dq^2}{q^2} A_a(\alpha_s(q^2)) \right\}$$



Final state identified parton: same as initial state pdf ↔ ff

Final state not-identified parton (jet): collinear (soft or hard)

$$J_N^a = \exp \left\{ \int_0^1 \frac{z^{N-1} - 1}{1-z} \left[\int_{(1-z)^2 Q^2}^{(1-z)Q^2} \frac{dq^2}{q^2} A_a(\alpha_s(q^2)) + \frac{1}{2} B_a(\alpha_s((1-z)Q^2)) \right] \right\}$$

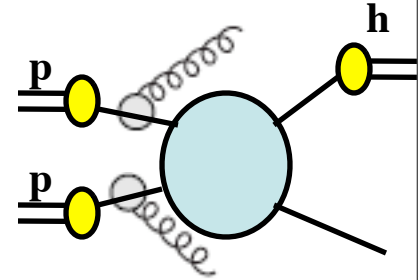
Large angle soft gluons: process dependent (color interference)

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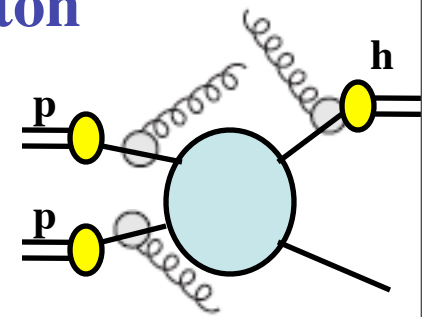
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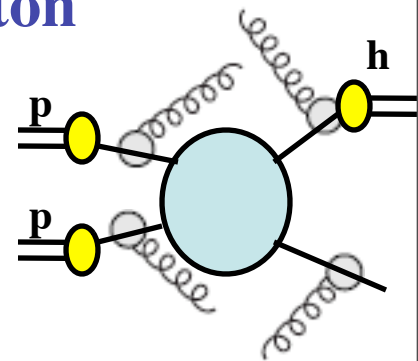
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Coefficients have a perturbative expansion (free of logs)

$$\mathbf{A}(\alpha_s) = \frac{\alpha_s}{\pi} \mathbf{A}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \mathbf{A}^{(2)} + \dots$$

$$A_q^{(1)} = C_F \quad A_g^{(1)} = C_A \quad B_q^{(1)} = -\frac{3}{2}C_F \quad B_g^{(1)} = -\beta_0$$

At leading log, exponents behave like (color interf. NLL)

$$\Delta_N^a = \exp \left[\underbrace{\frac{\alpha_s}{\pi} A_a^{(1)} \ln^2 N}_{\text{Sudakov enhancement}} \right] \quad J_N^a = \exp \left[\underbrace{-\frac{\alpha_s}{2\pi} A_a^{(1)} \ln^2 N}_{\text{Sudakov suppression}} \right]$$

Sudakov enhancement

(PDF or FF already much suppression in factorization)

Sudakov suppression

(final state not ID)

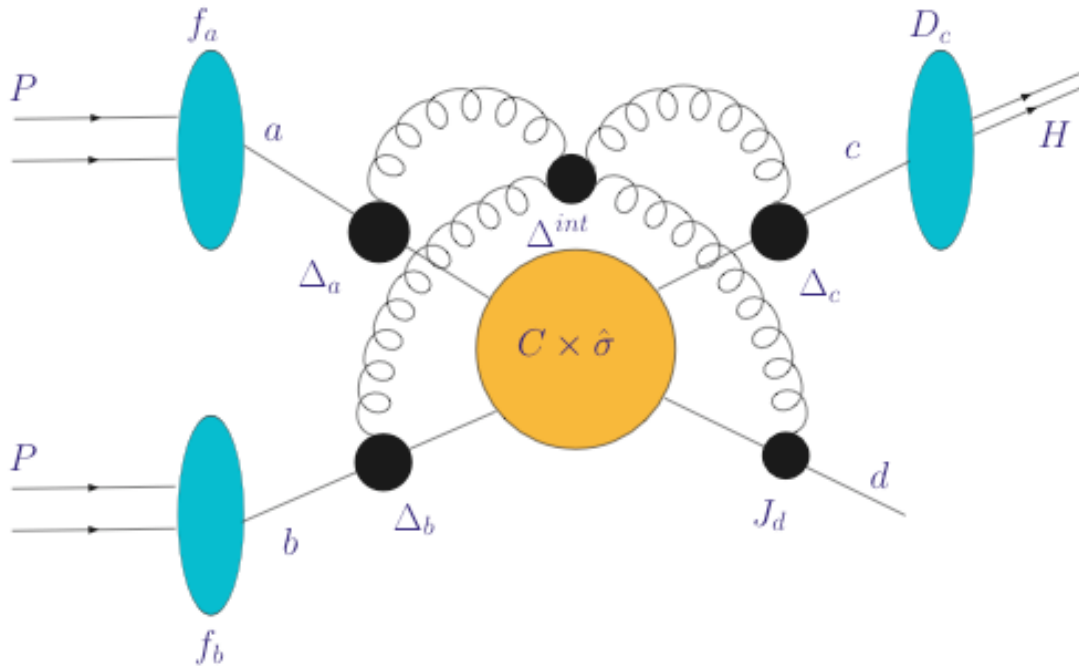
Important: the effect is amplified if hard gluons present

$$\frac{C_A}{C_F} = 2.25$$

Pion production in pp collisions

$$\log(1 - x_T)$$

$$x_T = \frac{2p_T}{\sqrt{S}}$$



After integration over rapidity

$$\sigma(N) = \sum_{a,b,c} f_{a/H_1}(N+1, \mu_{FI}^2) f_{b/H_2}(N+1, \mu_{FI}^2) D_{h/c}(2N+3, \mu_{FF}^2) \hat{\sigma}_{ab \rightarrow cd}(N)$$

$$\hat{\sigma}_{ab \rightarrow cd}^{(res)}(N) = C_{ab \rightarrow cd} \underbrace{\Delta_N^a \Delta_N^b \Delta_N^c J_N^d}_{\text{Hard partons}} \left[\sum_I \underbrace{G_{ab \rightarrow cd}^I \Delta_{IN}^{(int)ab \rightarrow cd}}_{\text{Color interferences}} \right] \hat{\sigma}_{ab \rightarrow cd}^{(Born)}(N)$$

Hard partons

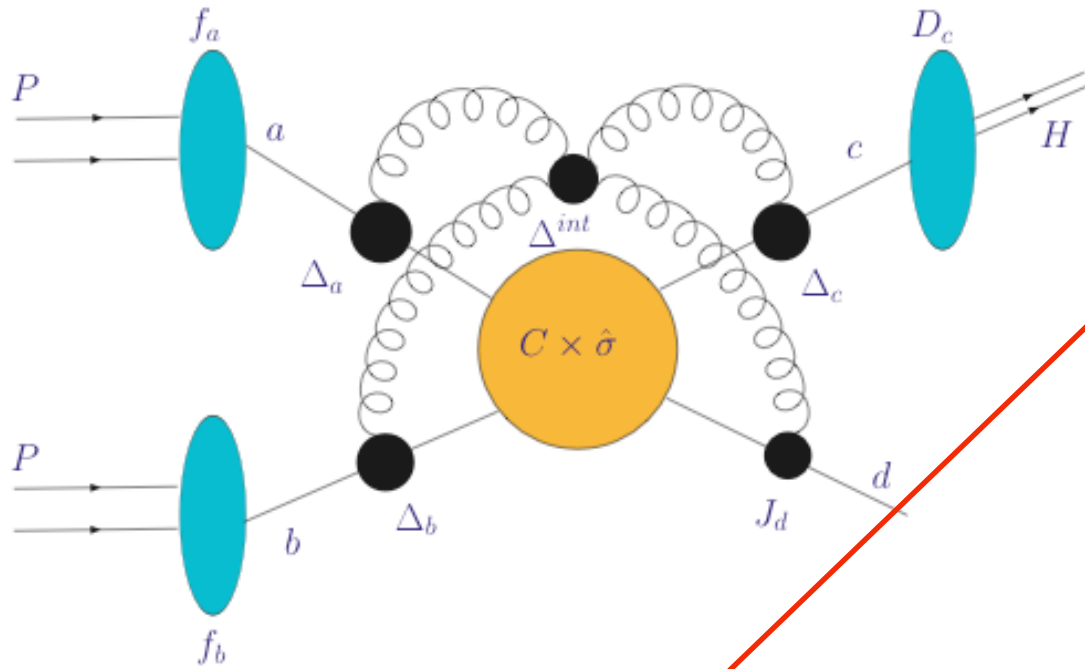
Color interferences

Kidonakis, Oderda,
Serman

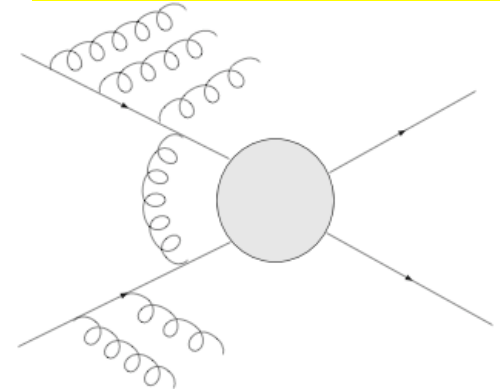
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C contains virtual corrections



After integration over rapidity

$$\sigma(N) = \sum_{a,b,c} f_{a/H_1}(N+1, \mu_{FI}^2) f_{b/H_2}(N+1, \mu_{FI}^2) D_{h/c}(2N+3, \mu_{FF}^2) \hat{\sigma}_{ab \rightarrow cd}(N)$$

$$\hat{\sigma}_{ab \rightarrow cd}^{(res)}(N) = C_{ab \rightarrow cd} \underbrace{\Delta_N^a \Delta_N^b \Delta_N^c J_N^d}_{\text{Hard partons}} \left[\sum_I \underbrace{G_{ab \rightarrow cd}^I \Delta_{IN}^{(int)ab \rightarrow cd}}_{\text{Color interferences}} \right] \hat{\sigma}_{ab \rightarrow cd}^{(Born)}(N)$$

Hard partons

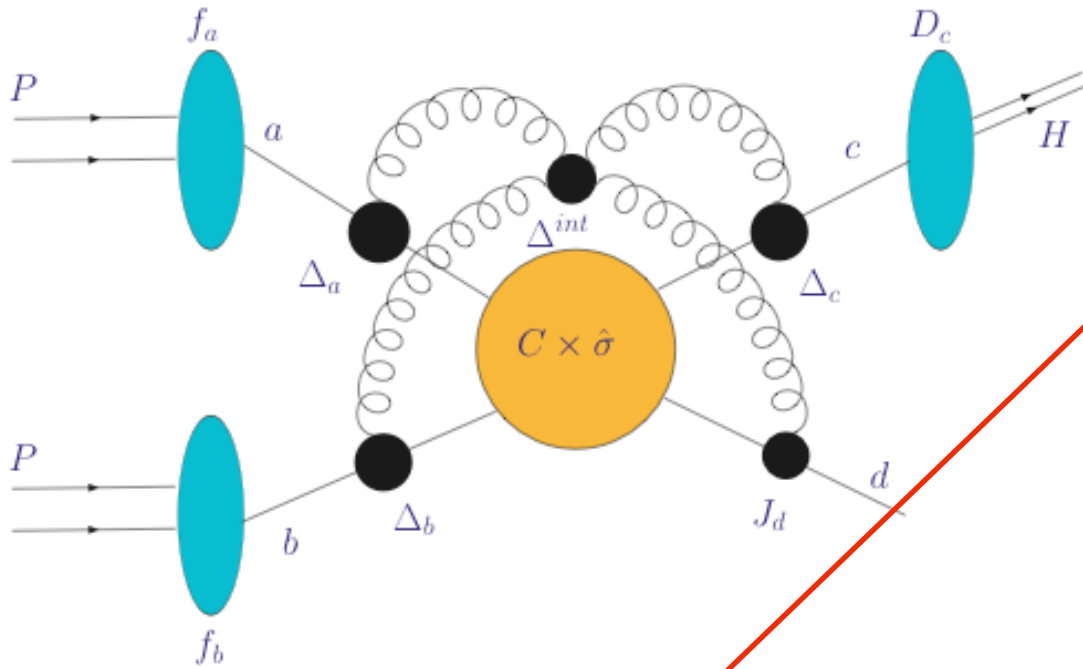
Color interferences

Kidonakis, Oderda, Serman

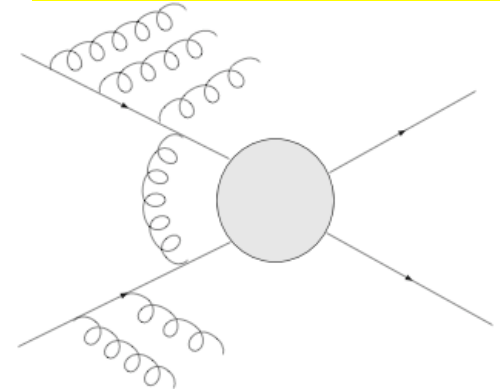
Pion production in pp collisions

$$\log(1 - x_T)$$

$$x_T = \frac{2p_T}{\sqrt{S}}$$



C contains virtual corrections



After integration over rapidity

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Enhancement

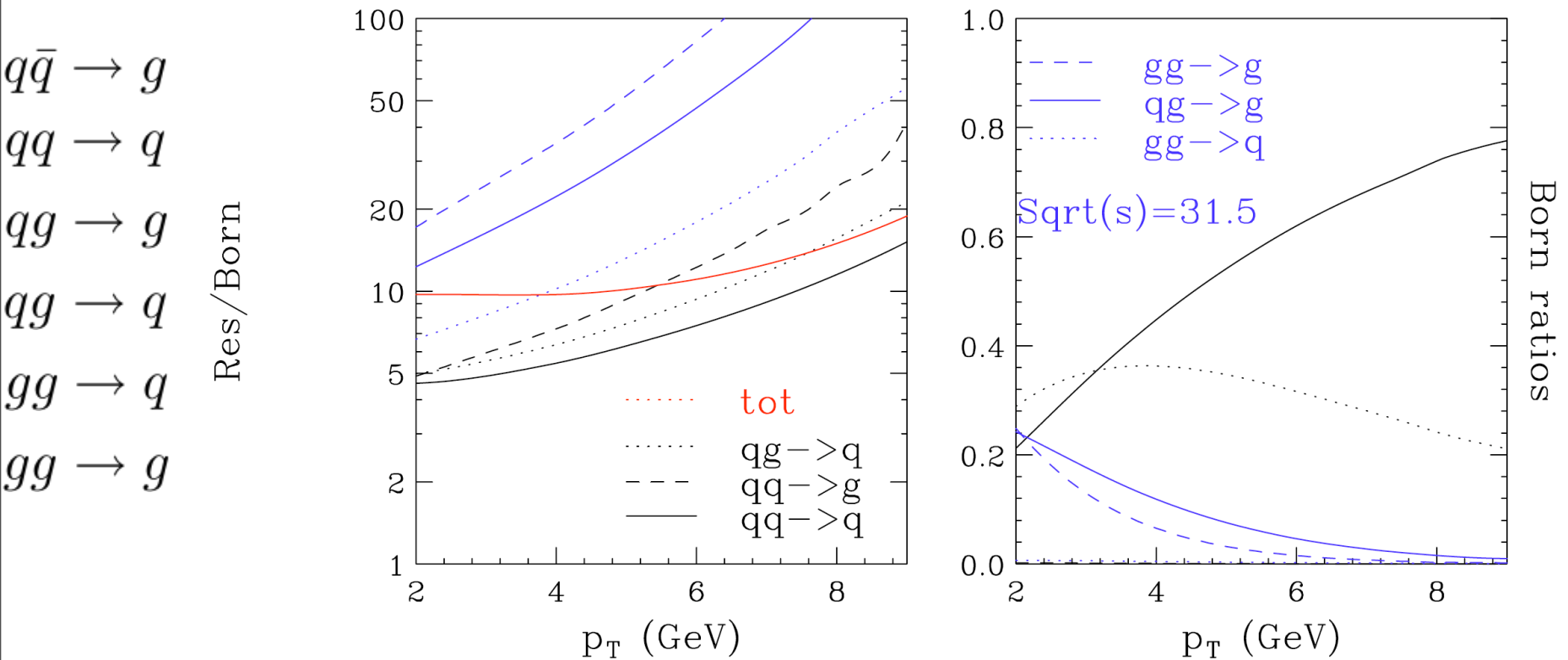
Hard partons

Color interferences

Kidonakis, Oderda, Serman

Several subprocesses, largest enhancement from

$$gg \rightarrow gg \quad \exp \left[\underbrace{\left(C_A + C_A + C_A - \frac{1}{2} C_A \right)}_{\frac{15}{2}} \frac{\alpha_s}{\pi} \ln^2(\mathbf{N}) \right]$$



NLL included D.de F. & W. Vogelsang

Amplified by fragmentation $p_T^{\text{parton}} = \frac{p_T^h}{z}$

- **always want to keep benefits of full fixed-order calculation:**

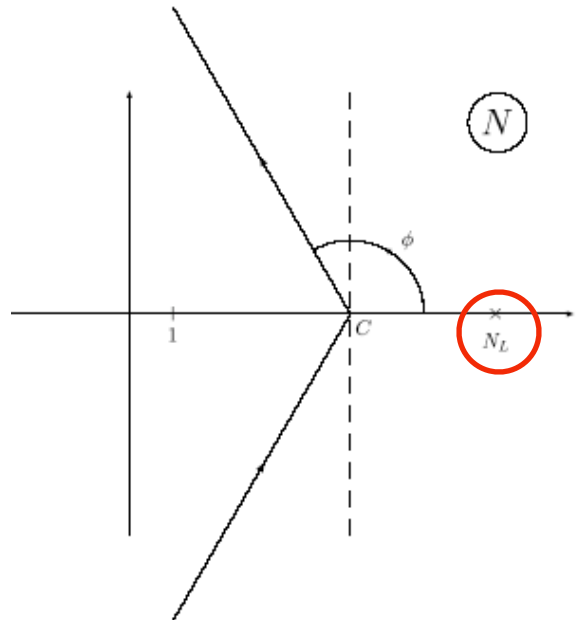
→ **“matching”:**

$$\sigma^{\text{matched}} = \sigma^{\text{res.}} \Big|_{\mathcal{O}(\alpha_s^n)} - \sigma^{\text{res.}} \Big|_{\mathcal{O}(\alpha_s^n)} + \sigma^{\text{f.o.}} \Big|_{\mathcal{O}(\alpha_s^n)}$$

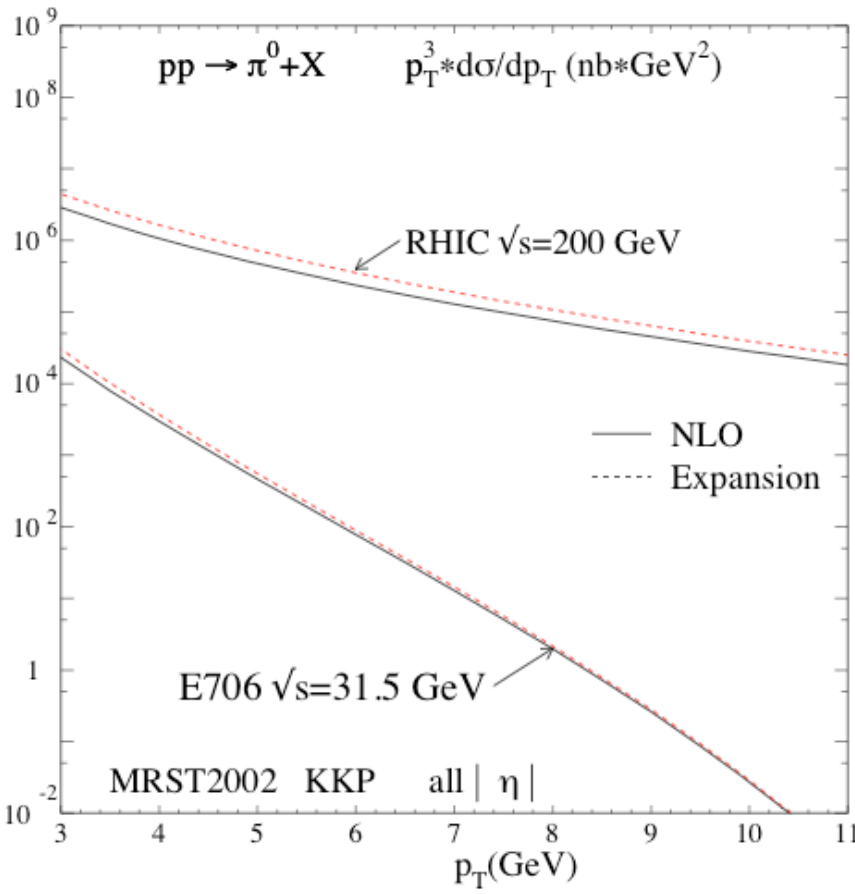
in phenomenological applications

- **Avoid Landau Pole: Mellin contour as**

Minimal prescription
(Catani, Mangano, Nason, Trentadue)

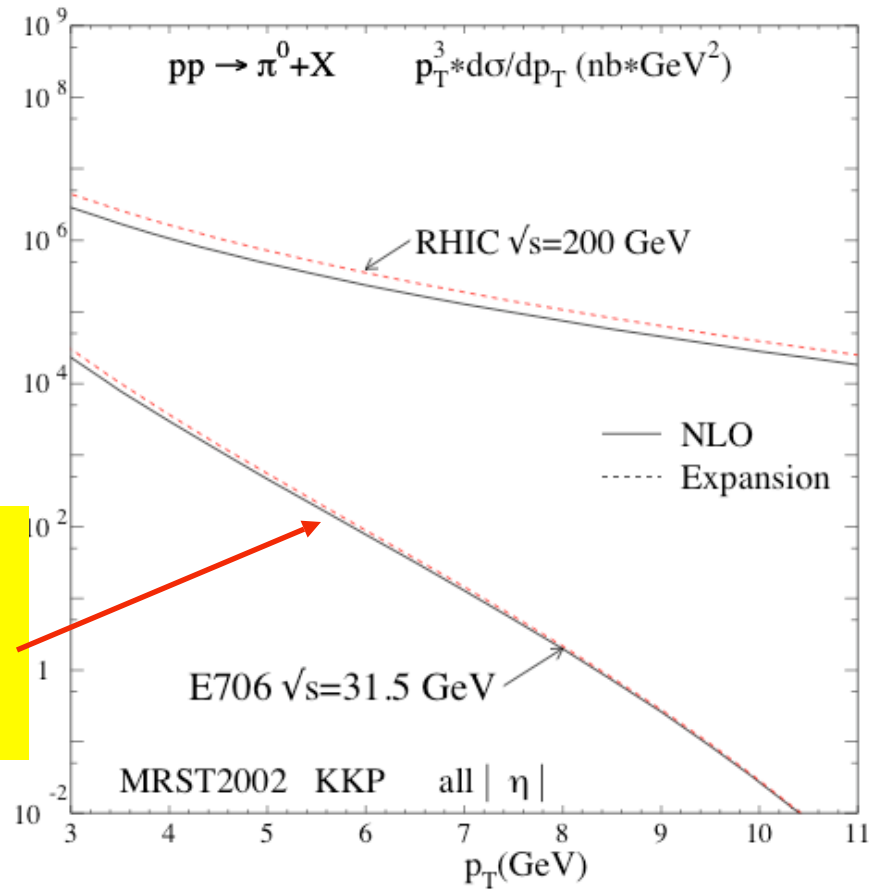


•Validity of resummation (kinematical range) :
Compare full NLO to soft approximation (expansion of resummed)



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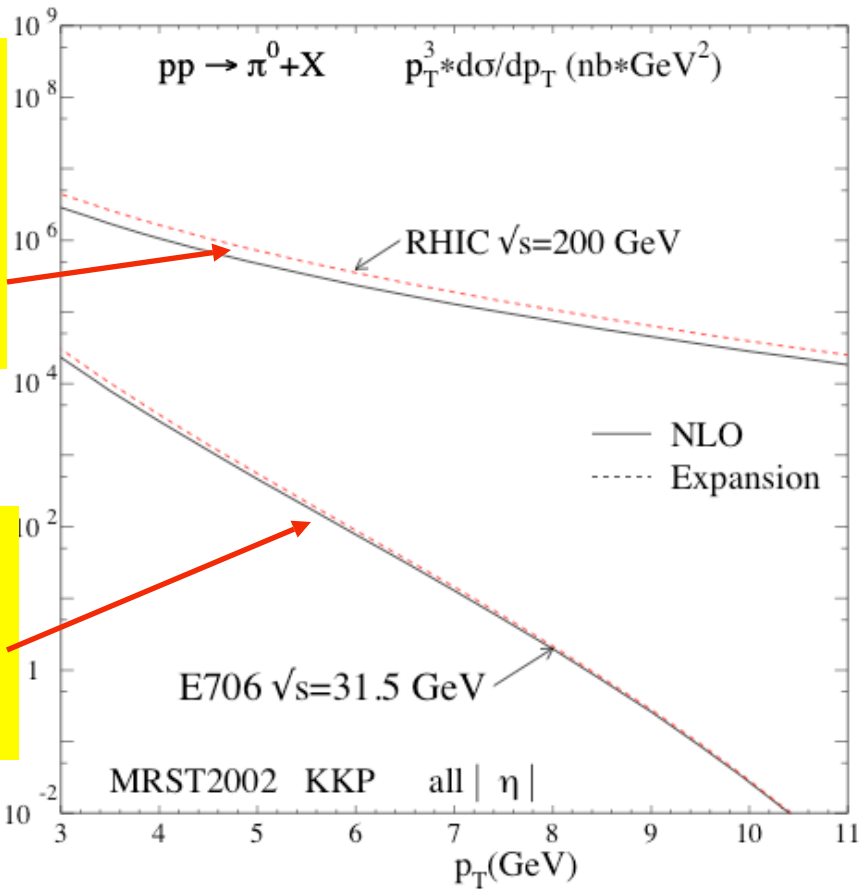
At “lower” energies very good agreement: dominance of soft contributions ✓



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 Compare full NLO to soft approximation (expansion of resummed)

At collider energies soft app. overestimates NLO: hard corrections and **subleading** terms more important ↓

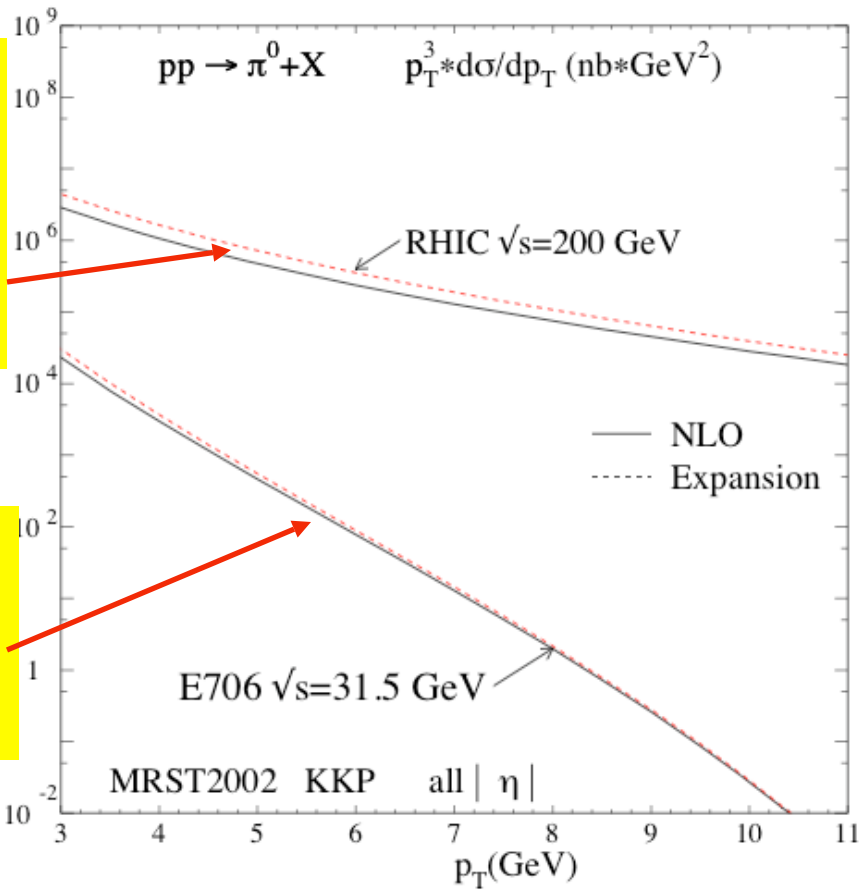
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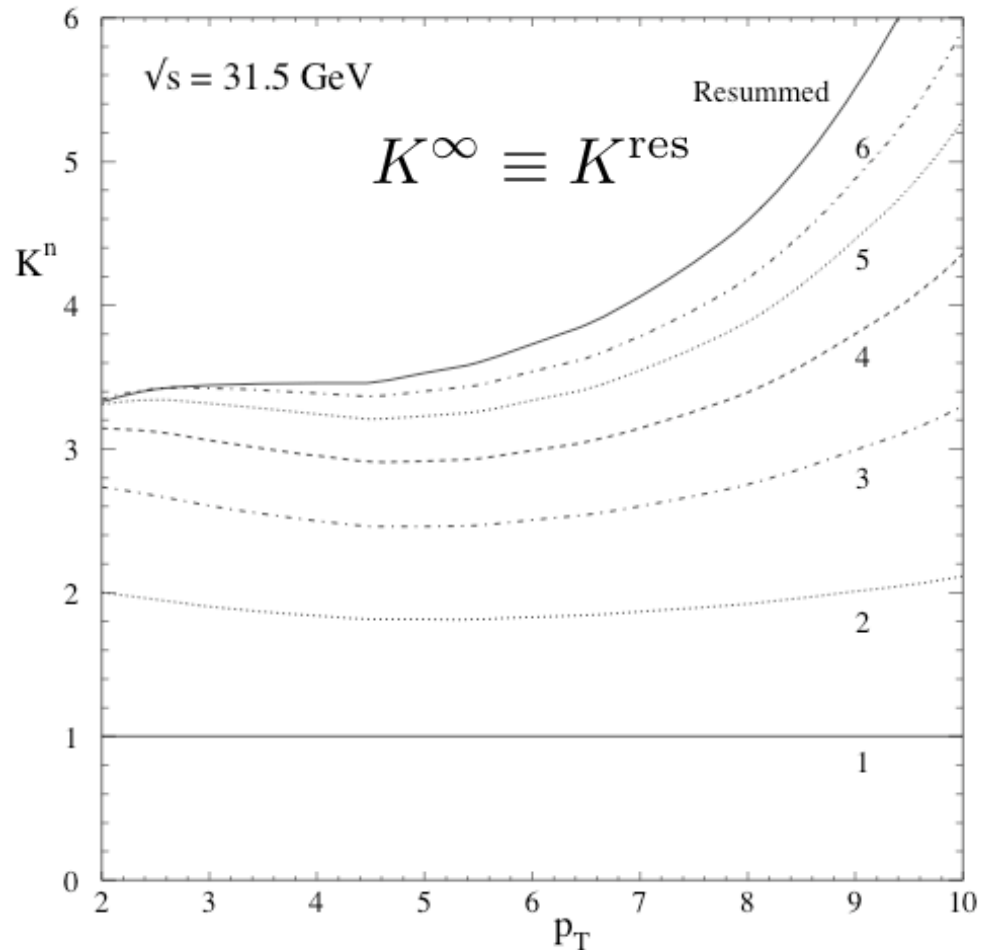
Failure at RHIC not a surprise: x_T much smaller at colliders

•Far away from threshold

•K-factors and convergence of resummed expression

$$K^n = \frac{d\sigma^{(\text{match})}/dp_T \big|_{\mathcal{O}(\alpha_s^{2+n})}}{d\sigma^{(NLO)}/dp_T}$$

Very slow convergence even for fixed target and low transverse momentum: all orders needed!



•K-factors and convergence of resummed expression

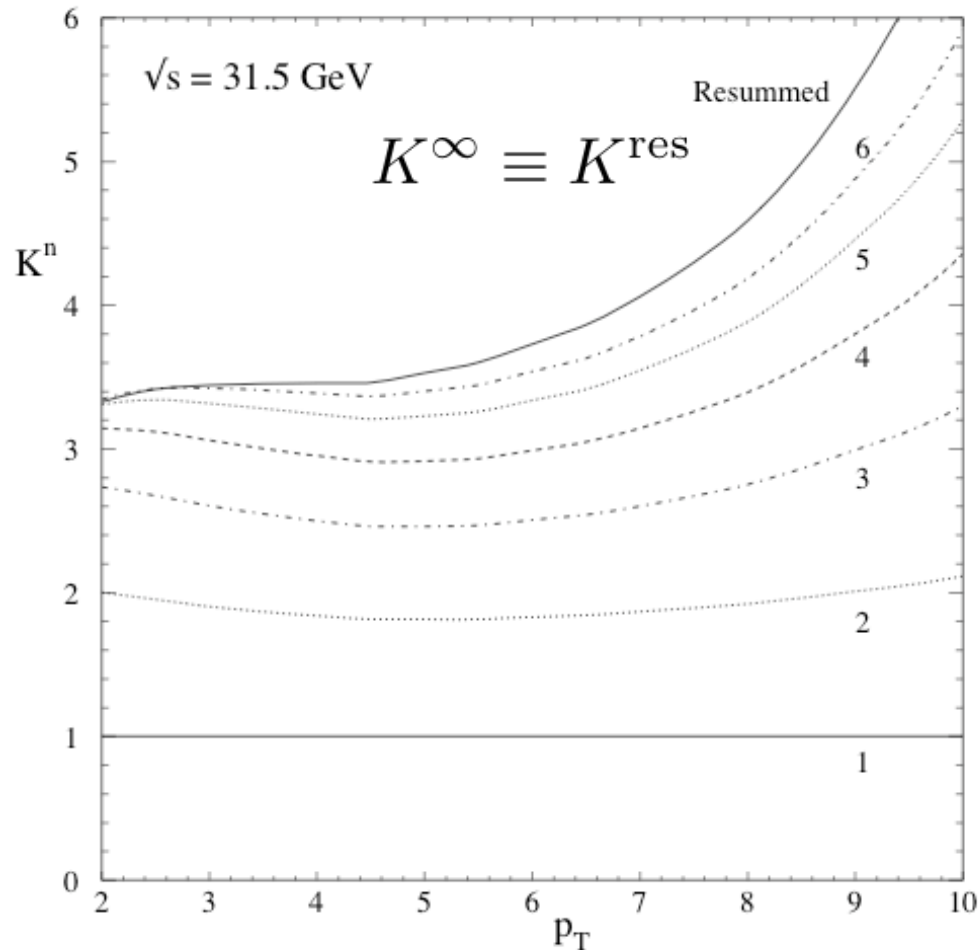
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**Comparison to data
only approximation: rapidity**

$$\frac{p_T^3 d\sigma^{(\text{match})}}{dp_T} (\eta \text{ in exp. range}) = K^{(\text{res})} \frac{p_T^3 d\sigma^{(NLO)}}{dp_T} (\eta \text{ in exp. range})$$

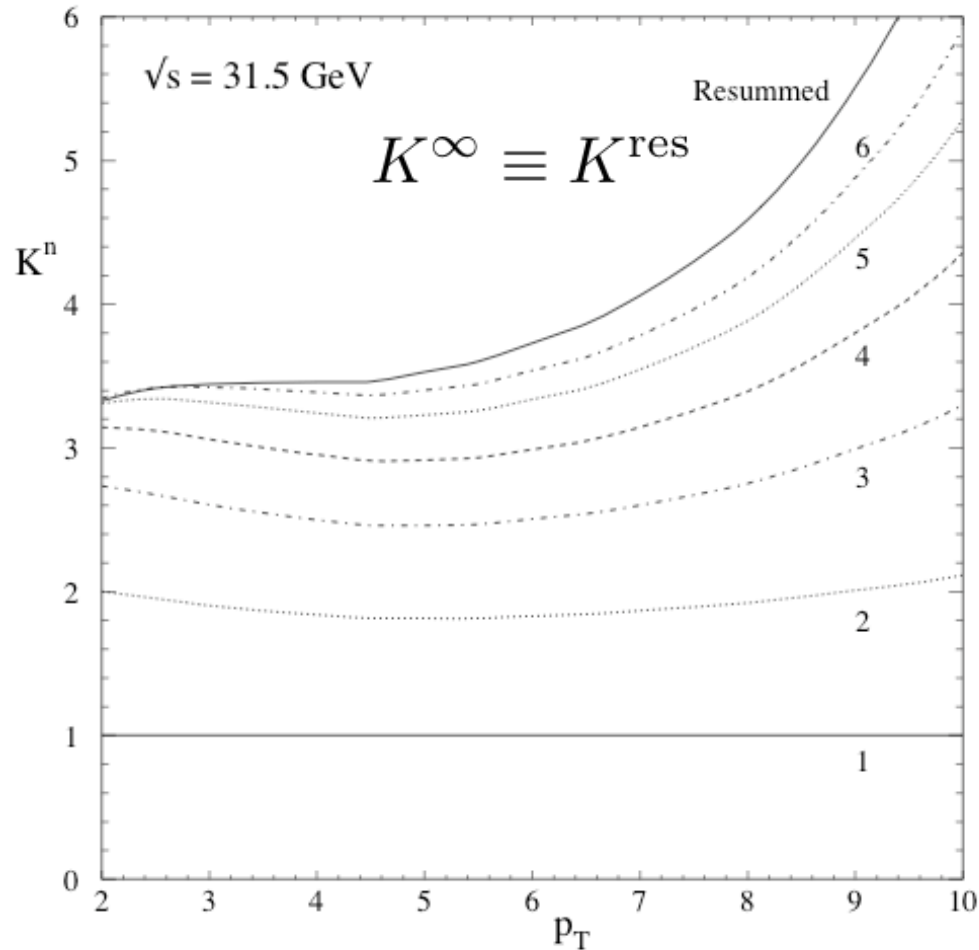
η integrated 



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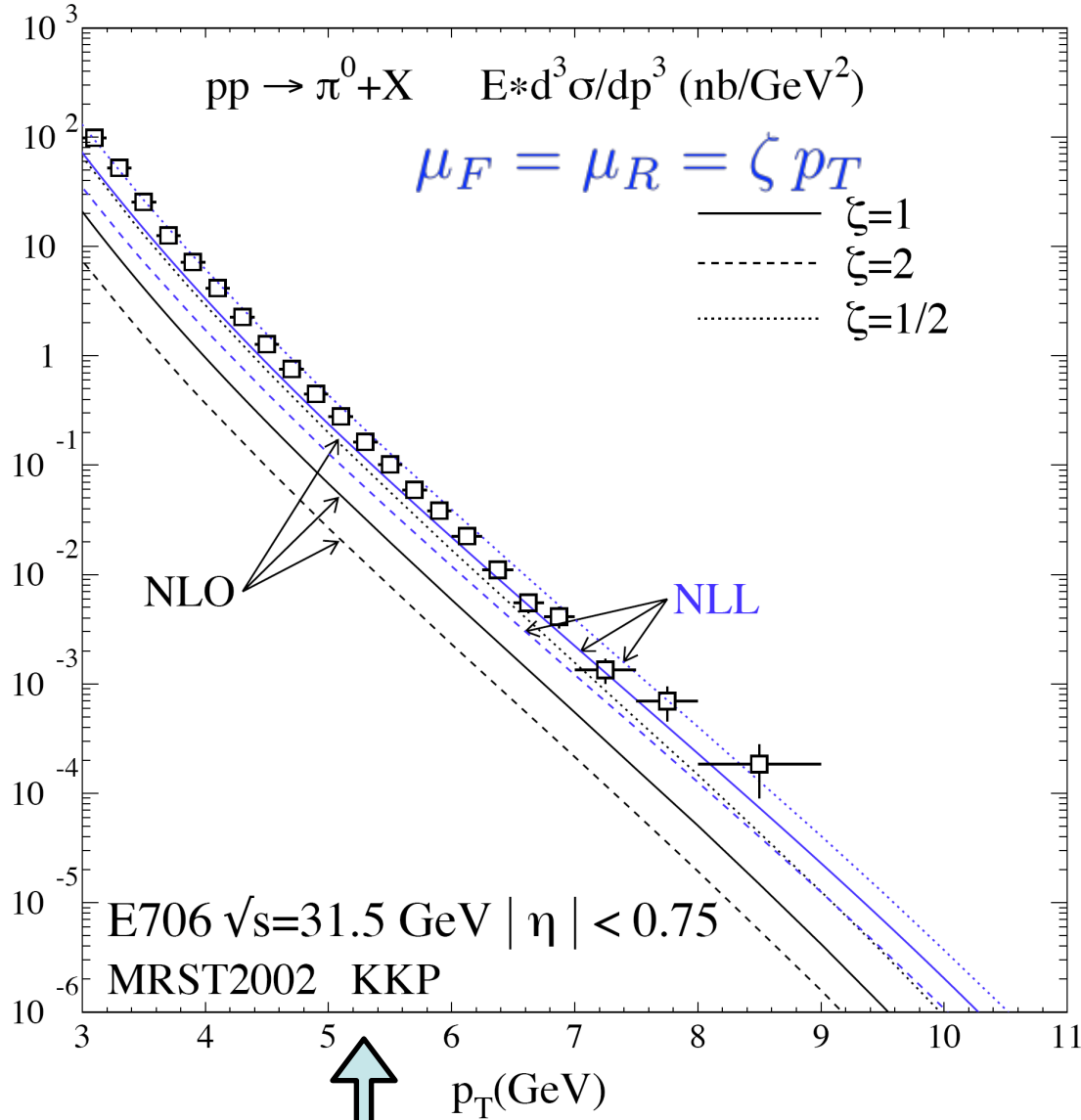
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η integrated

OK for prompt photons!

E706

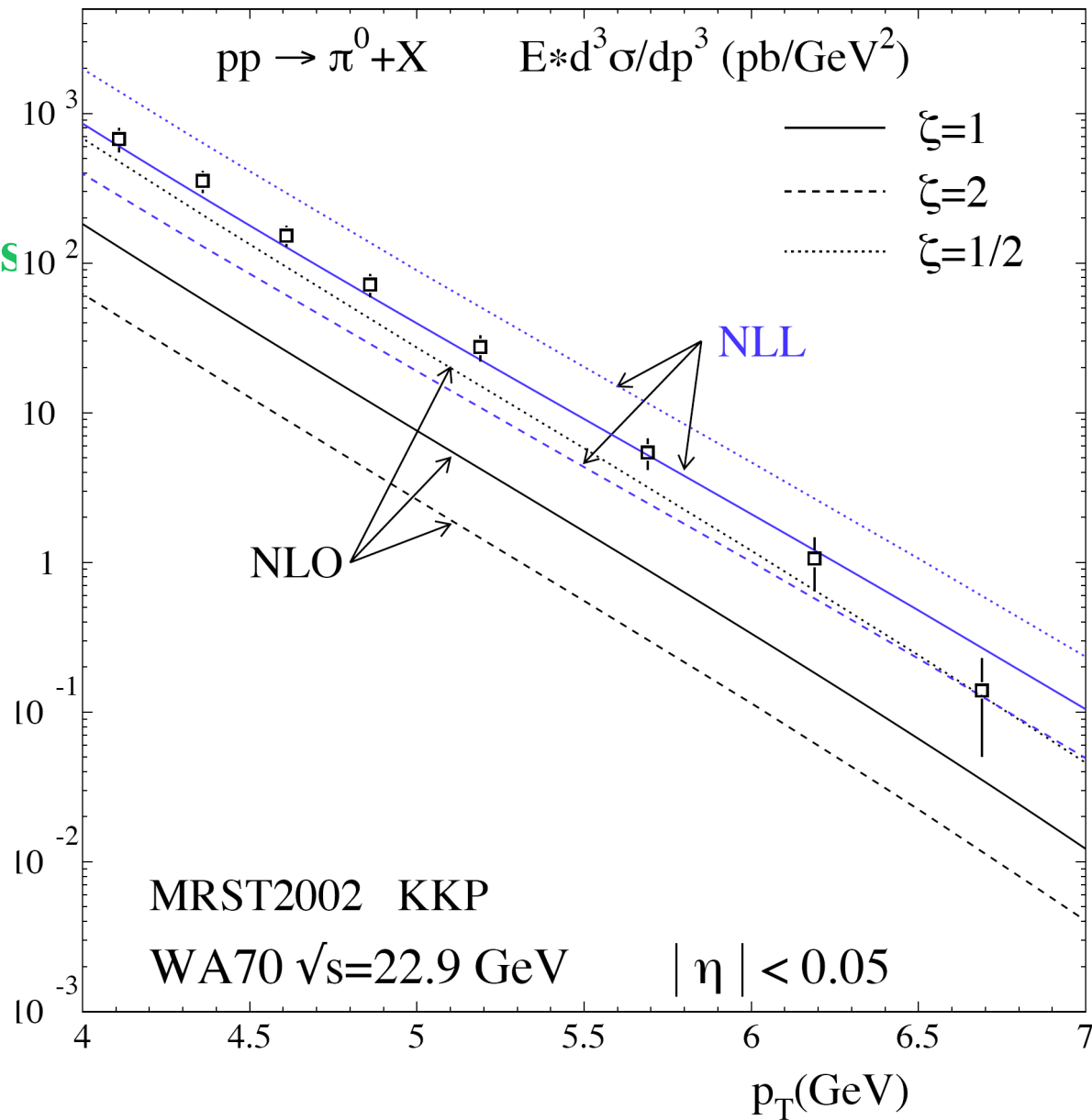
- Large enhancement
- Reduction of scale dep.
- Nice agreement with data!
- No need of intrinsic k_T



Kniesl, Kramer and Potter

Same for WA70

No need of exotic scales
 $\mu = p_T$ is enough

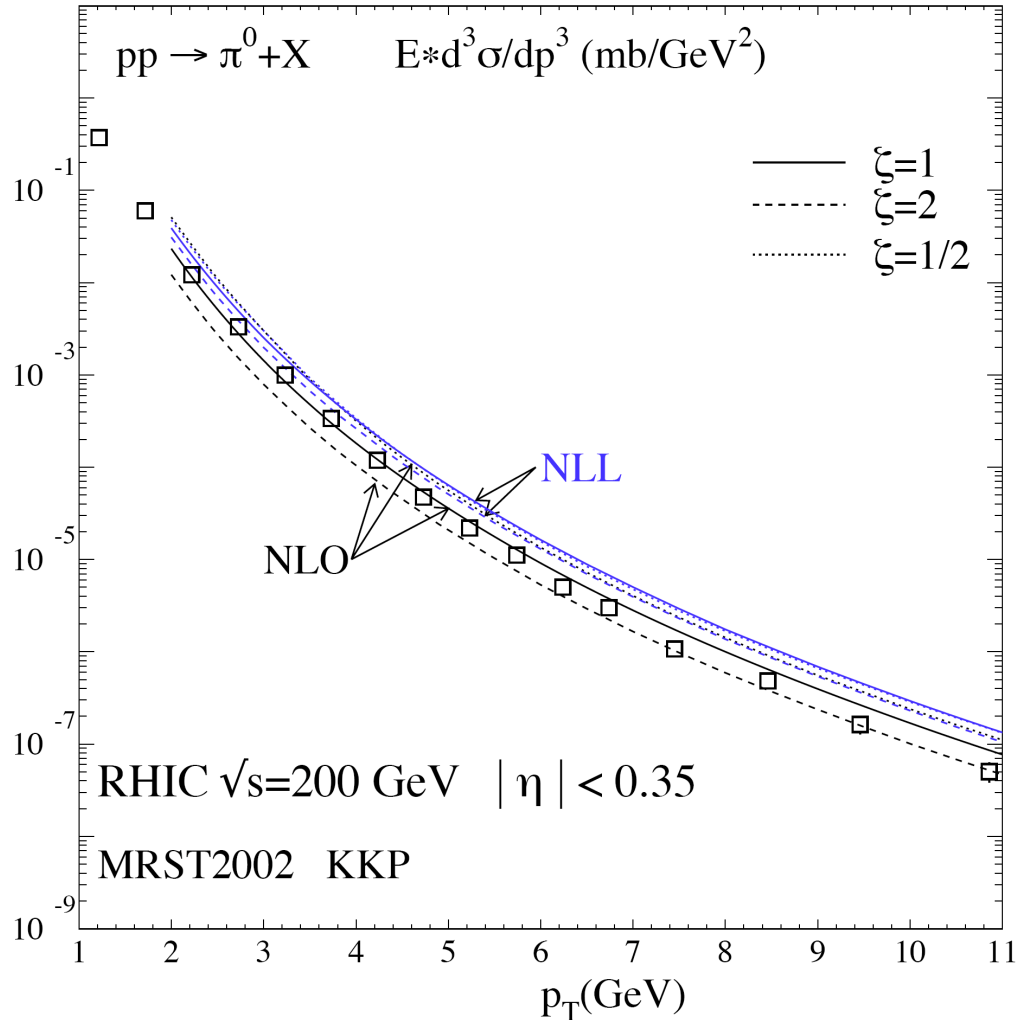


At RHIC energies resummed cross section overestimates data

Hard effects dominate over soft, not expected to work

$$x_T \ll 1$$

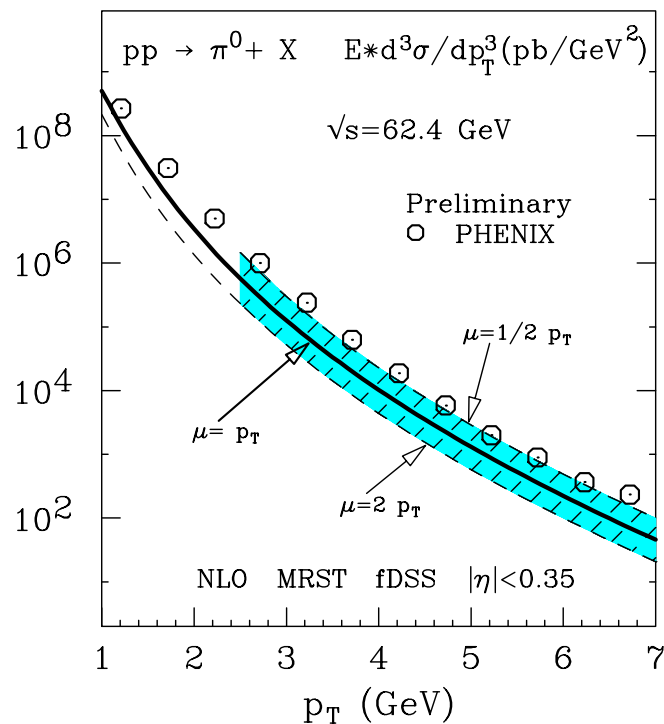
$$\sqrt{S} = 200 \text{ GeV}$$



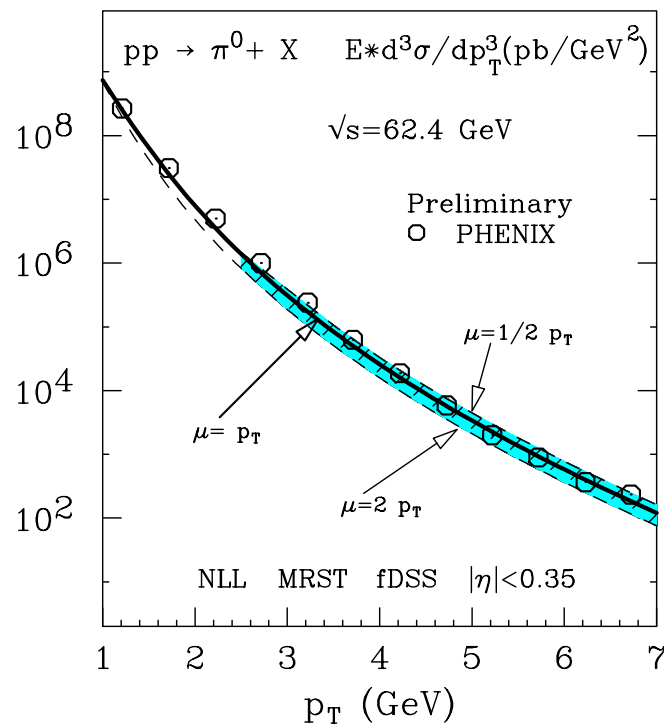
But at lower energies the situation is much better even at RHIC

$$\sqrt{S} = 62.4 \text{ GeV}$$

NLO



NLL

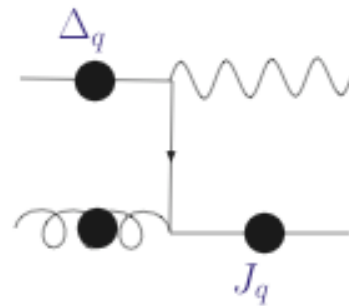
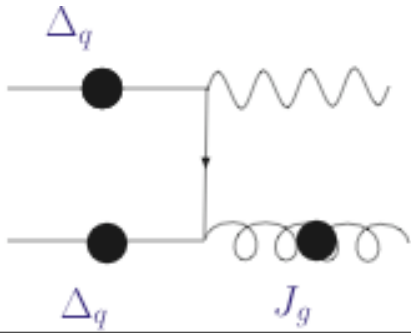
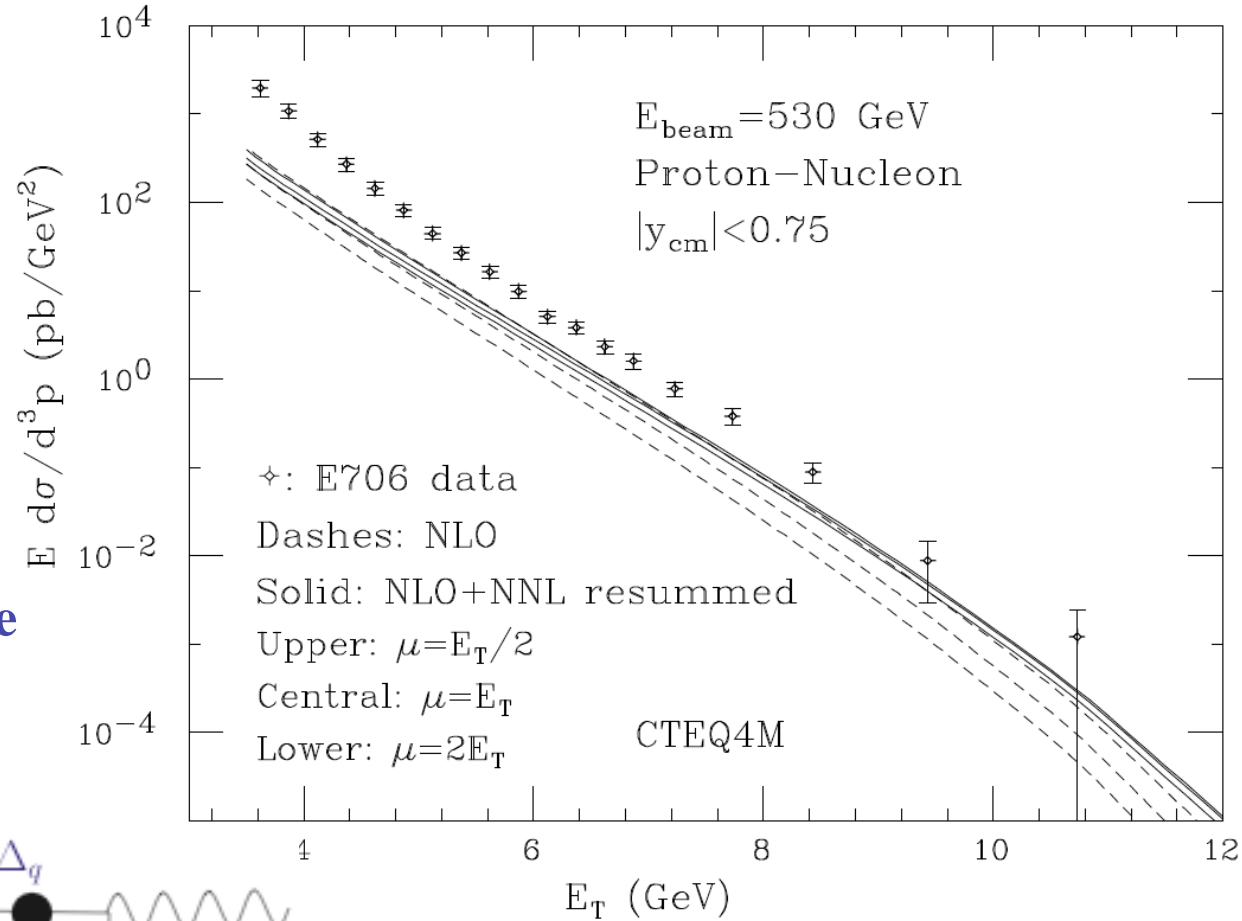


At 62.4 GeV (half way between fixed target and “colliders”)
soft logs still dominate

Prompt Photons: **direct** contribution resummed some time ago

Some enhancement, not enough for E706
resolved contribution only at fixed order

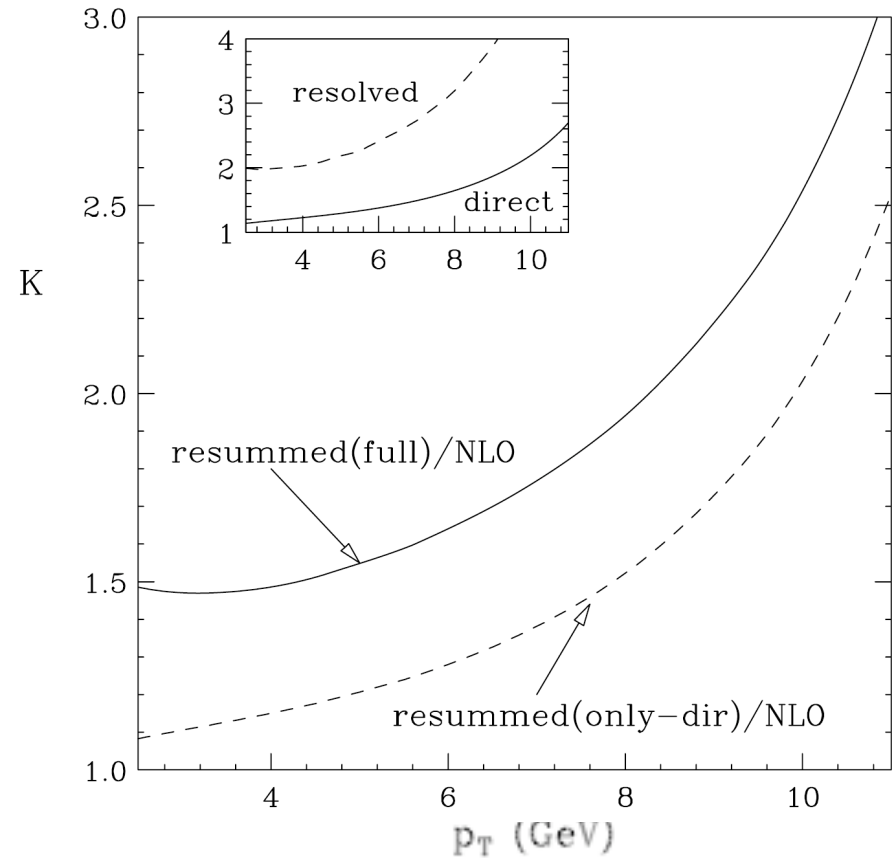
Soft emission less important
for direct contribution: more
quarks and no identified
final state parton



Catani, Mangano, Nason, Oleari, Vogelsang

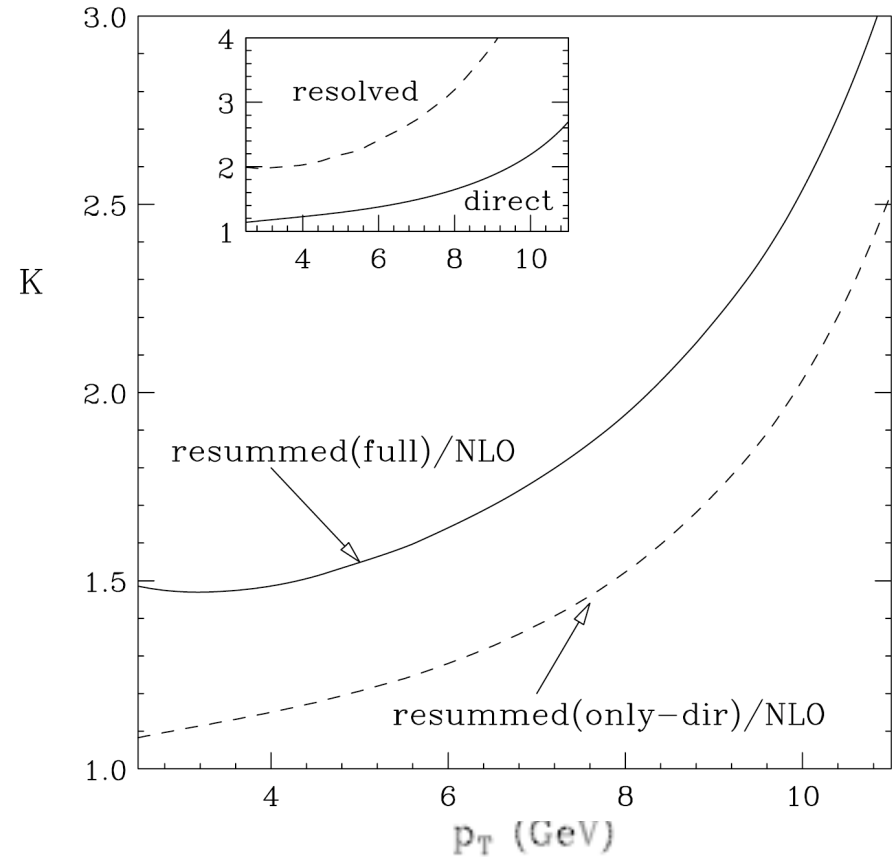
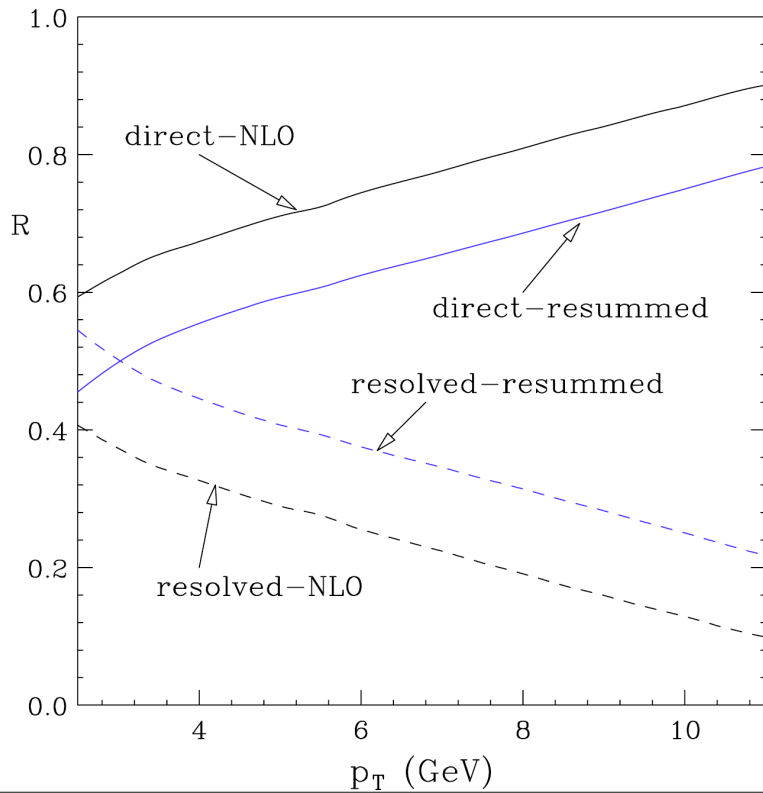
Resolved component similar to pion production: just use photon fragmentation functions (GRV)

~ 40% increase at low p_T
K-Resolved not as big as for pions
Less gluon to photon fragmentation



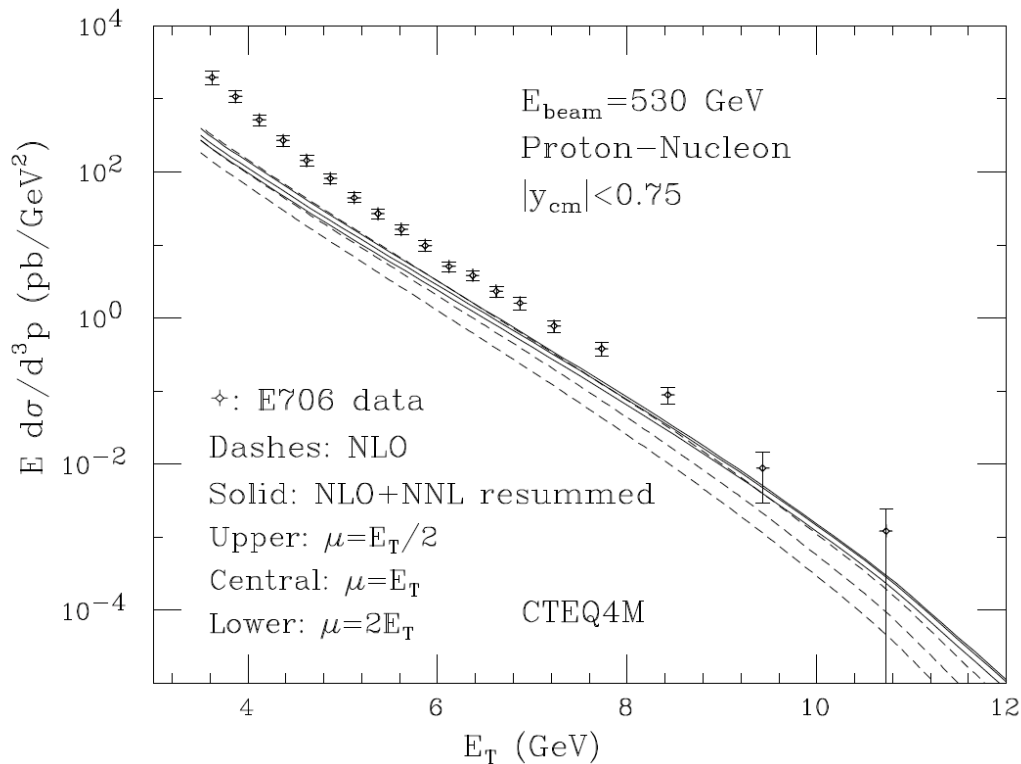
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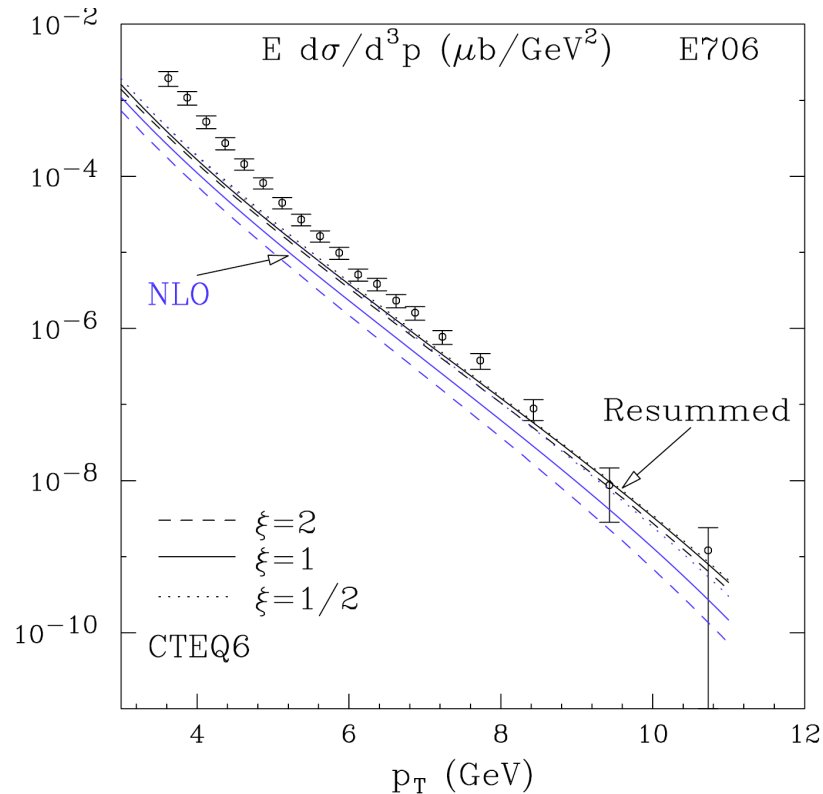


Resolved contribution becomes more important (see ratios)

Increase not enough, but helps at large transverse momentum

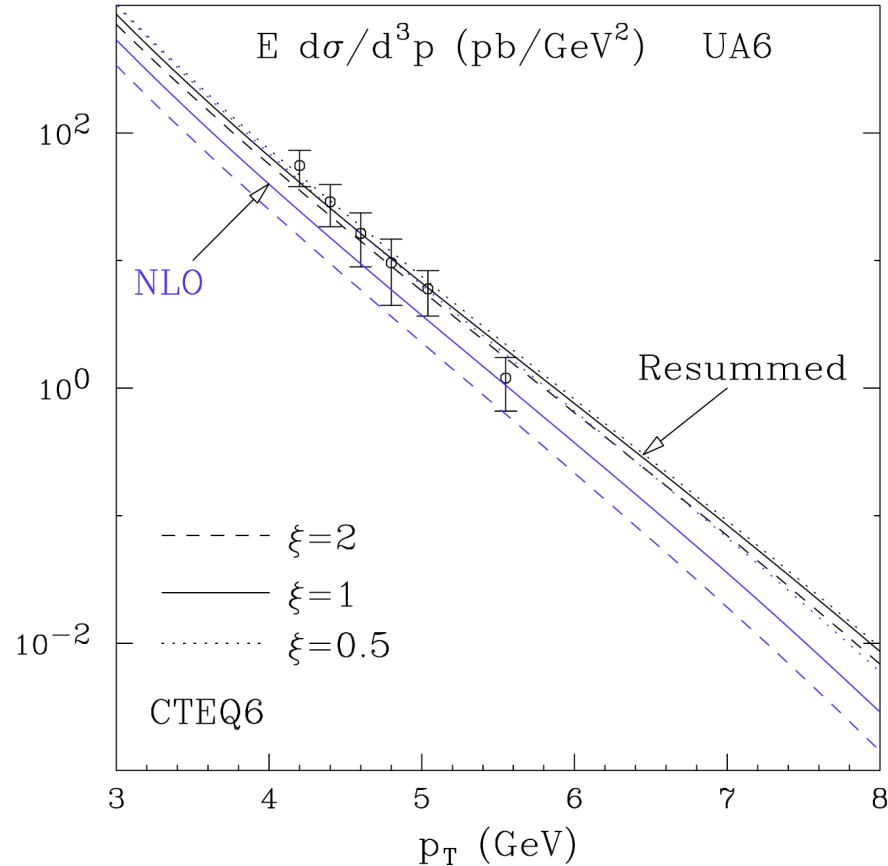
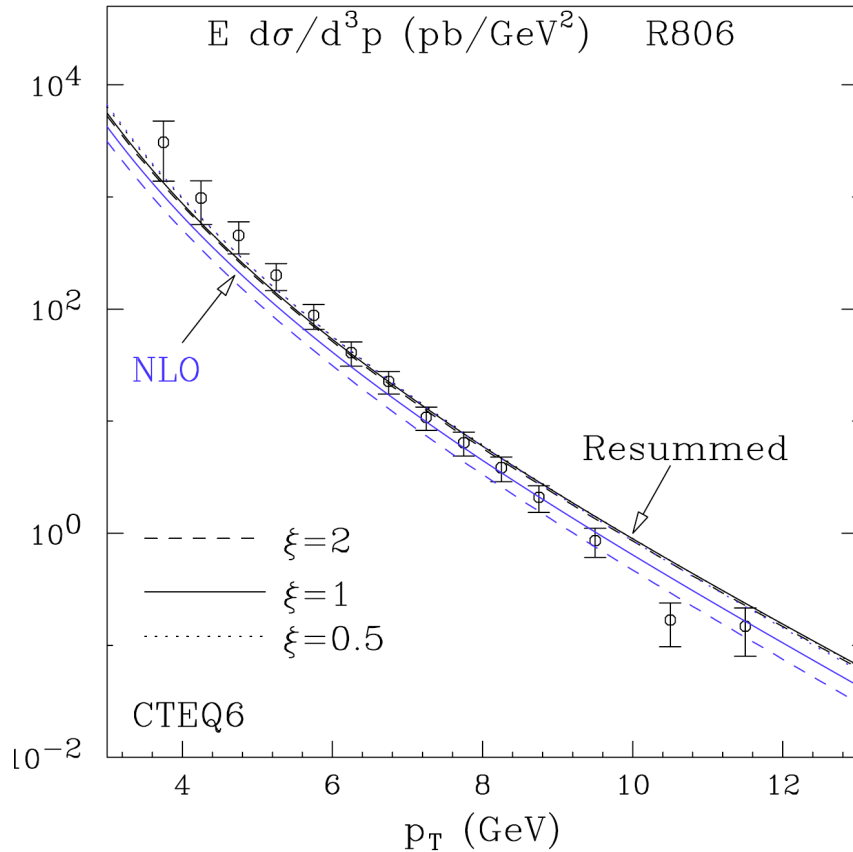


Only direct resummed



Direct+resolved resummed

Nice improvements for R806 and UA6

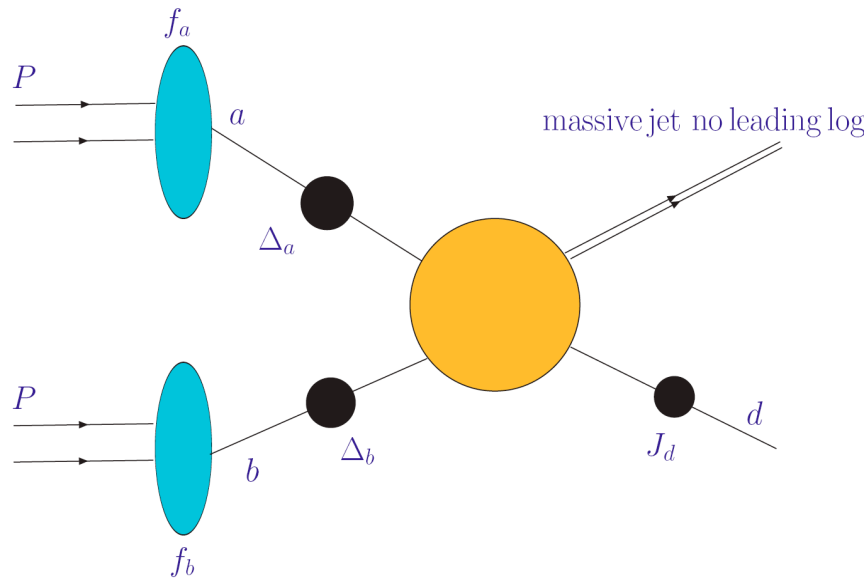


Threshold resummation can not “solve” prompt photons but certainly helps to improve agreement, besides providing more reliable predictions

• Jet production

Same partonic subprocesses but different logarithmic structure:
final state is an observed jet, not an “isolated” hadron

f.s. singularities regularized by jet mass/cone size



Some issues about jet definition
might even be

Only slight enhancement expected

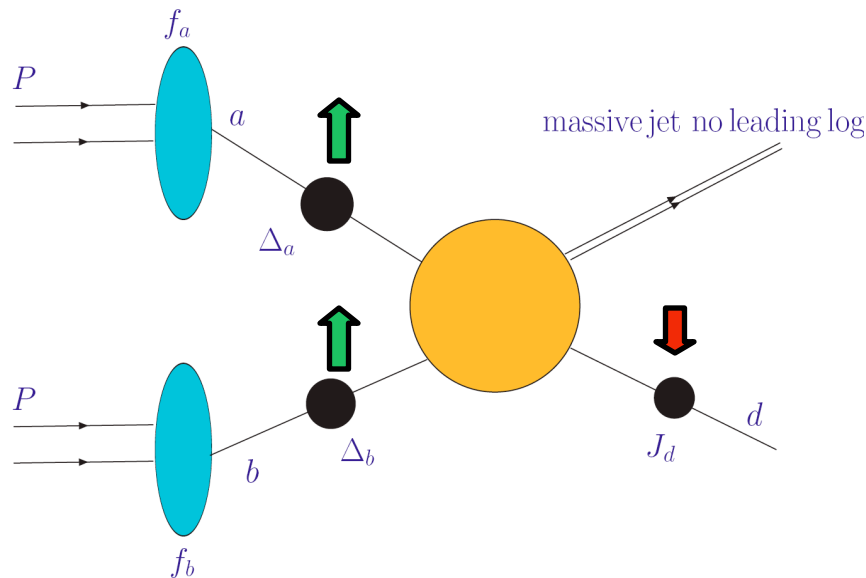
Calculation within the small cone approximation D. de F & W.Vogelsang

Full NLL with matching to NLO

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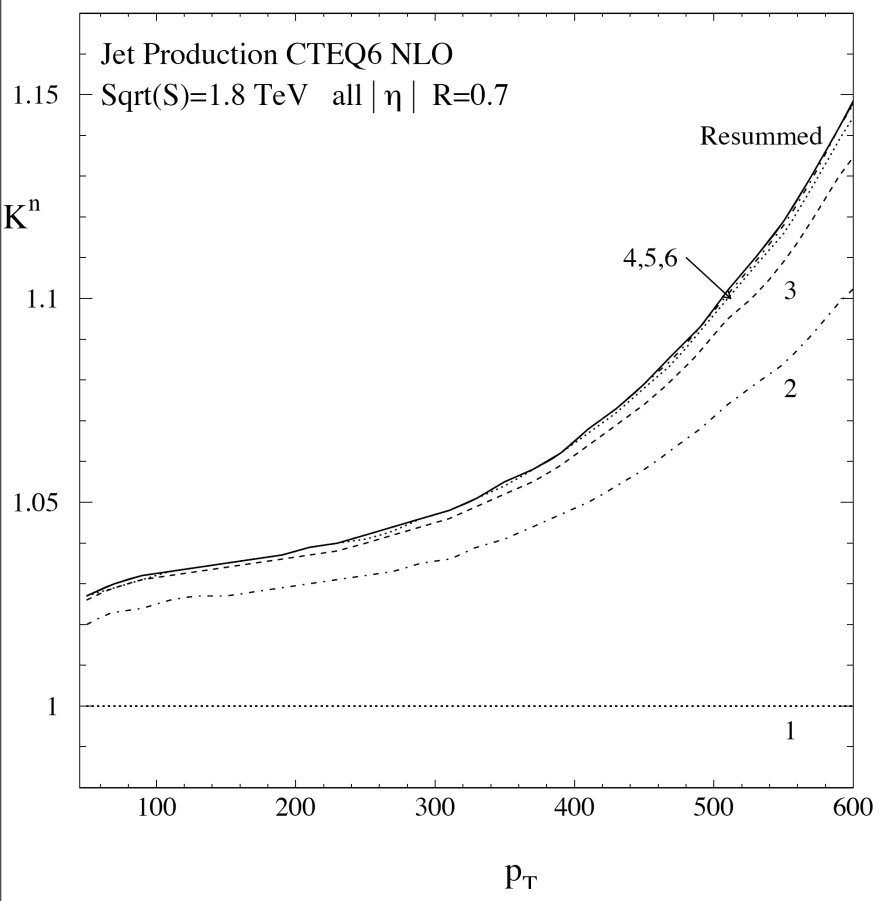
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Calculation within the small cone approximation D. de F & W.Vogelsang

Full NLL with matching to NLO

Effect rather small, < 10% at largest transverse momentum CDF

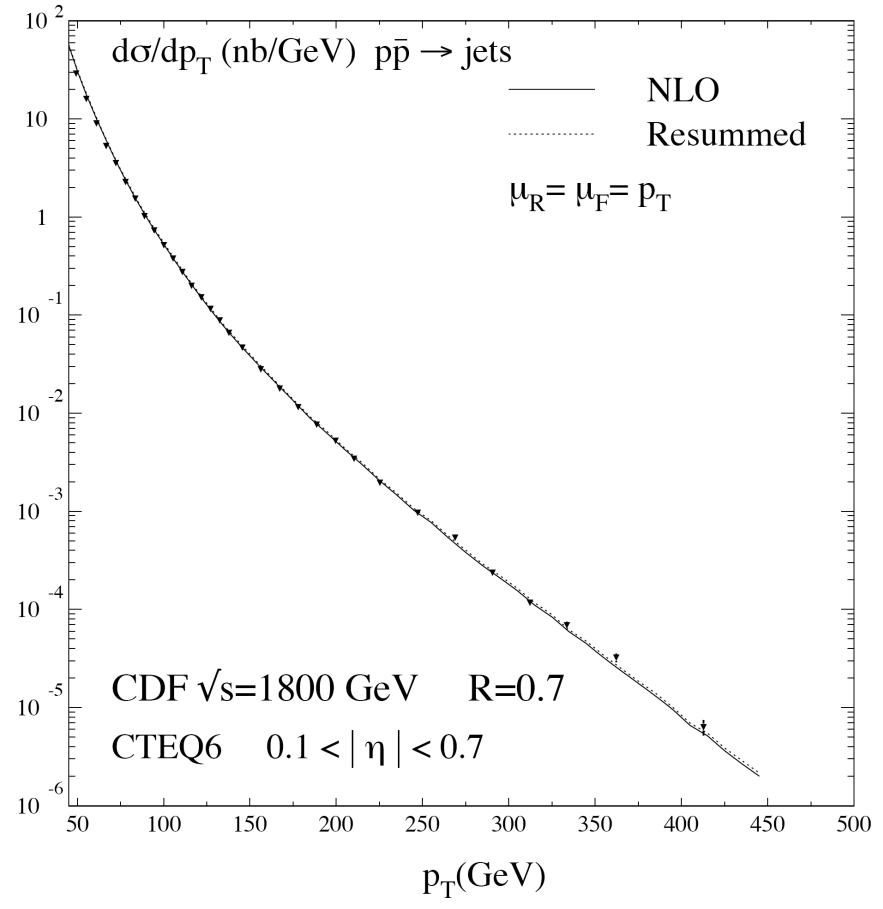
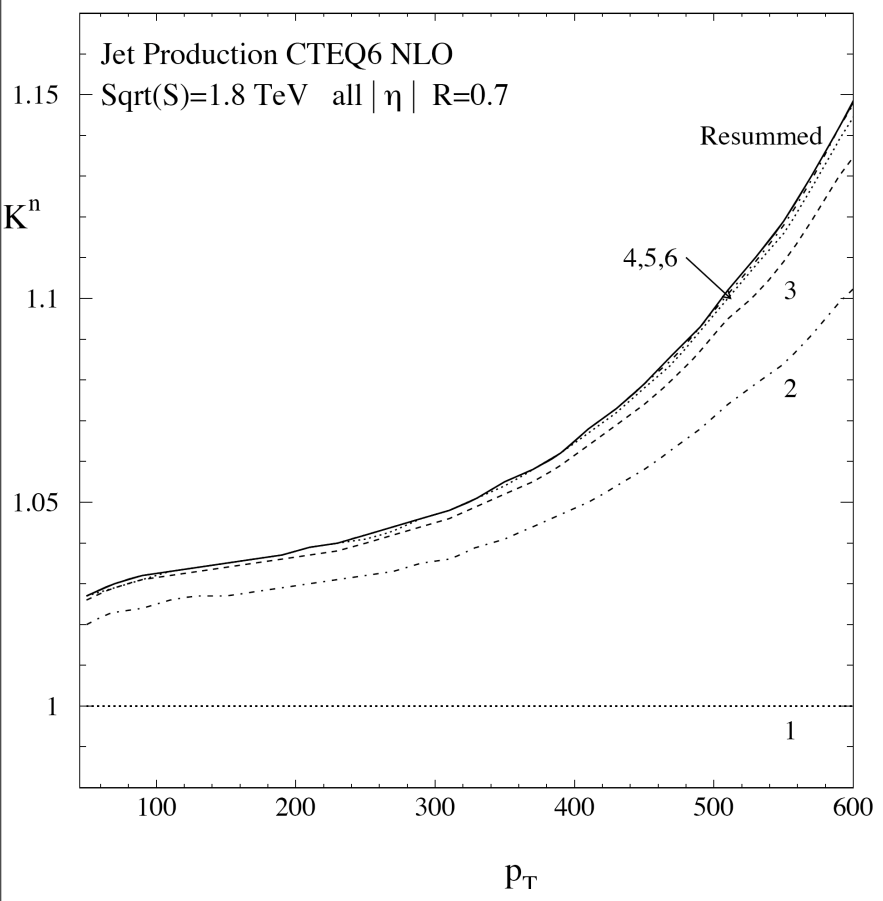
1.8 TeV



Very good convergence already at 3rd order

Effect rather small, < 10% at largest transverse momentum CDF

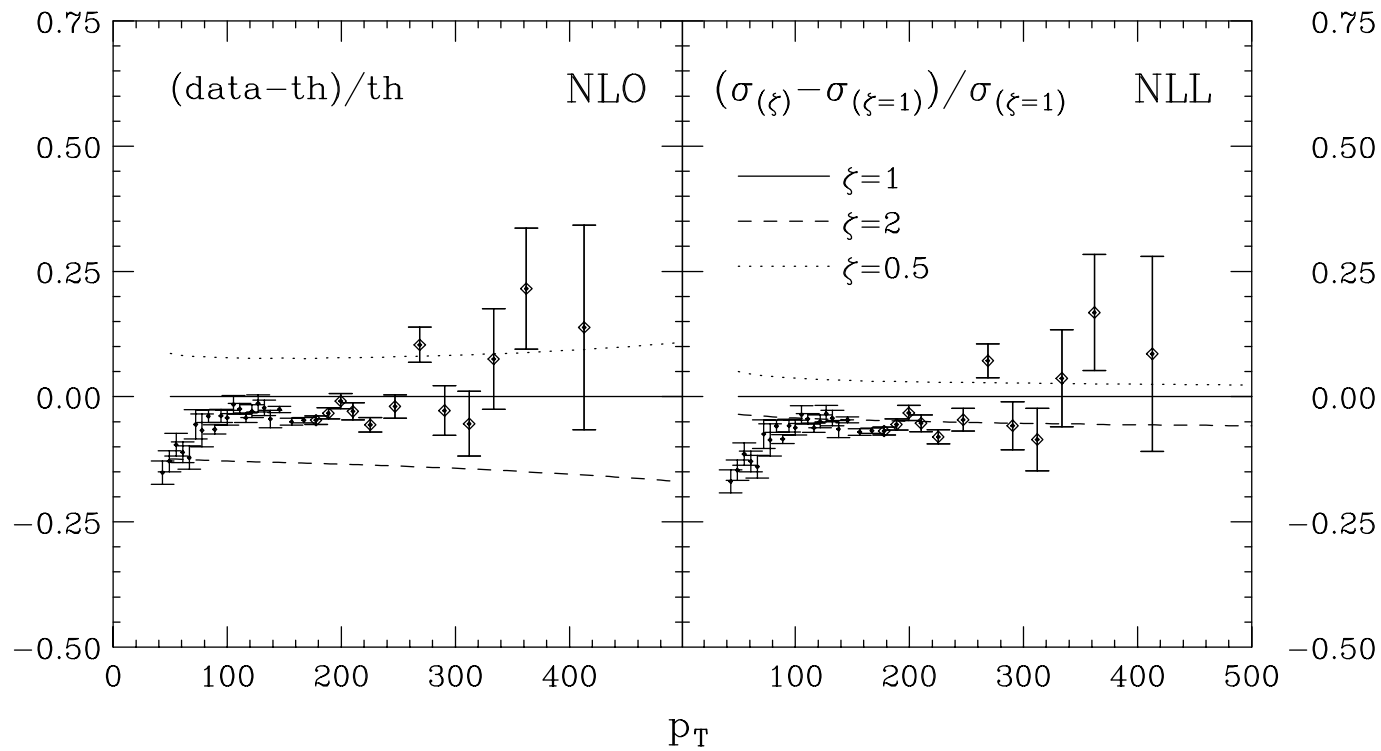
1.8 TeV



Very good convergence already at 3rd order

Hardly noticeable in Log plot

(data-th)/th



Small improvement at large transverse momentum

Conclusions

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- **Hadron production: corrections are large enough to bring agreement with fixed target data**
- **Photon production: resolved contribution increased.
Not enough to solve all problems but reduction of theoretical deficit at small transverse momentum**
- **Jet production: 10% increase at very large transverse momentum**

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Some phenomena usually explained by non-perturbative arguments (intrinsic k_T for hadrons and photons, gluon distribution for jets) can actually be (at least partially) understood in terms of (all orders) pQCD: take care!

RHIC
E706 Tevatron
WA70 CERN
UA6
R806
NA 24