Correlação entre estrelas de nêutrons e peles de nêutrons

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Motivation

- The properties of neutron stars (very isospin asymmetric matter in β- equilibrium) are obtained from appropriate EoS whose symmetry energy depends on the density;
- The symmetry energy also controls the size of the neutron skin thickness in heavy and asymmetric nuclei, as ^{208} Pb;
- Neutron skin thickness = difference between the neutron and the proton radii; depends on a precise measurement of both the charge and the neutron radius. The charge radius is already known within a precision of 1% for most stable nuclei; for the neutron radius, our present knowledge has an uncertainty of about 0.2 fm;
- The Parity Radius Experiment (PREX) at the Jefferson Laboratory is currently running to measure the ^{208}Pb neutron radius with an accuracy of less than 0.05 fm, using polarized electron scattering.

- Accurate experimental measurement of the neutron skin thickness can provide constraints to the EoS that describe neutron star matter.
- We use relativistic models with density dependent and constant couplings to calculate both the energy symmetry and the neutron skin thickness.

Formalism

The asymmetry for polarized electron scattering of a hadronic target is

$$\mathcal{A} = \frac{d\sigma_+/d\Omega - d\sigma_-/d\Omega}{d\sigma_+/d\Omega + d\sigma_-/d\Omega}, \tag{1}$$

where $d\sigma_\pm/d\Omega$ is the differential cross section for initially polarized electrons with positive($+$) and negative ($-$) helicities.

The electromagnetic interaction is not sensitive to the above difference. The asymmetry depends only on the weak interaction between the electron and the target. Accordind to the Standard Model the neutral Z-boson couples more strongly to the neutron than to the proton. For elastic scattering on an even-even target nucleus, the asymmetry can be written in the form:

$$\mathcal{A} = \frac{Gq^2}{2\pi\alpha\sqrt{2}} a[\beta_V^p + \beta_V^n \frac{\rho_n(q)}{\rho_p(q)}]. \tag{2}$$

G, α, a and $\beta_V^{p,n}$ = Standard Model coupling constants

q = transferred momentum by the electron to the nucleus

In the PREX experiment, the asymmetry is expected to be measured at $q \approx 0.4\ fm^{-1}$.

$$\rho_{n(p)}(q) = \int \ d^3r\ j_0(qr)\rho_{n(p)}(\mathbf{r}), \tag{3}$$

$$R_i^2 = \frac{\int \ d^3r r^2 \rho_i(\mathbf{r})}{\int \ d^3r \rho_i(\mathbf{r})}, \quad i = p, n. \tag{4}$$

The neutron skin thickness is

$$\theta = R_n - R_p. \tag{5}$$

Symmetry energy

$\mathcal{E}_{sym} = \frac{1}{2}\frac{\partial^2 \mathcal{E}/\rho}{\partial \delta^2}\Big|_{\delta=0}$, with $\delta = -\rho_3/\rho$ being the assymetry of the system with $\rho_3 = \rho_p - \rho_n$ and $\rho = \rho_p + \rho_n$.

The symmetry energy can be expanded around the nuclear saturation density and reads

$$\mathcal{E}_{sym}(\rho) = \mathcal{E}_{sym}(\rho_0) + \frac{L}{3}\left(\frac{\rho - \rho_0}{\rho_0}\right) + \frac{K_{sym}}{18}\left(\frac{\rho - \rho_0}{\rho_0}\right)^2, \tag{6}$$

L = the slope, K_{sym} = curvature of the nuclear symmetry energy at ρ_0

$$L = 3\rho_0 \frac{\partial \mathcal{E}_{sym}(\rho)}{\partial \rho}|_{\rho=\rho_0} \quad K_{sym} = 9\rho_0^2 \frac{\partial^2 \mathcal{E}_{sym}(\rho)}{\partial^2 \rho}|_{\rho=\rho_0}. \tag{7}$$

The DDH model (TW)

$$\mathcal{L} = \bar{\psi}\left[\gamma_\mu\left(i\partial^\mu - \Gamma_v V^\mu - \frac{\Gamma_\rho}{2}\boldsymbol{\tau}\cdot\mathbf{b}^\mu - e\frac{(1+\tau_{3i})}{2}A^\mu\right) - (M - \Gamma_s\phi)\right]\psi$$

$$+\frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2) - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu}$$

$$+\frac{1}{2}m_v^2 V_\mu V^\mu - \frac{1}{4}\mathbf{B}_{\mu\nu}\cdot\mathbf{B}^{\mu\nu} + \frac{1}{2}m_\rho^2\mathbf{b}_\mu\cdot\mathbf{b}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad (8)$$

ϕ =scalar-isoscalar meson field

V^μ =vector-isoscalar meson field

$\mathbf{b}^\mu$ =vector-isovector meson field

A^μ= photon field

$\Omega_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, $\mathbf{B}_{\mu\nu} = \partial_\mu\mathbf{b}_\nu - \partial_\nu\mathbf{b}_\mu - \Gamma_\rho(\mathbf{b}_\mu\times\mathbf{b}_\nu)$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$\tau_{3p} = 1$ $\tau_{3n} = -1$.

$$\Gamma_i(\rho) = \Gamma_i(\rho_{sat}) h_i(x), \quad x = \rho/\rho_{sat}, \tag{9}$$

with

$$h_i(x) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2}, \quad i = s, v \tag{10}$$

and

$$h_\rho(x) = \exp[-a_\rho(x - 1)], \tag{11}$$

The Thomas-Fermi Approximation

mesonic equations of motion:

$$\nabla^2\phi = m_s^2\phi - \Gamma_s\rho_s, \quad \nabla^2 V_0 = m_v^2 V_0 - \Gamma_v\rho, \tag{12}$$

$$\nabla^2 b_0 = m_\rho^2 b_0 - \frac{\Gamma_\rho}{2}\rho_3, \quad \nabla^2 A_0 = -e\rho_p, \tag{13}$$

$\rho_s =< \bar{\psi}\psi > =$ scalar density

$$\Omega = E - \mu_p B_p - \mu_n B_n \rightarrow EoS(\epsilon(\rho), P(\rho)) \tag{14}$$

$$\mu_p = \sqrt{k_{Fp}^2 + M^{*2}} + \Gamma_v V_0 + \frac{\Gamma_\rho}{2} b_0 + e A_0 + \Sigma_0^R, \tag{15}$$

$$\mu_n = \sqrt{k_{Fn}^2 + M^{*2}} + \Gamma_v V_0 - \frac{\Gamma_\rho}{2} b_0 + \Sigma_0^R, \tag{16}$$

$$\Sigma_0^R = \frac{\partial \Gamma_v}{\partial \rho} \rho \, V_0 + \frac{\partial \Gamma_\rho}{\partial \rho} \rho_3 \, \frac{b_0}{2} - \frac{\partial \Gamma_s}{\partial \rho} \rho_s \, \phi.$$

NL$\omega\rho$ model

$$\mathcal{L} = \bar{\psi}\left[\gamma_\mu\left(i\partial^\mu - g_v V^\mu - \frac{g_\rho}{2}\boldsymbol{\tau}\cdot\mathbf{b}^\mu - e\frac{(1+\tau_{i3})}{2}A^\mu\right) - (M - g_s\phi)\right]\psi$$

$$+\frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2) - \frac{1}{3!}\kappa\phi^3 - \frac{1}{4!}\lambda\phi^4 - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu}$$

$$+\frac{1}{2}m_v^2 V_\mu V^\mu - \frac{1}{4}\mathbf{B}_{\mu\nu}\cdot\mathbf{B}^{\mu\nu} + \frac{1}{2}m_\rho^2\mathbf{b}_\mu\cdot\mathbf{b}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$+g_\rho^2\mathbf{b}_\mu\cdot\mathbf{b}^\mu[\Lambda_s g_s^2\phi^2 + \Lambda_v g_v^2 V_\mu V^\mu], \tag{17}$$

$\Lambda_s = 0$, NL3 parametrization of the NLWM

Revisiting the symmetry energy

TW model:

$$\mathcal{E}_{sym} = \frac{k_F^2}{6\epsilon_F} + \frac{\Gamma_\rho^2}{8m_\rho^2}\rho, \tag{18}$$

NL$\omega\rho$ model:

$$\mathcal{E}_{sym} = \frac{k_F^2}{6\epsilon_F} + \frac{g_\rho^2}{8m_\rho^{*2}}\rho \tag{19}$$

$$m_\rho^{*2} = m_\rho^2 + 2g_v^2 g_\rho^2 \Lambda_v V_0^2$$

For both:

$$k_{Fp} = k_F(1+\delta)^{1/3}, \quad k_{Fn} = k_F(1-\delta)^{1/3}, \quad \delta = -\rho_3/\rho = 1 - 2y_p$$

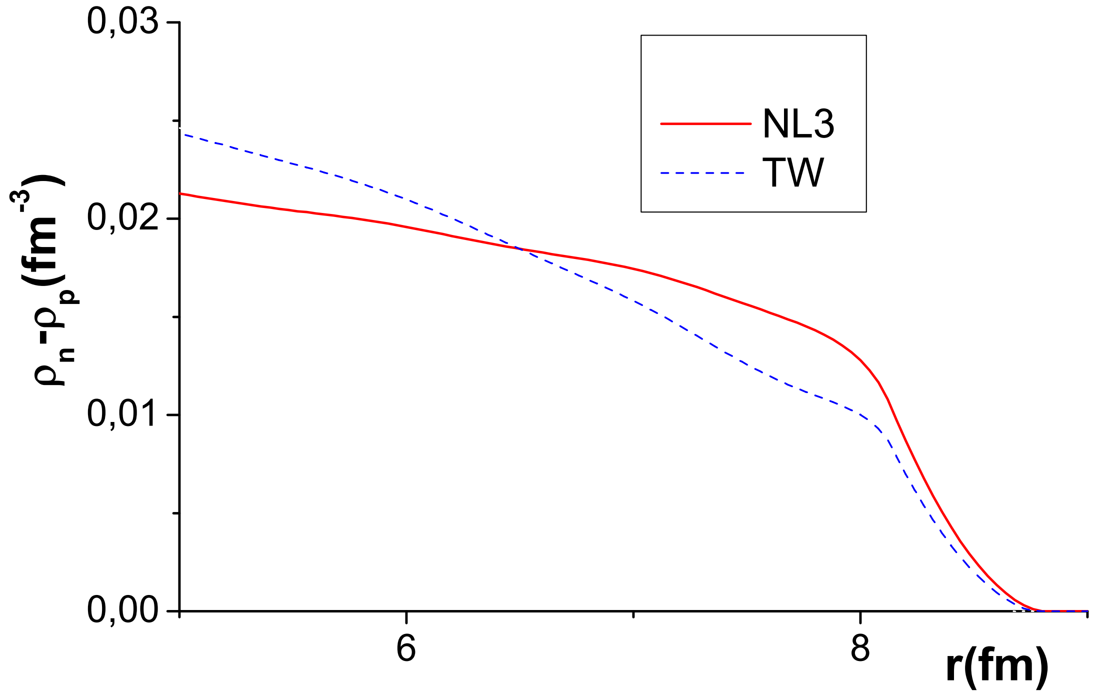
$k_F = (1.5\pi^2\rho)^{1/3}$, $\epsilon_F = \sqrt{k_F^2 + M^{*2}}$, $M^* =$ effective mass.

Surface energy

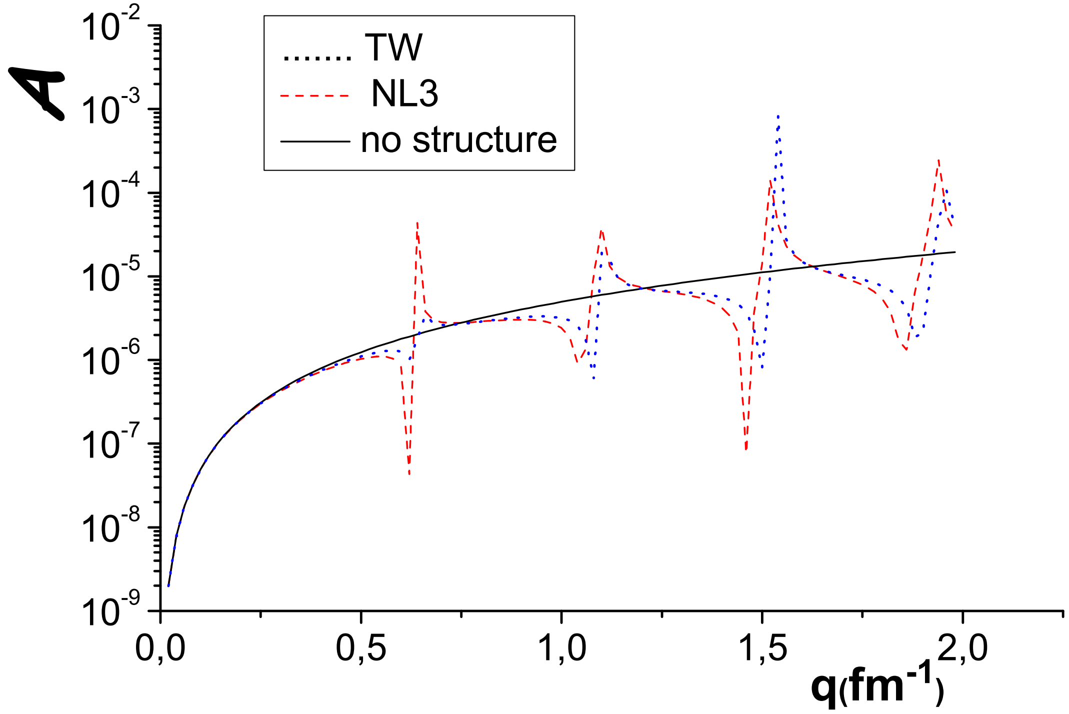
The surface energy per unit area, excluding the electromagnetic field, reads

$$\sigma = \int_0^\infty \mathrm{d}r \left[\left(\frac{\mathrm{d}\phi}{\mathrm{d}r} \right)^2 - \left(\frac{\mathrm{d}V_0}{\mathrm{d}r} \right)^2 - \left(\frac{\mathrm{d}b_0}{\mathrm{d}r} \right)^2 \right]. \tag{20}$$

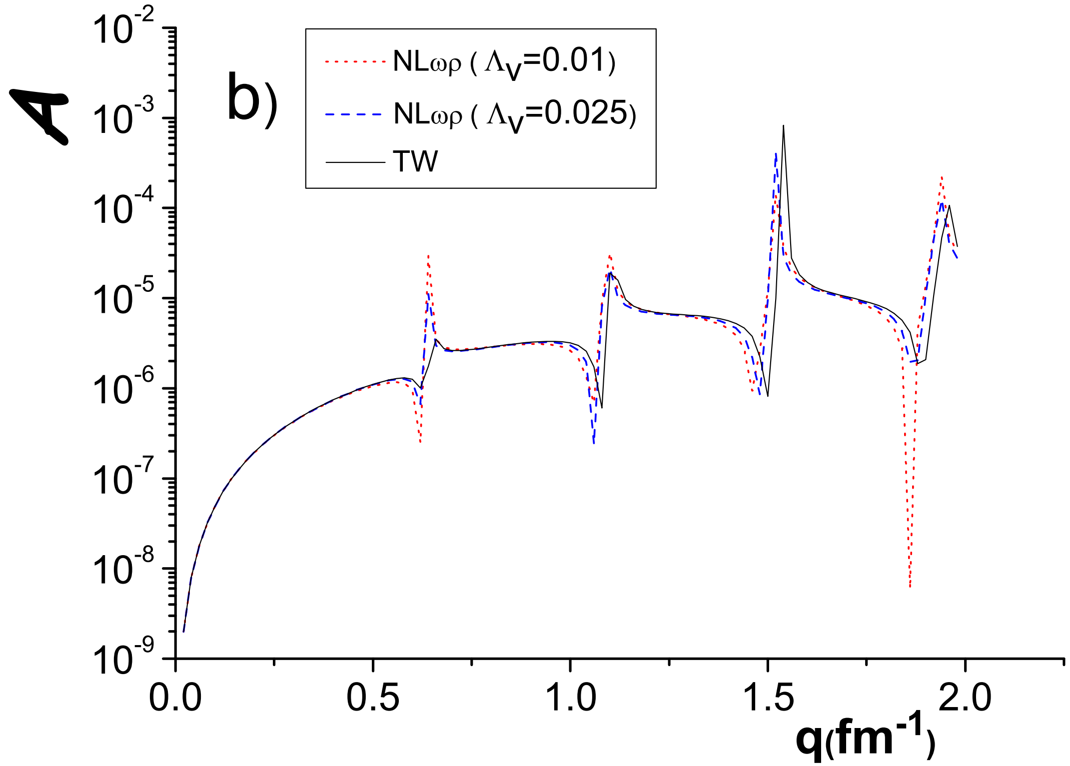
Results



Difference between neutron and proton densities obtained with the Thomas-Fermi approach.



Asymmetry versus transfered momentum.



Asymmetry versus transfered momentum.

^{208} Pb properties

model	R_n fm	R_p fm	θ fm	B/A MeV	σ Mev/fm^2	$\mathcal{E}_{sym.}$ MeV	L MeV
NL3	5.79	5.57	0.22	-7.79	0.96	37.4	118
NL$\omega\rho$,$\Lambda_v = 0.01$	5.77	5.57	0.20	-7.73	0.98	34.9	88
NL$\omega\rho$,$\Lambda_v = 0.02$	5.75	5.57	0.17	-7.65	0.99	33.1	68
NL$\omega\rho$,$\Lambda_v = 0.025$	5.74	5.58	0.16	-7.63	1.00	32.3	55
TW	5.68	5.52	0.16	-7.46	1.10	32.0	55
exp.		5.44					
exp.				-7.87			
exp.			0.12 ± 0.07				
exp.			0.20 ± 0.04				

Conclusions I

- Larger values of L correspond to larger values of the neutron skin; a correlation between the slope of the symmetry energy and the neutron skin thickness was found as in Skyrme-type models.

- Although the neutron skin thickness is model dependent, the asymmetry at low momentum transfers (below 0.5 fm^{-1}) is very similar for all models.

- As q increases, the asymmetry also becomes model dependent.

- Once accurate results for the neutron skin are obtained, some of the models can be ruled out and work as a constraint to neutron star EoS.

- A recalculation with Dirac solutions is necessary.

Dirac solution of DDHM (TW and DDME1)

$$[\gamma^{\mu}(i\partial_{\mu} - \Sigma_{\mu}) - (M - \Sigma_s)]\Psi = 0 \quad , \tag{21}$$

where

$$\Sigma_s = \Gamma_s \phi, \quad \Sigma_{\mu} = \Sigma_{\mu}^{(0)} + \Sigma_{\mu}^{R}, \tag{22}$$

with

$$\Sigma_{\mu}^{(0)} = \Gamma_v V_{\mu} + \frac{\Gamma_{\rho}}{2}\vec{\tau} \cdot \mathbf{b}_{\mu} + e\frac{(1+\tau_3)}{2}A_{\mu} \quad , \tag{23}$$

$$\Sigma_{\mu}^{R} = \frac{j_{\mu}}{\rho}\left(\frac{\partial \Gamma_v}{\partial \rho}\bar{\psi}\gamma^{\nu}\psi V_{\mu} + \frac{1}{2}\frac{\partial \Gamma_{\rho}}{\partial \rho}\bar{\psi}\gamma^{\nu}\vec{\tau}\psi \cdot \mathbf{b}_{\nu} - \frac{\partial \Gamma_s}{\partial \rho}\bar{\psi}\psi\phi\right).$$

The ansatz for the nucleon spinor is

$$\psi(\vec{\mathrm{r}}, t) = \psi(\vec{\mathrm{r}}) \exp(-iEt), \tag{24}$$

Only spherically symmetric nuclei are considered and the usual notation for the expected values of the meson fields are used:

$$\Sigma_0 = \Gamma_v V_0(\mathrm{r}) + \frac{1}{2}\Gamma_\rho \tau_3 \mathrm{b}_0(\mathrm{r}) + \mathrm{e}\frac{(1+\tau_3)}{2}\mathrm{A}_0(\mathrm{r}) + \Sigma_0^{\mathrm{R}} \quad , \tag{25}$$

$$\Sigma_0^R = \left(\frac{\partial \Gamma_v}{\partial \rho}\rho(r)V_0 + \frac{1}{2}\frac{\partial \Gamma_\rho}{\partial \rho}\rho_3(r)b_0 - \frac{\partial \Gamma_s}{\partial \rho}\rho_s(r)\phi\right), \tag{26}$$

$$\psi = \begin{pmatrix} g_\kappa(r)\mathcal{Y}_\kappa^{jm} \\ i f_\kappa(r)\mathcal{Y}_{-\kappa}^{jm} \end{pmatrix} \otimes \xi \tag{27}$$

$\mathcal{Y}_{\pm\kappa}^{jm}$ = spinorial spherical harmonics; ξ = isospin wavefunctions

$$\left(M^\star(r) + V(r)\right) g_\kappa(r) - \left(\frac{\partial}{\partial r} - \frac{\kappa - 1}{r}\right) f_\kappa(r) = E g_\kappa(r)$$

$$\left(\frac{\partial}{\partial r} + \frac{\kappa + 1}{r}\right) g_\kappa(r) - \left(M^\star(r) - V(r)\right) f_\kappa(r) = E f_\kappa(r) \quad , \tag{28}$$

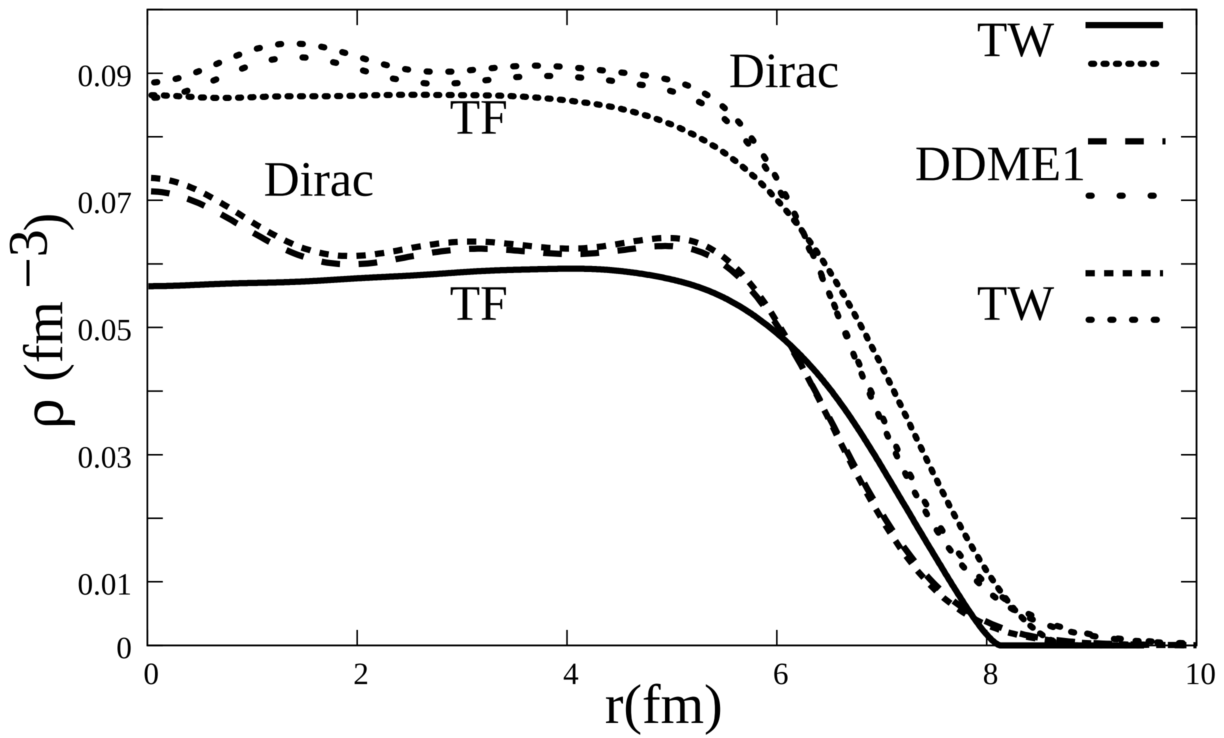
$$M^\star(r) = M - \Gamma_s(r)\phi(r),$$

and

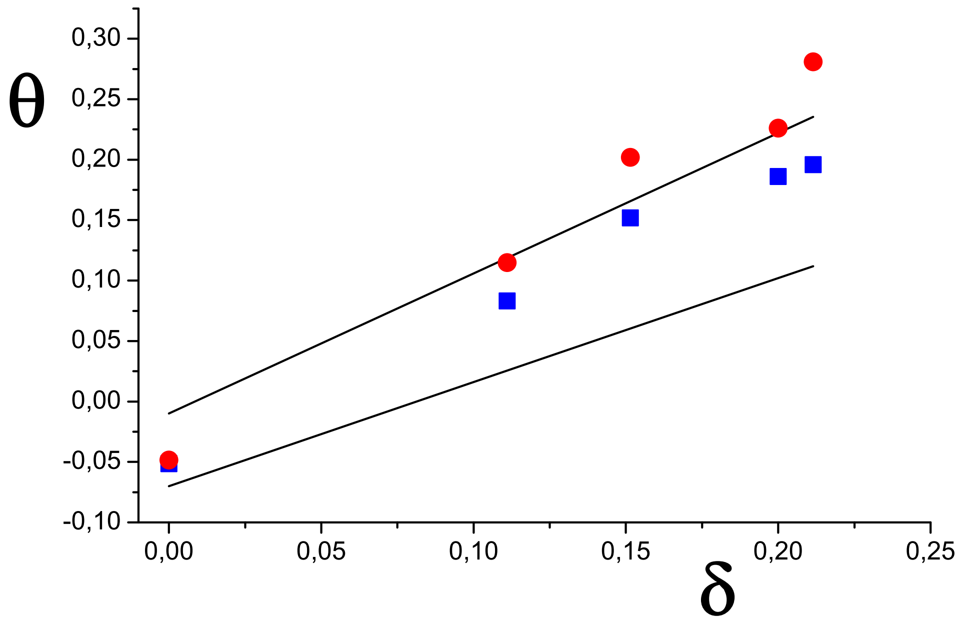
$$V(r) = \Gamma_v(r)V_0(r) + \frac{\Gamma_\rho(r)}{2}\tau_3 b_0(r) + e\frac{(1+\tau_3)}{2}A_0(r) + \Sigma_0^R(r).$$

$f_\kappa(r)$ and $g_\kappa(r)$ are expanded in the harmonic oscillator basis of dimensions N and M respectively as in *Y.K. Gambhir, P. Ring and A. Thimet, Ann. Phys.* **198**, *132 (1990)*

More results



Neutron and proton densities.

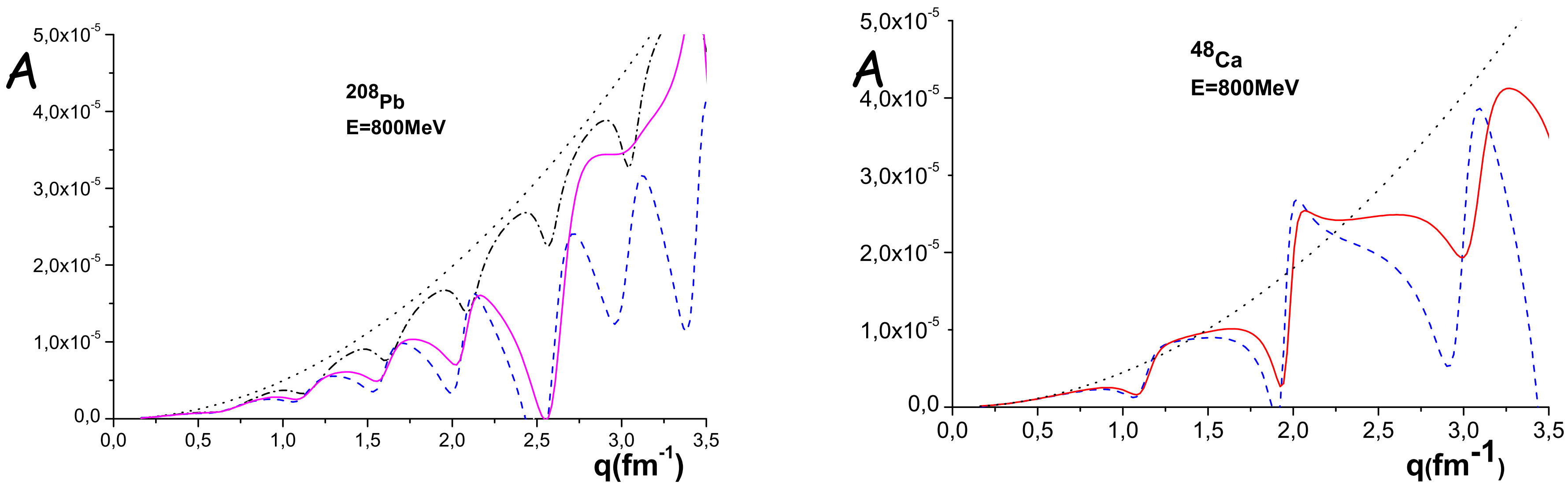


The lines represent the limits for the fitting from *A. Trzcinska, J. Jastrzebbski, P. Lubinski, F.J. Hartmann, R. Schmidt, T. von Egidy and B. Klos, Phys. Rev. Lett.* **87**, *082501 (2001)*:

$\theta = (-0.04 \pm -0.03) + (1.01 \pm 0.15)\delta$

$\delta = (N - Z)/A$

Squares = DDHM ; full circles = NL3; From lower to higher asymmetries: ^{40}Ca, ^{90}Zr, ^{66}Ni, ^{48}Ca and ^{208}Pb.



Asymmetry obtained for ^{208}Pb and ^{48}Ca in the NL3 (dashed line) and TW (full line). The doted line is the PWBA result and the dash-doted one is the three parameter Fermi result (*Fermi3p*). The incident electron energy was chosen as $E = 800MeV$.

^{208} Pb properties

model	R_n (fm)	R_p (fm)	R_c (fm)	θ (fm)	B/A MeV	σ Mev/fm^2
NL3-TF	5.79	5.57		0.22	-7.79	0.96
NL3-Dirac	5.74	5.46	5.51	0.28	-7.91	1.13
TW-TF	5.68	5.52		0.16	-7.46	1.10
TW-Dirac	5.61	5.42	5.48	0.20	-7.78	1.30
DDME1-Dirac	5.66	5.46	5.51	0.20	-7.91	1.18
exp.			5.50			
exp.					-7.87	
exp.				0.12 ± 0.07		
exp.				0.20 ± 0.04		
exp.				$0.16 \pm 0.02 \pm 0.04$		

Finite nuclei properties

model	nuclei	R_n (fm)	R_p (fm)	R_c (fm)	θ (fm)	B/A MeV	σ Mev/fm^2
NL3	^{40}Ca	3.32	3.37	3.43	-0.05	-8.62	1.48
TW	^{40}Ca	3.28	3.33	3.39	-0.05	-8.36	1.60
DDME1	^{40}Ca	3.32	3.37	3.43	-0.05	-8.62	1.45
exp.	^{40}Ca			3.48		-8.55	
NL3	^{48}Ca	3.60	3.37	3.44	0.23	-8.72	1.54
TW	^{48}Ca	3.54	3.35	3.42	0.19	-8.49	1.70
DDME1	^{48}Ca	3.58	3.39	3.46	0.19	-8.66	1.53
exp.	^{48}Ca					-8.67	
exp.	^{48}Ca			3.48			
NL3	^{90}Zr	4.30	4.19	4.25	0.11	-8.86	1.37
TW	^{90}Zr	4.24	4.15	4.22	0.08	-8.55	1.52
DDME1	^{90}Zr	4.28	4.19	4.25	0.08	-8.73	1.38
exp.	^{90}Zr					-8.71	
NL3	^{66}Ni	3.96	3.76	3.82	0.20	-8.74	1.47
TW	^{66}Ni	3.89	3.74	3.81	0.15	-8.56	1.63
DDME1	^{66}Ni	3.93	3.77	3.84	0.16	-8.72	1.49
exp.	^{66}Ni					-8.74	

Conclusions II

- We have improved previous calculations by considered the full solution of the Dirac equation and used an exact calculation for the scattered electron wavefunction.

- The model dependence of the electron scattering asymmetry is confirmed by the more exact calculation, although it is still very hard to be extracted from small momentum transfer data.

- If a linear relation between the neutron skin thickness and the proton-neutron asymmetry of the considered nuclei is really to be satisfied, the DDHM model parametrizations provide results within the appropriate range while the NL3 results for very asymmetric nuclei are outside the upper limit imposed by present data.

Obrigada !