High energy diffractive $\gamma\gamma$ scattering

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Summary

- Introduction
- Formalism: BFKL Pomeron
- Results: $\gamma\gamma$
- Applications:
  - ultra-peripheral heavy ion collisions
  - e^+e^- collisions
- Forthcoming studies
  - Higher conformal spins
  - Virtual photons
  - NLO BFKL
  - Polarization, helicity and non-colinear quarks
- Conclusions

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Introduction

- Understanding of high energy hadron processes from a fundamental perspective within Quantum Chromodynamics (QCD)
- Regge limit ($s \gg |t|$) in QCD $\Rightarrow$ Lipatov *et al.* $\Rightarrow$ **QCD Pomeron**, described by BFKL equation

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- Regge limit ($s \gg |t|$) in QCD $\Rightarrow$ Lipatov *et al.* $\Rightarrow$ **QCD Pomeron**, described by BFKL equation
- off-shell photon scattering at high energy in $e^+ e^-$ colliders $\Rightarrow$ advantage: not involve a non-perturbative target
- Vector meson pairs production in $\gamma\gamma$ collisions (Ginzburg *et al.*) as a test of BFKL Pomeron.
- Perturbative calculation for heavy mesons. Light meson production only in the case of large momentum transfer.

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- Vector meson pairs production in $\gamma\gamma$ collisions (Ginzburg *et al.*) as a test of BFKL Pomeron.
- Perturbative calculation for heavy mesons. Light meson production only in the case of large momentum transfer.
- $\gamma\gamma \rightarrow V_1 V_2$ in BFKL framework (V.G. & W.S., *Eur. Phys. J.* **C44**(2005) 515) with $V_i = \rho, \omega, \phi, J/\Psi, \Upsilon$ vector mesons in the large t exchange $\Rightarrow$ large rapidity gap in the final state.
- Successful description of meson photo-production in HERA with the BFKL formalism (Enberg, Motyka, Poludniowsky), even in the case of light mesons.
- Applicability on another processes with photon scattering: ultra peripheral heavy ion collisions, e^+e^- collisions.

Formalism

- Scattering amplitude, differential cross section and total cross section:

$$\mathrm{Im}\,\mathcal{A}(s,t) = \frac{16\pi}{9t^2}\mathcal{F}(z,\tau), \quad \frac{d\sigma}{dt} = \frac{1}{16\pi}|\mathcal{A}(s,t)|^2, \quad \sigma(\gamma\gamma \to V_1V_2) = \int_{|t|_{min}}^{\infty} d|t| \; \frac{d\sigma(\gamma\gamma \to V_1V_2)}{d|t|}$$
(1)

$$with\ z = \frac{3\alpha_s}{2\pi}\ln\left(\frac{s}{\Lambda^2}\right),\ \tau = \frac{|t|}{M_V^2 + Q_\gamma^2}$$

where M_V is the mass of the vector meson, Q_γ^2 is the photon virtuality and Λ^2 is a characteristic scale related to M_V^2 and $|t|$

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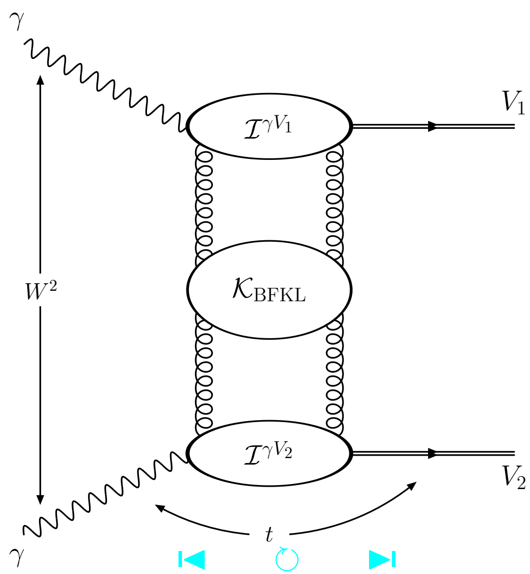
- In this work $Q_\gamma = 0$ and in LLA Λ is arbitrary (see below) and α_s is constant.



Formalism

- BFKL amplitude in LLA approximation and lowest conformal spin $(n = 0)$

(2) $$\mathcal{F}_{\mathrm{BFKL}}(z,\tau) = \frac{t^2}{(2\pi)^3} \int d\nu \frac{\nu^2}{(\nu^2 + 1/4)^2} e^{\chi(\nu) z} I_\nu^{\gamma V_1}(Q_\perp) I_\nu^{\gamma V_2}(Q_\perp)^*$$

where

$$t = -Q_\perp^2, \quad \chi(\nu) = 4\mathcal{R}\mathrm{e}\left(\psi(1) - \psi\left(\frac{1}{2} + i\nu\right)\right)$$

and

$$I_\nu^{\gamma V_i}(Q_\perp) = \int \frac{d^2 k_\perp}{(2\pi)^2} \mathcal{I}_{\gamma V_i}(k_\perp, Q_\perp) \int d^2\rho_1 d^2\rho_2 \left(\frac{(\rho_1 - \rho_2)^2}{\rho_1^2 \rho_2^2}\right)^{1/2 + i\nu} e^{ik_\perp \cdot \rho_1 + i(Q_\perp - k_\perp)\cdot \rho_2}$$

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- The impact factors $\mathcal{I}_{\gamma V_i}$ in pQCD (leading terms) are

(3) $$\mathcal{I}_{\gamma V_i} = \frac{\mathcal{C}_i\alpha_s}{2}\left(\frac{1}{\bar{q}^2} - \frac{1}{q_\parallel^2 + k_\perp^2}\right)$$

where

$$\mathcal{C}_i^2 = \frac{3\Gamma_{ee}^{V_i} M_{V_i}^3}{\alpha_{\rm em}}, \quad \bar{q}^2 = q_\parallel^2 + Q_\perp^2/4, \quad q_\parallel^2 = (Q_\gamma^2 + M_{V_i}^2)/4$$

Formalism & Results: $\gamma\gamma$

$$I_\nu^{\gamma V_i}(Q_\perp) = -\mathcal{C}_i\,\alpha_s \frac{16\pi}{Q_\perp^3}\frac{\Gamma(1/2-i\nu)}{\Gamma(1/2+i\nu)}\left(\frac{Q_\perp^2}{4}\right)^{i\nu}\int_{1/2-i\infty}^{1/2+i\infty}\frac{du}{2\pi i}\left(\frac{Q_\perp^2}{4M_{V_i}^2}\right)^{1/2+u}\Gamma^2(1/2+u)$$

$$\times\frac{\Gamma(1/2-u/2-i\nu/2)\Gamma(1/2-u/2+i\nu/2)}{\Gamma(1/2+u/2-i\nu/2)\Gamma(1/2+u/2+i\nu/2)} \quad (4)$$

- Comparison with Born level (two gluon exchange):

$$\mathcal{F}_{\mathrm{Born}}(s,t) = 2\pi t^2\int\frac{d^2k}{(2\pi)^2}\,\frac{\mathcal{I}_{\gamma V_1}\,\mathcal{I}_{\gamma V_2}}{k_\perp^2(k_\perp - Q_\perp)^2}$$

Note: Non-perturbative gluon propagators contributions in MBG,WS (*Phys. Lett.* **B521** (2001) 259).

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Formalism & Results: $\gamma\gamma$

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Results:

- Choice of the parameters: $\alpha_s = 0.2$, $\Lambda^2 = \beta_1 M_{V_1}^2 + \beta_2 M_{V_2}^2 + \gamma|t|$, with $\beta_1 = \beta_2 = 1/2$
- Combinations with light mesons have a total cross section cut in $|t|_{min} = 1$ GeV^2. With heavy mesons, $|t|_{min} = 0$ GeV^2

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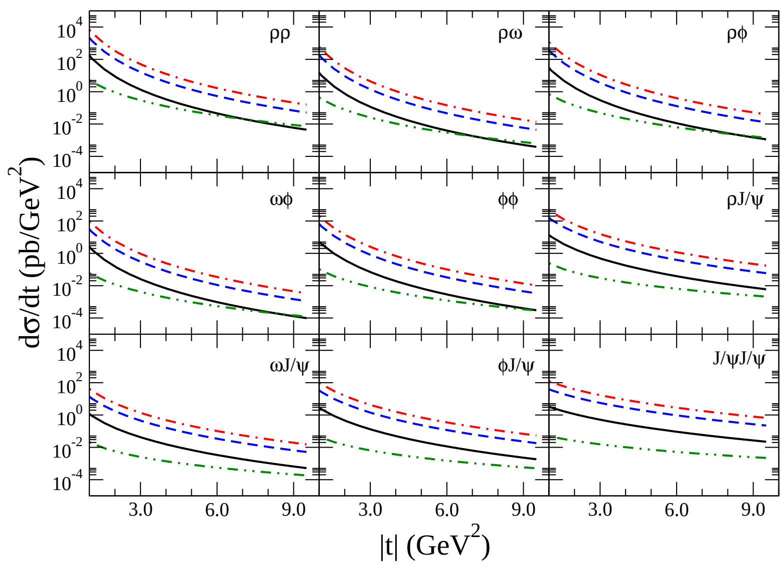
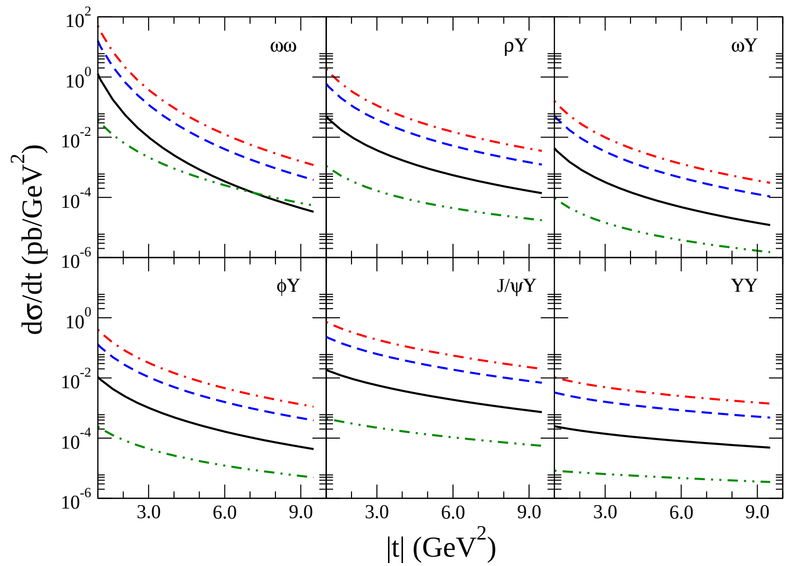
Results: $\gamma\gamma$

- Differential cross section (Solid line: $W = 100$ GeV; dashed line: $W = 500$ GeV; dot-dashed line: $W = 1000$ GeV; dot-dot-dashed: Born level)





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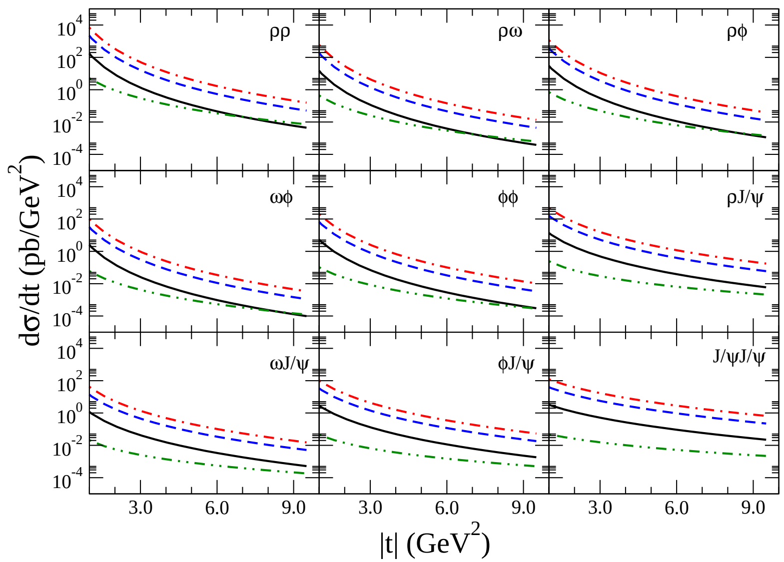
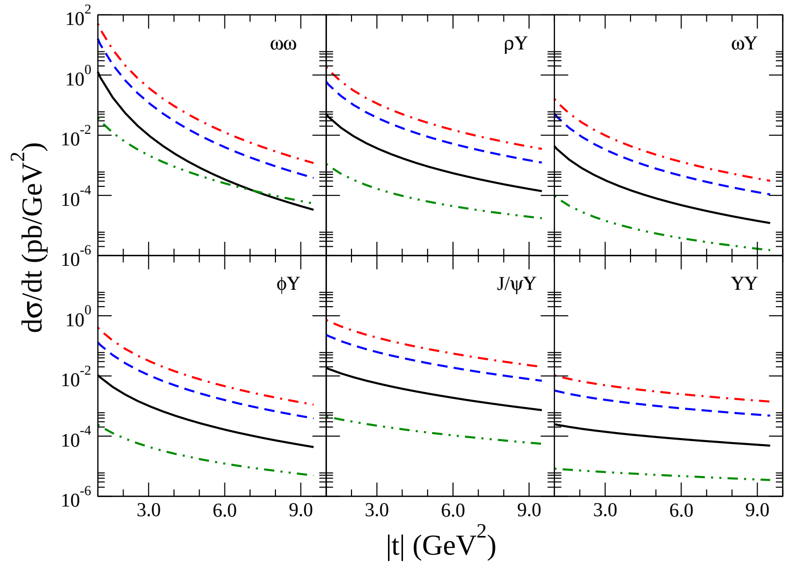
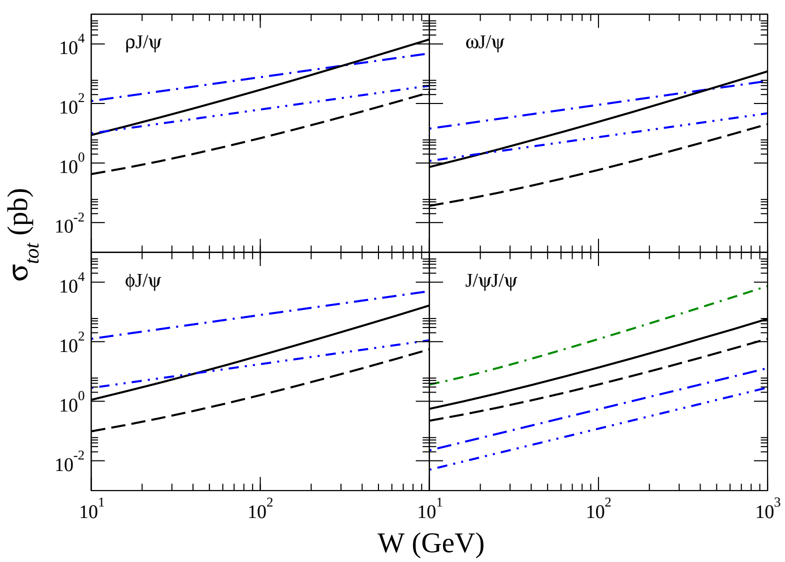


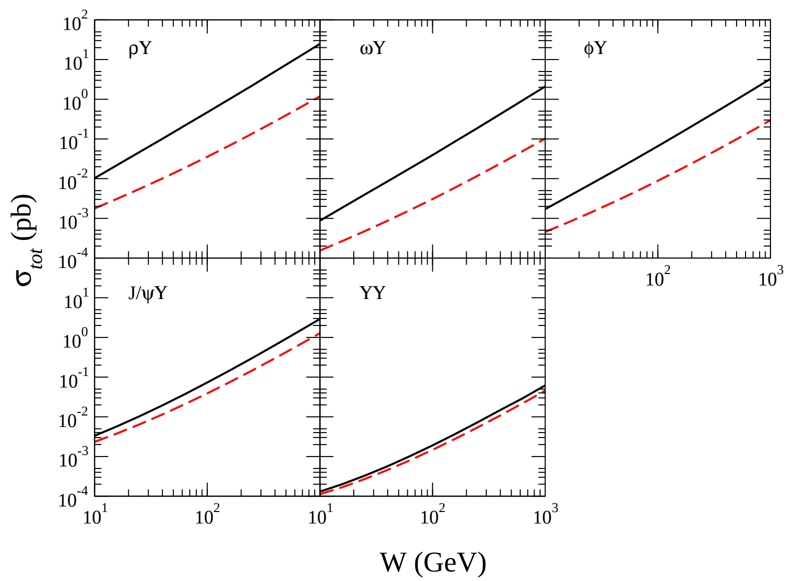


- $d\sigma/dt$ behavior: decrease when the masses increase; become flatter
- The inclusion of BFKL machinery result: growth of the cross section and a different t-behavior; steeper t-dependence in comparison with the Born calculation.

Results: $\gamma\gamma$

- Total cross sections. Left: Solid line: BFKL ($|t|_{min} = 0$ GeV^2); dashed line: BFKL ($|t|_{min} = 1$ GeV^2); dot-dashed line: FELIX ($|t|_{min} = 0$ GeV^2); dot-dot-dashed line: FELIX ($|t|_{min} = 1$ GeV^2); dot-dashed-dashed line BFKL-fit from FELIX. Right: solid line: BFKL ($|t|_{min} = 0$ GeV^2); dashed line: BFKL ($|t|_{min} = 1$ GeV^2).





Results: $\gamma\gamma$

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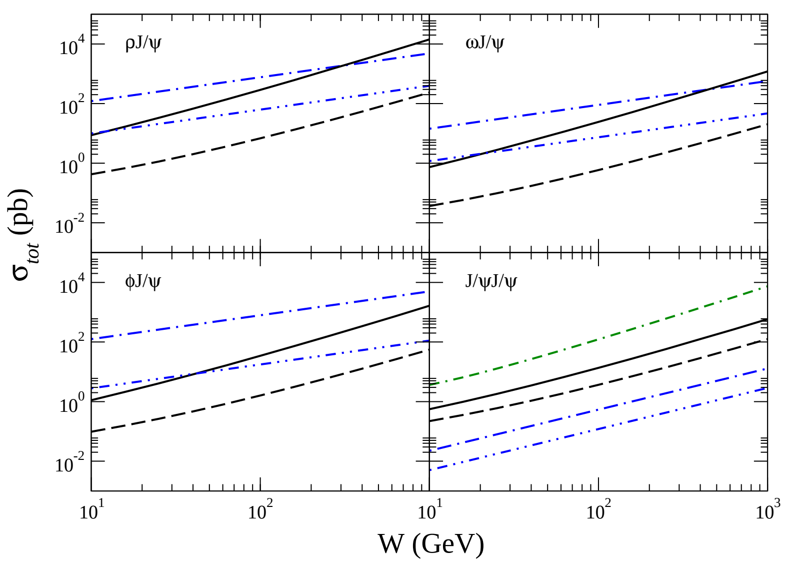

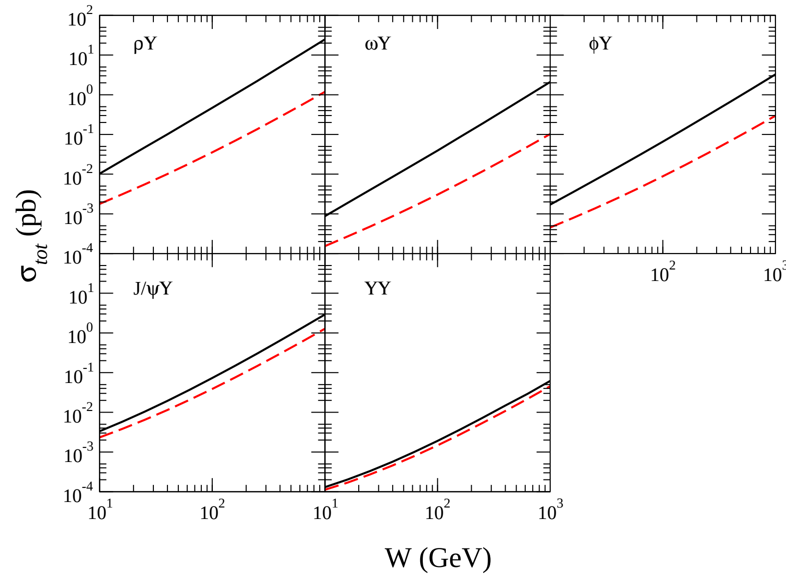

- $\sigma_{\rm tot}$: decrease when mass increase; small difference when mass increase between $|t|_{min} = 0$ and 1.
- BFKL effects: strongly increases with the energy $\Rightarrow \sigma_{\rm tot} \approx W^{\lambda}$, $|t_{min}| = 0 \rightarrow \lambda \approx 1.6$, $|t_{min}| = 1 \rightarrow \lambda \approx 1.4$

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Results: $\gamma\gamma$

- Comparison with another attempts:
  - Ginzburg *et al.*: Born level $\Rightarrow$ present results agree for light and heavy mesons.
  - Gonçalves & Machado: small-t approx., ansatz for the slope and approx. solution of BFKL eq. $\Rightarrow$ similar W dep., $\neq$ normalization. NLO correction gives a softer W dep.
  - FELIX Coll.: Pomeron exchange factorization $\Rightarrow$ $\neq$ normalization & energy dep. $\Rightarrow$ violation of this factorization, to be tested in future colliders.
  - Dosch *et al.*: Stochastic Vacuum model needs energy dependence *by hand* $\Rightarrow$ BFKL have steeper dependence. Heavy mesons: agree in low energy, but not in high W due the parametrization.
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- Estimations of number of events in new linear colliders (number of events per second)

	TESLA	CLIC	ILC
$\rho\rho$	8.0	2.0	28.0
$\rho J/\Psi$	28.0	5.0	100.0
$J/\Psi J/\Psi$	2.0	0.4	8.0
$\Upsilon\Upsilon$	0.0002	0.00004	0.0007

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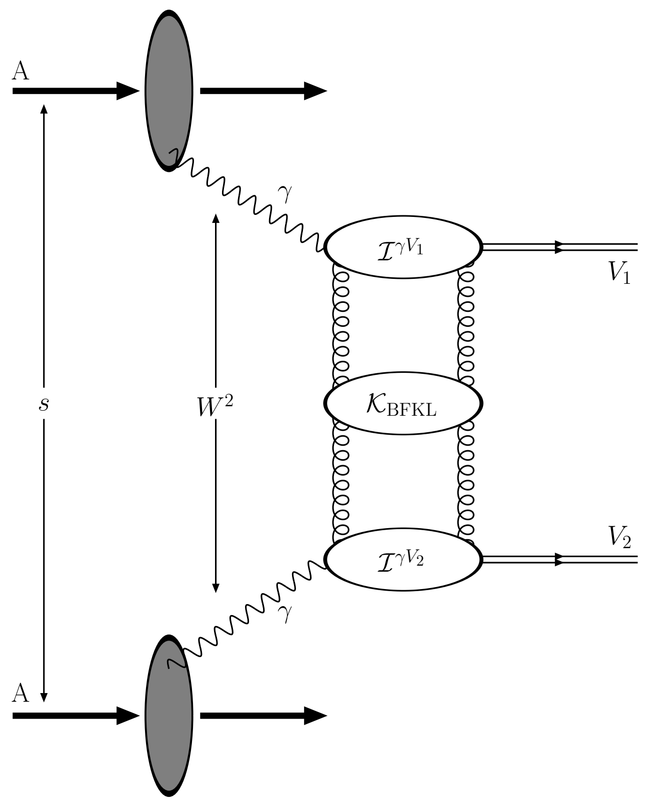
- The number may be consider a lower bound in number of events $\Rightarrow$ The process could be used to constraint the QCD dynamics.

Applications: ultra-peripheral heavy ion collisions

- Long range electromagnetic interaction $\Rightarrow \sigma \approx Z^4\alpha^4$.
- Total cross-section in factorized form,

$$\sigma_{AA\to AA\,V_1\,V_2}(s) = \int d\tau\, \frac{d\mathcal{L}_{\gamma\gamma}}{d\tau}\, \hat{\sigma}_{\gamma\gamma\to V_1\,V_2}(\hat{s})$$

where $d\mathcal{L}_{\gamma\gamma}/d\tau$ is the $\gamma\gamma$ luminosity (Cahn/Jackson)



Applications: ultra-peripheral heavy ion collisions

Results:

Cross section in nb (events/mouth)

$t_{min} = 1\ GeV^2$

$V_1 V_2$	RHIC		LHC	
	Si	Au	Ca	Pb
$\rho\rho$	10^{-2} (0.42)	1.2 (0.2)	4.3 (2×10^3)	432 (181)
$\omega\omega$	6×10^{-5} (3×10^{-3})	8×10^{-3} (10^{-3})	0.03 (15)	3 (1.2)
$\phi\phi$	2×10^{-3} (0.01)	0.02 (4×10^{-3})	0.14 (74)	14 (6)
$\rho\omega$	7×10^{-4} (0.03)	0.1 (0.02)	0.3 (174)	34 (14)
$\rho\phi$	10^{-3} (0.06)	0.16 (0.03)	0.7 (378)	73 (31)
$\omega\phi$	6×10^{-4} (0.03)	0.013 (3×10^{-3})	0.06 (32)	6 (3)

$t_{min} = 0\ GeV^2$

$V_1 V_2$	RHIC		LHC	
	Si	Au	Ca	Pb
$\rho\rho$	3.2 (140)	340 (68)	2821 (10^6)	2×10^5 (10^5)
$\omega\omega$	0.02 (1)	2 (0.5)	20 (10^5)	1.6×10^3 (690)
$\phi\phi$	0.02 (0.9)	1.6 (0.3)	20 (10^5)	1.6×10^3 (704)
$\rho\omega$	0.5 (23)	55 (11)	500 (2.10^5)	4×10^5 (1.6×10^5)
$\rho\phi$	0.2 (10)	22 (4.5)	230 (10^5)	2×10^5 (1.6×10^5)
$\omega\phi$	0.02 (0.9)	2 (0.4)	19 (10^5)	1.6×10^3 (667)

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Applications: ultra-peripheral heavy ion collisions

- large difference $\Rightarrow$ contributions of small t
- huge light meson production at LHC energies

Light-heavy mesons

$V_1 V_2$	RHIC		LHC	
	Si	Au	Ca	Pb
$\rho J/\Psi$	8×10^{-3} (0.34)	0.25 (0.05)	15 (7×10^{3})	1200 (504)
$\omega J/\Psi$	6×10^{-4} (0.03)	0.02 (4×10^{-3})	1.3 (657)	101 (42)
$\phi J/\Psi$	4×10^{-3} (0.18)	0.08 (0.01)	2×10^{4} (10^{7})	8×10^{4} (3×10^{4})
$\rho \Upsilon$	10^{-6} (5×10^{-5})	3×10^{-6} (6×10^{-7})	0.02 (10)	1.3 (0.5)
$\omega \Upsilon$	10^{-7} (4×10^{-6})	2×10^{-7} (5×10^{-8})	10^{-3} (0.85)	0.1 (0.04)
$\phi \Upsilon$	2×10^{-7} (8×10^{-6})	4×10^{-7} (10^{-7})	2×10^{-3} (1.5)	0.2 (0.08)

Heavy-heavy mesons

	HEAVY ION	$J/\Psi\ J/\Psi$	$J/\Psi \Upsilon$	$\Upsilon\Upsilon$
RHIC	SiSi	2×10^{-3} (10^{-2})	1.6×10^{-7} (7×10^{-6})	8×10^{-10} (4×10^{-8})
	AuAu	2×10^{-3} (5×10^{-4})	2×10^{-7} (3×10^{-8})	6×10^{-11} (10^{-11})
LHC	CaCa	0.74 (387)	3×10^{-3} (1.8)	8×10^{-5} (0.05)
	PbPb	61 (26)	0.25 (0.1)	5×10^{-3} (2×10^{-3})

- Decrease of cross-section with the increase of the meson mass
- Increase of the cross sections in comparison with the Born level $\Rightarrow$ QCD Pomeron effects

Applications: e^+e^- collisions

- Simplicity of the initial state
- Observable in future linear colliders (TESLA,CLIC,ILC)
  - photon emission from lepton beams $\Rightarrow$ Weizsäcker-Williams (WW) energy distribution
  - electron beam into photon beam by backscattering of photons off an intense laser beam
- Cross section

$$\sigma_{e^+e^-\to e^+e^-V_1V_2}(\sqrt{s_{ee}}) = \int_0^1 dx_a \int_0^1 dx_b \, \Theta(\hat{s}-\hat{s}_{min}) f_{\gamma/e}(x_a) f_{\gamma/e}(x_b) \sigma_{\gamma\gamma\to V_1V_2}(\hat{s})$$

with the WW energy spectrum

$$f_{\gamma/e}(x) = \frac{\alpha_{\rm em}}{2\pi}\left\{\frac{1+(1-x)^2}{x}\ln\left(\frac{Q^2_{max}}{Q^2_{min}}\right) - 2m^2x\left[\frac{1}{Q^2_{min}} - \frac{1}{Q^2_{max}}\right]\right\}$$

where α_{em} is the EM coupling constant, $x = E_\gamma/E$, $Q^2_{min} = (m^2x^2)/(1-x)$, $Q^2_{max} = E^2(1-x)\theta^2_{max}$ and

$$\sigma_{\gamma\gamma\to V_1V_2}(\hat{s}) = \int_{t_{min}}^{\infty} dt \, \frac{d\sigma_{\gamma\gamma\to V_1V_2}}{dt}(\hat{s})$$

Applications: e^+e^- collisions

Results:

$|t|_{min} = 0$ and $\theta_{max} = 30$ mrad, BFKL Pomeron (Two-gluon). Cross sections in pb.

	$\sqrt{s_{ee}} = 200$ GeV	$\sqrt{s_{ee}} = 500$ GeV	$\sqrt{s_{ee}} = 1000$ GeV	$\sqrt{s_{ee}} = 3000$ GeV
$\rho J/\Psi$	0.90 (0.015)	5.80 (0.049)	21.87 (0.097)	178.19 (0.22)
$\phi J/\Psi$	0.11 (0.0023)	0.69 (0.0073)	2.58 (0.014)	20.77 (0.033)
$\omega J/\Psi$	0.075 (0.0013)	0.48 (0.0041)	1.85 (0.0081)	15.03 (0.019)
$J/\Psi J/\Psi$	0.045 (0.0021)	0.27 (0.0066)	0.98 (0.012)	7.56 (0.031)
$\rho\Upsilon$	0.0013 (0.000055)	0.0093 (0.00017)	0.036 (0.00034)	0.31 (0.00080)
$\omega\Upsilon$	0.00011 (0.0000055)	0.00078 (0.000017)	0.0030 (0.000034)	0.026 (0.000080)
$\phi\Upsilon$	0.0002 (0.000011)	0.0013 (0.000034)	0.0050 (0.000068)	0.040 (0.00016)
$J/\Psi\Upsilon$	0.00025 (0.000027)	0.0015 (0.000086)	0.0052 (0.00017)	0.038 (0.00040)
$\Upsilon\Upsilon$	0.0000072 (0.0000014)	0.000038 (0.0000045)	0.00012 (0.0000088)	0.0008 (0.000020)

$|t|_{min} = 1$ GeV^2 and $\theta_{max} = 30$ mrad, BFKL Pomeron (Two-gluon). Cross sections in pb

	$\sqrt{s_{ee}} = 200$ GeV	$\sqrt{s_{ee}} = 500$ GeV	$\sqrt{s_{ee}} = 1000$ GeV	$\sqrt{s_{ee}} = 3000$ GeV
$\rho\rho$	0.18 (0.035)	1.03 (0.11)	3.60 (0.21)	26.62 (0.51)
$\rho\phi$	0.033 (0.0053)	0.19 (0.016)	0.66 (0.032)	4.87 (0.077)
$\rho\omega$	0.015 (0.0030)	0.088 (0.0093)	0.309 (0.018)	2.28 (0.043)
$\phi\phi$	0.0067 (0.00084)	0.038 (0.0026)	0.130 (0.0051)	0.951 (0.012)
$\phi\omega$	0.0029 (0.00044)	0.016 (0.0013)	0.057 (0.0027)	0.41 (0.0064)
$\omega\omega$	0.0013 (0.00025)	0.007 (0.00080)	0.026 (0.0025)	0.19 (0.0036)

Applications: e^+e^- collisions

Number of events per year at TESLA, CLIC and ILC ($\sqrt{s_{ee}}$ = 500 GeV and $\theta_{max} = 30$ mrad).

	TESLA	CLIC	ILC
$\rho\rho$	350200.0	206000.0	226600.0
$\rho J/\Psi$	1972000.0	1160000.0	1276000.0
$J/\Psi J/\Psi$	91800.0	54000.0	59400.0
$\Upsilon\Upsilon$	12.92	7.6	8.4

- Comparison with previous results:
  - Kwiecinski/Motyka, full BFKL with IR cut and NLO BFKL with kinematic cuts: similar results
  - Ziaja/Motyka, gluon distribution in $\rho J/\psi$
- Results are conservative (not include the soft Pomeron contribution)
  $\Rightarrow$ bound for real production

Forthcoming studies

- Higher conformal spins
  - BFKL kernel have conformal (Möbius) transformation symmetry
  - Fourier series solution for the equation $\Rightarrow$ summation of eigenfunctions $\Rightarrow$ usually only the $n = 0$ is consider $\Rightarrow$ $n \neq 0$ is sub-dominant $\Rightarrow$ small differences.

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  - same calculation $\Rightarrow$ modification in the energy scale
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Forthcoming studies

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- NLO BFKL
  - hard task to include next leading order contributions (BFKL $t = 0$: 1977 $\Rightarrow$ $t \neq 0$: 1986, NLO BFKL 1998) $\Rightarrow$ ***Large*** contributions
  - Use a kinematic veto (Kwiecinski/Motyka)
  - Problems on energy scale choice (renormalization schemes: $\overline{\mathrm{MS}}$, MOM) $\Rightarrow$ BLM scheme $\Rightarrow$ used in $\gamma^*\gamma^* \to \mathrm{hadrons}$ with LEP & LEPII data
  - previous calculations (Brodsky & The Russian Gang) use NLO BFKL kernel with LO photon impact factors $\Rightarrow$ a ***true*** calculation includes the NLO impact factors $\Rightarrow$ Ivanov, Kotsky, Papa: ***HUGE*** expressions

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Forthcoming studies

- Helicity contributions (Enberg, Motyka, Poludniowski)
  - more general impact factor $\Rightarrow$ polarization (odd, even) helicity amplitudes
  - include more complete meson wave functions
  - include conformal spins
  - complex analytical structure
  - describes the DESY data (HERA, ZEUS) in meson photo-production

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Conclusions

- Probe QCD dynamics in a new & unexplored kinematic regime
- $\gamma\gamma$ processes have a important rôle in background contributions in other processes.
- $\gamma\gamma$ processes can constrain the QCD dynamics in more clear process.
- Future photon colliders can be experimentally check this predictions.

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A Quick *introduction to BFKL Pomeron*

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- In Regge limit ($s \gg |t|$), exchange in t channel: $\mathcal{A}(s,t) \propto s^{\alpha(t)}, \; \alpha(t) = \alpha(0) + \alpha' \, t$

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- ***Result:*** gluon ladder with uncrossed rungs, vertical gluons *"reggeized"*, non-local vertexes, strong ordering of the momenta along the ladder.
- Integral equation for the Mellin transform of the scattering amplitude: **B**alitskiĭ-**F**adin-**K**uraev-**L**ipatov Equation.

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- Differences between the BFKL eq. solutions and the Regge theory: cut in the amplitude instead poles; large dominant singularity $\Rightarrow$ violation of the Froissart-Martin limit: $\sigma_{\rm tot} \lesssim \ln^2 s$;
- BFKL eq. as a diffusion eq.: IR divergences
- Solutions: NLO BFKL, unitarity corrections,...

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