

O Pomeron na QCD

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Outline

O Pomeron

Propostas para QCD

Pomeron da pQCD

Espalhamento quark-quark

Escada de Glúons Mais Simples

Octeto de Cor

Conclusões

Referências

O que é o Pomeron?

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- Teoria de Regge:

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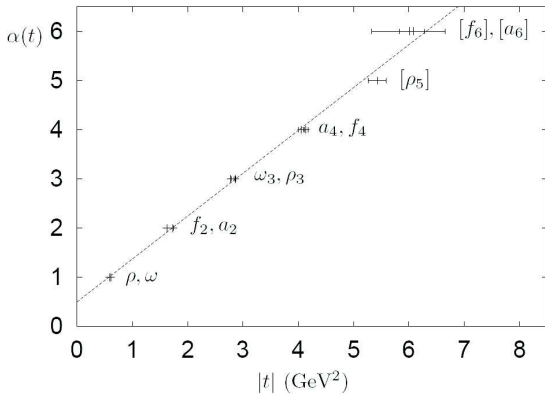
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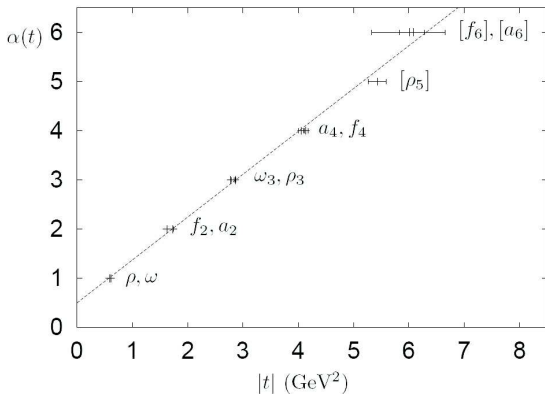
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⇒ Limite de **Froissart-Martin**: $\sigma_{tot} \leq C \ln^2 s$

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Trajatórias Mesônicas



Reggeon $\mathbb{R} \rightarrow$ Números quânticos do vácuo!

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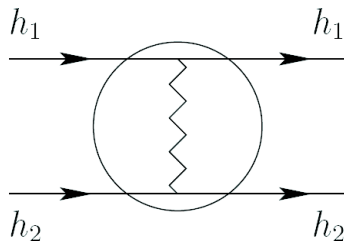
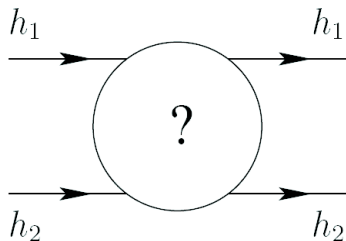
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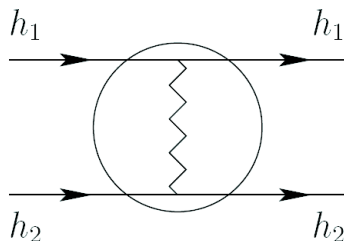
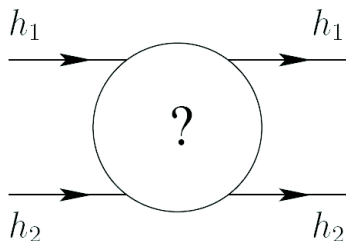
Processo Elásticos/Difrativos \rightarrow Números Quânticos do Vácuo!

A Física antes da QCD

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pré-QCD \Rightarrow Espalhamento Hádron-Hádron

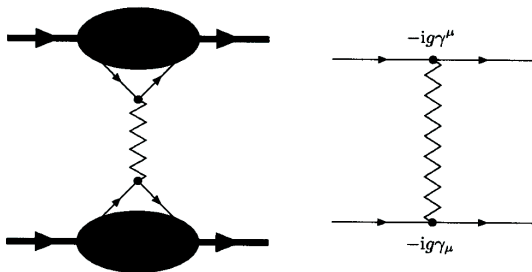
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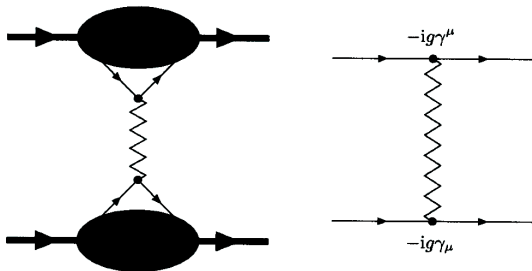
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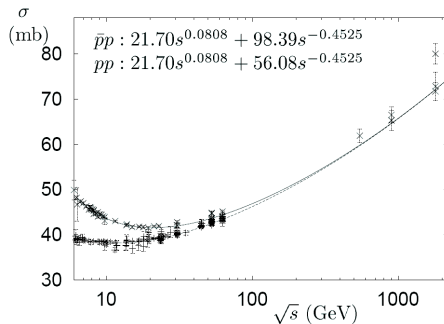
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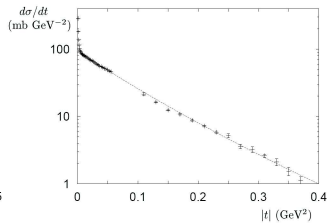
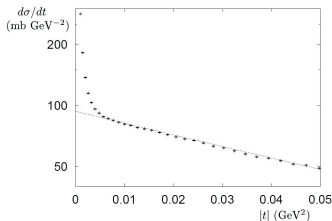
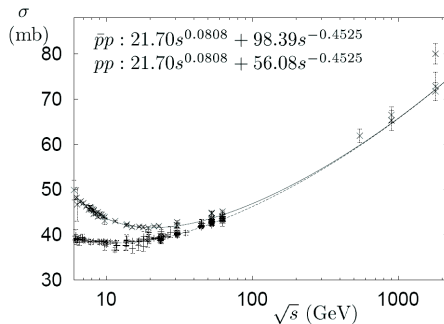
$$\frac{d\sigma_{el}^{pp}}{dt} = \frac{g^4 [3F_1]^4}{4\pi \text{sen}^2\left(\frac{\pi\alpha_P(t)}{2}\right)} \left(\frac{s}{s_0}\right)^{2\alpha_P(t)-2} \begin{cases} \alpha_P(0) = 1.08 \\ \alpha'_P = 0.25 \text{ GeV}^{-2} \\ g^4 = 3.21 \text{ GeV}^{-2} \end{cases}$$

[Donnachie, Landshoff](#) (ZPC 2, 55, 1979; NPB 244, 322, 1984)

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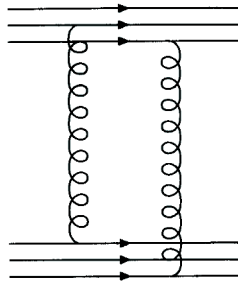
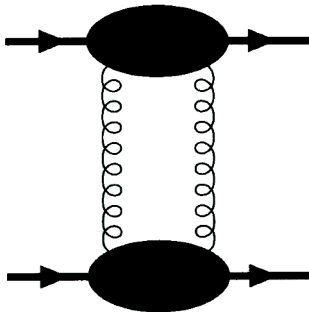


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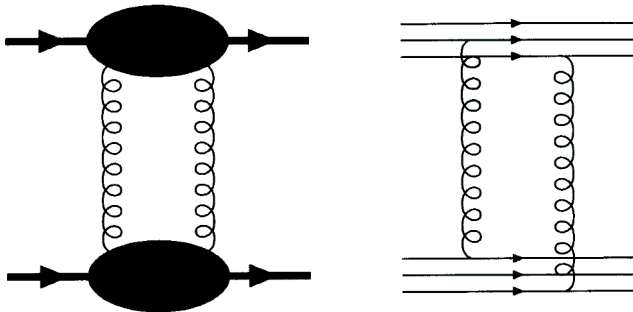
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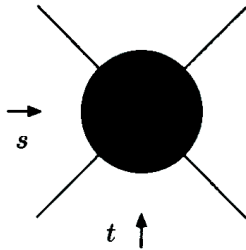
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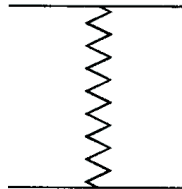
$$\text{LN} \xrightarrow{\text{n-pQCD}} \text{LP}$$

QCD Perturbativa

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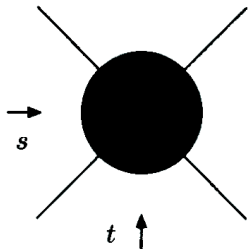
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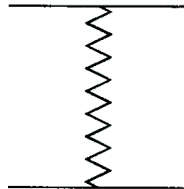
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Espalhamento quark-quark \Rightarrow Pomeron: $s \gg |t|$

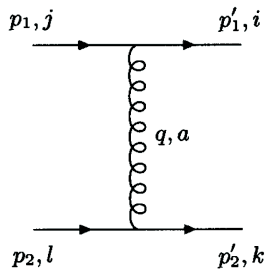
Troca de um glúon

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Mínimo **2** glúons \Rightarrow Pomeron sem carga de cor.

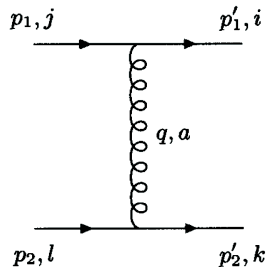
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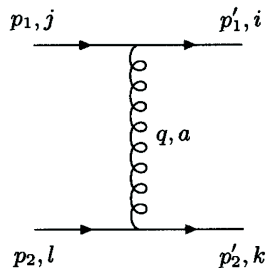


\Rightarrow Parametrização de Sudakov:

$$q = \alpha p_1 + \beta p_2 + q_{\perp} \quad \left\{ \begin{array}{l} q_{\perp} = (0, \mathbf{q}, 0) \\ s = 2(p_1 \cdot p_2) \end{array} \right.$$

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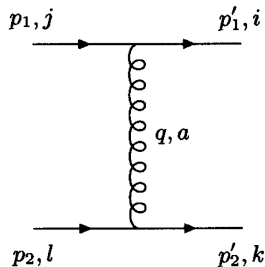
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\Rightarrow Condição de camada de massa:

$$\left. \begin{array}{l} (p_1 - q)^2 = -(1 - \alpha)\beta s - \mathbf{q}^2 = 0 \\ (p_2 + q)^2 = \alpha(1 + \beta)s - \mathbf{q}^2 = 0 \end{array} \right\} \quad \left. \begin{array}{l} \alpha = |\beta| \simeq \frac{\mathbf{q}^2}{s} \ll 1 \\ \mathbf{q}^2 \simeq -\mathbf{q}^2 \end{array} \right.$$

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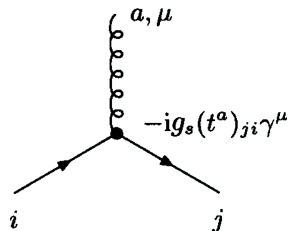
$$\frac{(t_{ij}^a t_{kl}^a)(t_{ij}^b t_{kl}^b)^*}{N_c^2} = \frac{\text{Tr}(t^a t^b) \text{Tr}(t^a t^b)}{N_c^2} = \frac{N_c^2 - 1}{4N_c^2} = \frac{2}{9}$$

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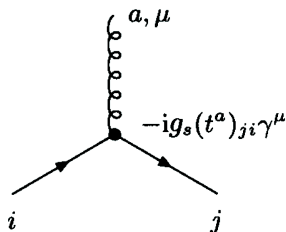
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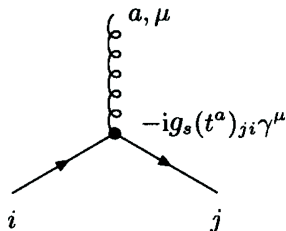
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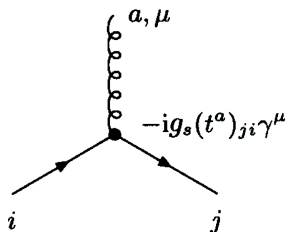
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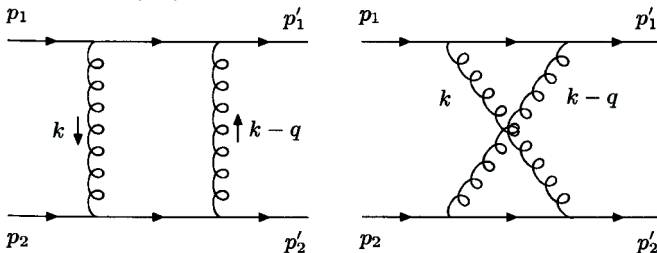
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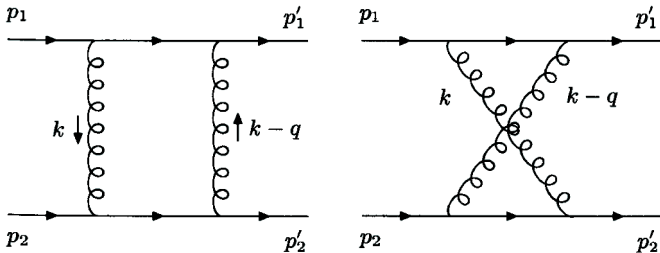
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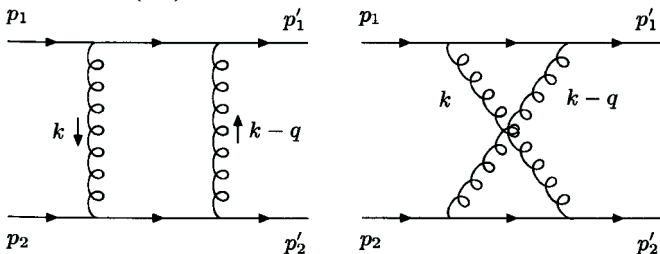
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- Regas de Cutkosky: $t \equiv q^2$ (JMP 1, 429, 1960)

$$\text{Im}\mathcal{A}^{(1)}(s, t) = \frac{1}{2} \int d\Pi_2 \mathcal{A}^{(0)}(s, k^2) \mathcal{A}^{(0)\dagger}(s, [k - q]^2)$$

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- Integral do Espaço de Fase para **2 corpos**:

$$\begin{aligned}\int d\Pi_2 &= \int \frac{d^4\kappa_1}{(2\pi)^3} \frac{d^4\kappa_2}{(2\pi)^3} \delta(\kappa_1^2) \delta(\kappa_2^2) (2\pi)^4 \delta(p_1 + p_2 - \kappa_1 - \kappa_2) \\ &= \int \frac{d^4k}{(2\pi)^2} \delta([p_1 - k]^2) \delta([p_2 + k]^2)\end{aligned}$$

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⇒ **Variáveis de Sudakov**:

$$k = \alpha p_1 + \beta p_2 + k_\perp \quad , \quad d^4k = \left(\frac{s}{2}\right) d\alpha d\beta d^2\mathbf{k}$$

Espaço de Fase

- Integral do Espaço de Fase para **2 corpos**:

$$\begin{aligned} \int d\Pi_2 &= \int \frac{d^4\kappa_1}{(2\pi)^3} \frac{d^4\kappa_2}{(2\pi)^3} \delta(\kappa_1^2) \delta(\kappa_2^2) (2\pi)^4 \delta(p_1 + p_2 - \kappa_1 - \kappa_2) \\ &= \int \frac{d^4k}{(2\pi)^2} \delta([p_1 - k]^2) \delta([p_2 + k]^2) \end{aligned}$$

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⇒ **Integral + Sudakov**:

$$\int d\Pi_2 = \frac{s}{8\pi^2} \int d\alpha d\beta d^2\mathbf{k} \delta(-\beta[1-\alpha]s + \mathbf{k}^2) \delta(\alpha[1+\beta]s - \mathbf{k}^2)$$

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$$k = -\left(\frac{\mathbf{k}^2}{s}\right) p_1 + \left(\frac{\mathbf{k}^2}{s}\right) p_2 + \mathbf{k}$$

Diagrama Quadrado

Diagrama Quadrado

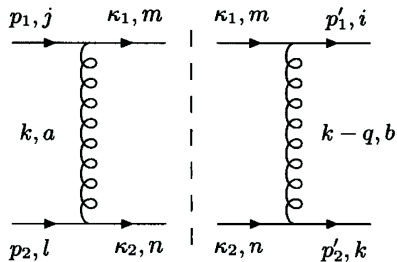
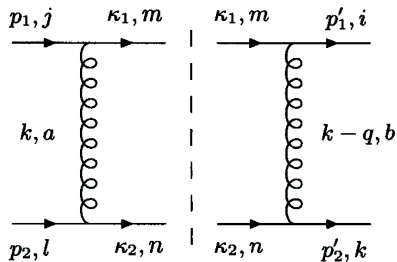


Diagrama Quadrado

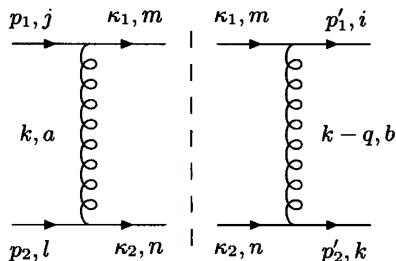


⇒ Amplitudes:

$$A^{(0)}(s, k^2) = -8\pi\alpha_s(t_{mj}^a t_{nl}^a) \left[\frac{s}{\mathbf{k}^2} \right]$$

$$A^{(0)\dagger}(s, (k-q)^2) = -8\pi\alpha_s(t_{mi}^b t_{nk}^b)^* \left[\frac{s}{(\mathbf{k}-\mathbf{q})^2} \right]$$

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⇒ Amplitude Imaginária:

$$\text{Im}\mathcal{A}_a^{(1)}(s, t) = 4\alpha_s^2(t^a t^b)_{ij}(t^a t^b)_{kl} s \int d^2\mathbf{k} \left[\frac{1}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^2} \right]$$

Amplitude Total

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⇒ Relação de Dispersão → ordem dominante em $\ln s$:

$$\mathcal{A}_a^{(1)}(s, t) = - \left(\frac{4\alpha_s^2}{\pi} \right) (t^a t^b)_{ij} (t^a t^b)_{kl} s \ln \left(\frac{s}{t} \right) \int \frac{d^2 \mathbf{k}}{\mathbf{k}^2 (\mathbf{k} - \mathbf{q})^2}$$

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⇒ Reescrevendo:

$$\mathcal{A}_a^{(1)}(s, t) = - \left(\frac{16\pi\alpha_s}{N_c} \right) (t^a t^b)_{ij} (t^a t^b)_{kl} \left(\frac{s}{t} \right) \ln \left(\frac{s}{t} \right) \epsilon(t)$$

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Divergente em Infravermelho

Quarks Confinados

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- **Consideração Inicial** \Rightarrow quarks na camada de massa

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Diagrama Cruzado

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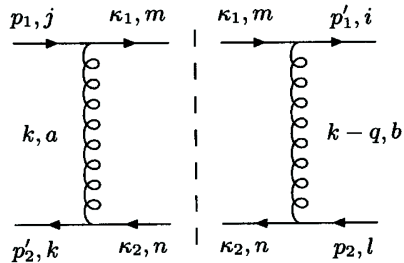
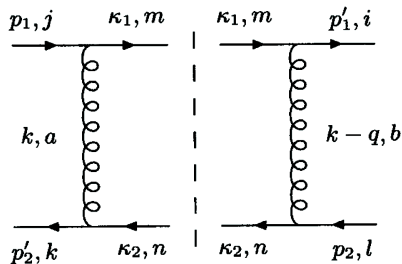


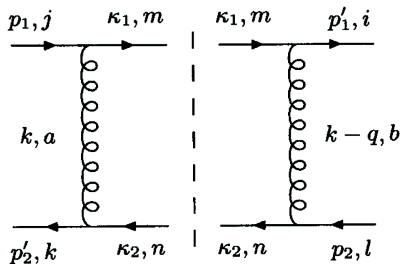
Diagrama Cruzado



⇒ Amplitude Total:

$$\mathcal{A}_b^{(1)}(s, t) = - \left(\frac{16\pi\alpha_2}{N_c} \right) (t^a t^b)_{ij} (t^b t^a)_{kl} \left(\frac{U}{t} \right) \ln \left(\frac{U}{t} \right) \epsilon(t)$$

Diagrama Cruzado



⇒ **Amplitude Total:**

$$\mathcal{A}_b^{(1)}(s, t) = - \left(\frac{16\pi\alpha_2}{N_c} \right) (t^a t^b)_{ij} (t^b t^a)_{kl} \left(\frac{u}{t} \right) \ln \left(\frac{u}{t} \right) \epsilon(t)$$

⇒ **Aproximação de Alta Energia:** $s \simeq -u$

$$\mathcal{A}_b^{(1)}(s, t) = \left(\frac{16\pi\alpha_2}{N_c} \right) (t^a t^b)_{ij} (t^b t^a)_{kl} \left(\frac{s}{t} \right) \ln \left(\frac{s}{|t|} \right) \epsilon(t)$$

Amplitude Total do Processo

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- Somando ambas as contribuições:

$$\begin{aligned}
 \mathcal{A}^{(1)}(s, t) &= \mathcal{A}_a^{(1)}(s, t) + \mathcal{A}_b^{(1)}(s, t) \\
 &= - \left(\frac{16\pi\alpha_s}{N_c} \right) (t^a t^b)_{ij} \left(\frac{s}{t} \right) \\
 &\times \left\{ [t^a, t^b]_{kl} \ln \left(\frac{s}{|t|} \right) - i\pi (t^a t^b)_{kl} \right\} \epsilon(t)
 \end{aligned}$$

Amplitude Total do Processo

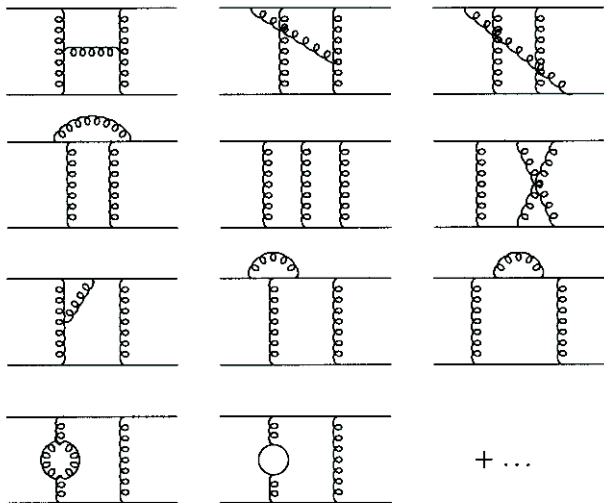
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 \end{aligned}$$

$\text{Im}\mathcal{A}^{(1)}(s, t) \rightarrow$ subdominantes em $\ln s \Rightarrow$ **Singleto de Cor!**

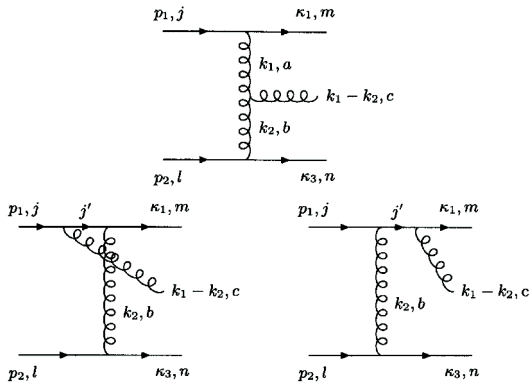
Diagramas de dois Loops

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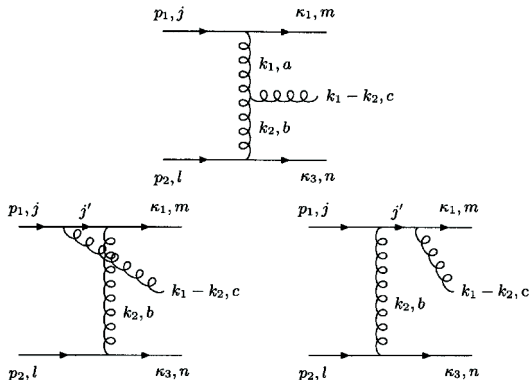


Diagramas de Emissão

Diagramas de Emissão



Diagramas de Emissão



⇒ **Variáveis de Sudakov:**

$$k_1 = \alpha_1 p_1 + \beta_1 p_2 + k_{1\perp}$$

$$k_2 = \alpha_2 p_1 + \beta_2 p_2 + k_{2\perp}$$

Limites Cinemáticos

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- Coeficientes de Sudakov:

$$1 \gg \alpha_1 \gg \alpha_2$$

$$1 \gg |\beta_2| \gg |\beta_1|$$

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$$1 \gg \alpha_1 \gg \alpha_2$$

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- Glúons na camada de massa:

$$\begin{aligned} (k_1 - k_2)^2 &= k_1^2 + k_2^2 - 2(k_1 \cdot k_2) = 0 \\ &= -\mathbf{k}_1^2 - \mathbf{k}_2^2 - \alpha_1\beta_2 s - \alpha_2\beta_1 s + \mathbf{k}_1 \cdot \mathbf{k}_2 = 0 \\ &\simeq -(\mathbf{k}_1 - \mathbf{k}_2)^2 - \alpha_1\beta_2 s = 0 \end{aligned}$$

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⇒ Logo,

$$\begin{aligned} \alpha_1\beta_2s &= -(\mathbf{k}_1 - \mathbf{k}_2)^2 \\ \mathbf{k}_1^2 &\simeq \mathbf{k}_2^2 \simeq \mathbf{q}^2 \end{aligned}$$

Emissão Central

Emissão Central

- Amplitude de Espalhamento:

$$\begin{aligned}
 i\mathcal{A}_{2\rightarrow 3,a}^{\rho} &= (-2ig_s p_1^{\mu}) t_{mj}^a \left(-\frac{i}{k_1^2} \right) \\
 &\times g_s f_{abc} [(k_1 + k_2)^{\rho} g^{\mu\nu} + (k_1 - 2k_2)^{\mu} g^{\nu\rho} + (k_2 - 2k_1)^{\nu} g^{\rho\mu}] \\
 &\times \left(-\frac{i}{k_2^2} \right) (-2ig_s p_2^{\nu}) t_{nl}^b
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 &\times \left(-\frac{i}{k_2^2} \right) (-2ig_s p_2^\nu) t_{nl}^b
 \end{aligned}$$

- Limites Cinemáticos:

$$\mathcal{A}_{2\rightarrow 3,a}^\rho = -2ig_s^3 f_{abc} (t_{mj}^a t_{nl}^b) \left(\frac{1}{\mathbf{k}_1^2 \mathbf{k}_2^2} \right) [\alpha_1 p_1^\rho + \beta_2 p_2^\rho - (k_1^\rho + k_2^\rho)]$$

Emissão Anterior

Emissão Anterior

- Amplitude de Espalhamento:

$$\begin{aligned}
 i\mathcal{A}_{2\rightarrow 3,b}^{\rho} &= (-2ig_s p_1^{\rho}) t_{j'j}^c \left[\frac{i}{(p_1 - k_1 + k_2)^2} \right] (-2ig_s)(p_1^{\mu} - k_1^{\mu} + k_2^{\mu}) t_{mj'}^b \\
 &\times \left(-\frac{i}{k_2^2} \right) (-2ig_s p_{2\mu}) t_{nl}^b
 \end{aligned}$$

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⇒ Limites Cinemáticos:

$$\mathcal{A}_{2\rightarrow 3,b}^{\rho} = -4g_s (t^b t^c)_{mj} t_{nl}^b \left[\frac{1}{\beta_2 s k_2^2} \right] p_1^{\rho}$$

Emissão Posterior

Emissão Posterior

- Amplitude de Espalhamento:

$$\begin{aligned}
 i\mathcal{A}_{2\rightarrow 3,c}^{\rho} &= (-2ig_s p_{1\mu}) t_{j'j}^b \left(-\frac{i}{k_2^2} \right) (-2ig_s p_2^{\mu}) t_{nl}^b \\
 &\times \left[\frac{i}{(p_1 - k_2)^2} \right] (-2ig_s) (p_1^{\rho} - k_2^{\rho}) t_{mj'}^c
 \end{aligned}$$

Emissão Posterior

- Amplitude de Espalhamento:

$$i\mathcal{A}_{2\rightarrow 3,c}^\rho = (-2ig_s p_{1\mu}) t_{j'j}^b \left(-\frac{i}{k_2^2}\right) (-2ig_s p_2^\mu) t_{nl}^b \\ \times \left[\frac{i}{(p_1 - k_2)^2}\right] (-2ig_s)(p_1^\rho - k_2^\rho) t_{mj'}^c$$

- Limites Cinemáticos:

$$\mathcal{A}_{2\rightarrow 3,c}^\rho = 4g_s^3 s f_{abc}(t_{mj}^a t_{nl}^b) \left(\frac{1}{\beta_2 s k_2^2 p_1^\rho}\right)$$

Emissão Posterior

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- Limites Cinemáticos:

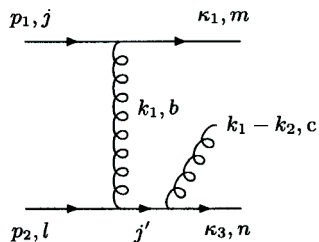
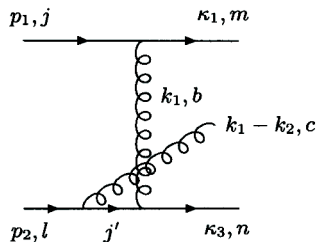
$$\mathcal{A}_{2\rightarrow 3,c}^\rho = 4g_s^3 s f_{abc}(t_{mj}^a t_{nl}^b) \left(\frac{1}{\beta_2 s k_2^2 p_1^\rho}\right)$$

- Amplitude Total de Espalhamento:

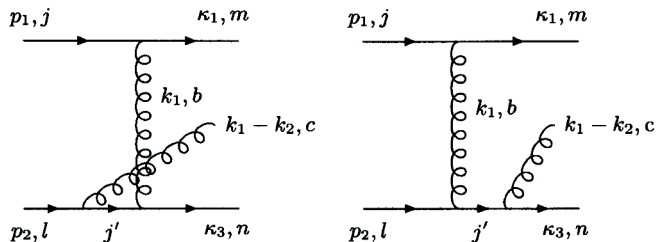
$$[t^b, t^c] = if_{abc} t^a \rightarrow \mathcal{A}_{2\rightarrow 3,b+c}^\rho = -4ig_s^3 s f_{abc}(t_{mj}^a t_{nl}^b) \left(\frac{1}{\beta_2 s k_2^2}\right) p_1^\rho$$

Emissão Inferior

Emissão Inferior



Emissão Inferior



- Amplitude Total de Espalhamento:

$$\mathcal{A}_{2 \rightarrow 3,d+e}^\rho = -4ig_s^3 f_{abc} (t_{mj}^a t_{nl}^b) \left(\frac{1}{\alpha_1 s k_1^2} \right) p_2^\rho$$

Vértice Efetivo

Vértice Efetivo

- Amplitude Total em $\mathcal{O}(g_s^3)$

$$\mathcal{A}_{2 \rightarrow 3}^\rho = -4ig_s^3 \left(\frac{p_1^\mu p_2^\nu}{k_1^2 k_2^2} \right) (t_{mj}^a t_{nl}^b) f_{abc} \Gamma_{\mu\nu}^\rho$$

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- Vértice Efetivo de Lipatov:

$$\Gamma_{\mu\nu}^\rho(k_1, k_2) = \left(\frac{2p_{2\mu} p_{1\nu}}{s} \right) \left[\left(\alpha_1 + \frac{2k_1^2}{\beta_2 s} \right) p_1^\rho + \left(\beta_2 + \frac{2k_2^2}{\alpha_1 s} \right) p_2^\rho - (k_{1\perp}^\rho + k_{2\perp}^\rho) \right]$$

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$$\Rightarrow \text{Invariante de Gauge:} \quad (k_{1\rho} - k_{2\rho}) \Gamma_{\mu\nu}^\rho(k_1, k_2) = 0$$

Diagrama Efetivo

Diagrama Efetivo

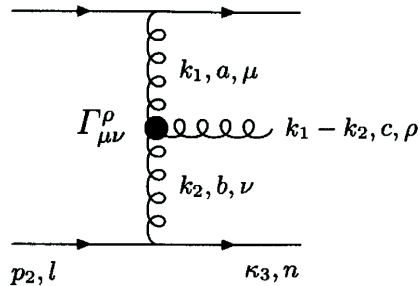
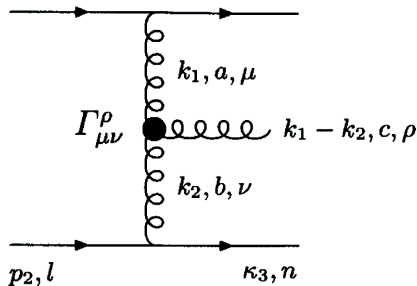


Diagrama Efetivo



- Amplitude Efetiva de Emissão:

$$\begin{aligned}
 i\mathcal{A}_{2\rightarrow 3}^\rho &= (-2ig_s p_1^\mu) t_{mj}^a \left(-\frac{i}{k_1^2}\right) f_{abc} g_s \Gamma_{\mu\nu}^\rho(k_1, k_2) \\
 &\times \left(-\frac{i}{k_2^2}\right) (-2ig_s p_2^\nu) t_{nl}^b
 \end{aligned}$$

Diagrama Efetivo

Diagrama Efetivo

- Amplitude Efetiva Total :

$$\mathcal{A}_{2 \rightarrow 3}^{\rho} = 2i g_s t_{mj}^a \left(\frac{i}{\mathbf{k}_1^2} \right) f_{abc} g_s C^{\rho}(k_1, k_2) \left(\frac{i}{\mathbf{k}_2^2} \right) g_s t_{nl}^b$$

Diagrama Efetivo

- Amplitude Efetiva Total :

$$\mathcal{A}_{2 \rightarrow 3}^{\rho} = 2ig_s t_{mj}^a \left(\frac{i}{\mathbf{k}_1^2} \right) f_{abc} g_s C^{\rho}(k_1, k_2) \left(\frac{i}{\mathbf{k}_2^2} \right) g_s t_{nl}^b$$

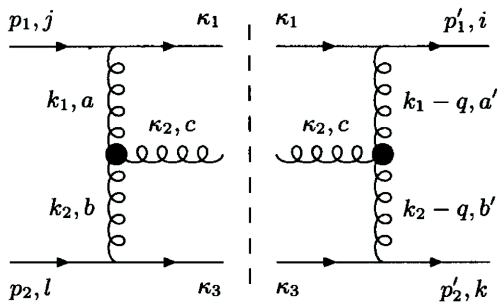
onde

$$\Gamma_{\mu\nu}^{\rho} = \left(\frac{2}{s} \right) p_{2\mu} p_{1\nu} C^{\rho}$$

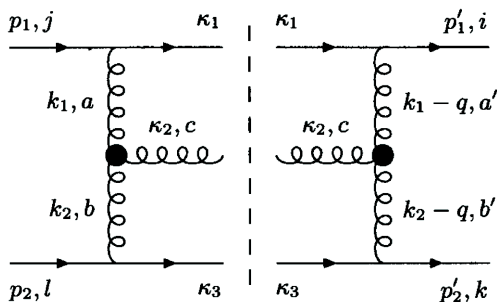
$$C^{\rho} = \left(\frac{2}{s} \right) p_1^{\mu} p_2^{\nu} \Gamma_{\mu\nu}^{\rho}$$

Contribuição de Glúon Real

Contribuição de Glúon Real



Contribuição de Glúon Real



- Amplitude Total \Rightarrow Regras de Cutkosky

$$\text{Im}\mathcal{A}_{\text{real}}^{(2)}(s, t) = -\frac{g_{\rho\sigma}}{2} \int d\Pi_3 \mathcal{A}_{2 \rightarrow 3}^{\rho}(k_1, k_2) \mathcal{A}_{2 \rightarrow 3}^{\sigma\dagger}(k_1 - q, k_2 - q)$$

Espaço de Fase

Espaço de Fase

⇒ Integral do Espaço de Fase de três corpos:

$$\begin{aligned}
 \int d\Pi_3 &= \int \frac{d^4\kappa_1}{(2\pi)^3} \frac{d^4\kappa_2}{(2\pi)^3} \frac{d^4\kappa_3}{(2\pi)^3} \delta(\kappa_1^2) \delta(\kappa_2^2) \delta(\kappa_3^2) (2\pi)^4 \delta^4(p_1 + p_2 - \kappa_1 - \kappa_2 - \kappa_3) \\
 &= \frac{1}{(2\pi)^5} \int d^4\kappa_1 d^4\kappa_3 \delta(\kappa_1^2) \delta(\kappa_3^2) \delta([p_1 + p_2 - \kappa_1 - \kappa_3]^2) \\
 &= \frac{1}{(2\pi)^5} \int d^4k_1 d^4k_2 \delta([p_1 - k_1]^2) \delta([p_2 + k_2]^2) \delta([k_1 - k_2]^2)
 \end{aligned}$$

Espaço de Fase

⇒ Integral do Espaço de Fase de três corpos:

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 \int d\Pi_3 &= \int \frac{d^4\kappa_1}{(2\pi)^3} \frac{d^4\kappa_2}{(2\pi)^3} \frac{d^4\kappa_3}{(2\pi)^3} \delta(\kappa_1^2) \delta(\kappa_2^2) \delta(\kappa_3^2) (2\pi)^4 \delta^4(p_1 + p_2 - \kappa_1 - \kappa_2 - \kappa_3) \\
 &= \frac{1}{(2\pi)^5} \int d^4\kappa_1 d^4\kappa_3 \delta(\kappa_1^2) \delta(\kappa_3^2) \delta([p_1 + p_2 - \kappa_1 - \kappa_3]^2) \\
 &= \frac{1}{(2\pi)^5} \int d^4k_1 d^4k_2 \delta([p_1 - k_1]^2) \delta([p_2 + k_2]^2) \delta([k_1 - k_2]^2)
 \end{aligned}$$

⇒ **Parametrização de Sudakov**:

$$\begin{aligned}
 \int d\Pi_3 &= \frac{s^2}{4(2\pi)^5} \int d\alpha_1 d\beta_1 d^2\mathbf{k}_1 \int d\alpha_2 d\beta_2 d^2\mathbf{k}_2 \\
 &\times \delta(-\beta_1[1 - \alpha_1]s - \mathbf{k}_1^2) \delta(\alpha_2[1 + \beta_2]s - \mathbf{k}_2^2) \\
 &\times \delta([\alpha_1 - \alpha_2][\beta_1 - \beta_2]s - [\mathbf{k}_1 - \mathbf{k}_2]^2)
 \end{aligned}$$

Aproximações

Aproximações

⇒ Aproximações: $1 \gg \alpha_1 \gg \alpha_2$, $1 \gg |\beta_2| \gg |\beta_1|$, $k_i^2 \simeq -\mathbf{k}_i^2$

Aproximações

⇒ Aproximações: $1 \gg \alpha_1 \gg \alpha_2$, $1 \gg |\beta_2| \gg |\beta_1|$, $k_i^2 \simeq -\mathbf{k}_i^2$

$$\begin{aligned}
 \int d\Pi_3 &= \frac{s^2}{4(2\pi)^5} \int d\alpha_1 d\beta_1 d^2\mathbf{k}_1 \int d\alpha_2 d\beta_2 d^2\mathbf{k}_2 \\
 &\times \delta(-\beta_1 s - \mathbf{k}_1^2) \delta(\alpha_2 s - \mathbf{k}_2^2) \delta(-\alpha_1 \beta_2 s - [\mathbf{k}_1 - \mathbf{k}_2]^2) \\
 &= \frac{1}{4(2\pi)^5} \int_{\alpha_2}^1 \frac{d\alpha_1}{\alpha_1} \int_0^1 d\alpha_2 \int d^2\mathbf{k}_1 \int d^2\mathbf{k}_2 \delta(\alpha_2 - \mathbf{k}_2^2) \\
 &= \frac{1}{4(2\pi)^5 s} \int_{\mathbf{q}^2/s}^1 \frac{d\alpha_1}{\alpha_1} \int d^2\mathbf{k}_1 \int d^2\mathbf{k}_2
 \end{aligned}$$

Amplitude Total

Amplitude Total

- Amplitude do lado direito do diagrama:

$$\begin{aligned} \mathcal{A}_{2 \rightarrow 3}^{\rho \dagger} &= -2ig_s t_{im}^{a'} \left[-\frac{i}{(k_1 - q)^2} \right] \\ &\times (-f_{a'bc'} g_s) C^\rho(-[k_1 - q], -[k_2 - q]) \left(\frac{-i}{\mathbf{k}_2^2} \right) g_s t_{kn}^{b'} \end{aligned}$$

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- Amplitude Total:

$$\begin{aligned} \mathcal{A}_{\text{total}} &= \mathcal{A}_{2 \rightarrow 3}^\rho(k_1, k_2) \mathcal{A}_{2 \rightarrow 3, \rho}^\dagger(k_1 - q, k_2 - q) = \\ &= 4g_s^6 s^2 \mathcal{G}_{\text{real}} \left[\frac{C^\rho(k_1, k_2) C_\rho(-k_1 + q, -k_2 + q)}{\mathbf{k}_1^2 \mathbf{k}_2^2 (\mathbf{k}_1 - \mathbf{q})^2 (\mathbf{k}_1 - \mathbf{q})^2} \right] \end{aligned}$$

Amplitude Total

- Amplitude do lado direito do diagrama:

$$\begin{aligned} \mathcal{A}_{2 \rightarrow 3}^{\rho \dagger} &= -2i g_s t_{im}^{a'} \left[-\frac{i}{(k_1 - q)^2} \right] \\ &\times (-f_{a'bc'} g_s) C^\rho(-[k_1 - q], -[k_2 - q]) \left(\frac{-i}{\mathbf{k}_2^2} \right) g_s t_{kn}^{b'} \end{aligned}$$

- Amplitude Total:

$$\begin{aligned} \mathcal{A}_{\text{total}} &= \mathcal{A}_{2 \rightarrow 3}^\rho(k_1, k_2) \mathcal{A}_{2 \rightarrow 3, \rho}^\dagger(k_1 - q, k_2 - q) = \\ &= 4g_s^6 s^2 \mathcal{G}_{\text{real}} \left[\frac{C^\rho(k_1, k_2) C_\rho(-k_1 + q, -k_2 + q)}{\mathbf{k}_1^2 \mathbf{k}_2^2 (\mathbf{k}_1 - \mathbf{q})^2 (\mathbf{k}_1 - \mathbf{q})^2} \right] \end{aligned}$$

$$\Rightarrow \text{Fator de cor: } \mathcal{G}_{\text{real}} = -(t^{a'} t^a)_{ij} (t^{b'} t^b)_{kl} f_{abc} f_{a'b'c}$$

Amplitude Imaginária

Amplitude Imaginária

- $(d\Pi_3) \oplus \mathcal{A}_{\text{total}} e,$

$$\begin{aligned}
 C &= C^\rho(k_1, k_2) C_\rho(-k_1 + q, -k_2 + q) = \\
 &= -2 \left[\mathbf{q}^2 - \frac{\mathbf{k}_1^2(k_2^2 - \mathbf{q})^2}{(\mathbf{k}_1 - \mathbf{k}_2)^2} - \frac{\mathbf{k}_2^2(\mathbf{k}_1 - \mathbf{q})^2}{(\mathbf{k}_1 - \mathbf{k}_2)^2} \right]
 \end{aligned}$$

Amplitude Imaginária

- $(d\Pi_3) \oplus \mathcal{A}_{\text{total}}$ e,

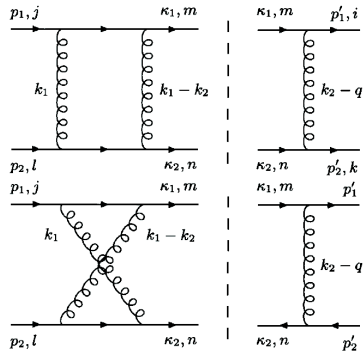
$$\begin{aligned} C &= C^\rho(k_1, k_2) C_\rho(-k_1 + q, -k_2 + q) = \\ &= -2 \left[\mathbf{q}^2 - \frac{\mathbf{k}_1^2(k_2^2 - \mathbf{q})^2}{(\mathbf{k}_1 - \mathbf{k}_2)^2} - \frac{\mathbf{k}_2^2(\mathbf{k}_1 - \mathbf{q})^2}{(\mathbf{k}_1 - \mathbf{k}_2)^2} \right] \end{aligned}$$

obtemos,

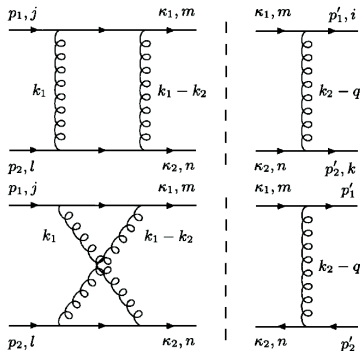
$$\begin{aligned} \text{Im}\mathcal{A}_{\text{real}}^{(2)}(s, t) &= \left(\frac{2\alpha_s^3}{\pi^2} \right) \mathcal{G}_{\text{real}} s \ln \left(\frac{s}{|t|} \right) \int d^2\mathbf{k}_1^2 \int d^2\mathbf{k}_2^2 \\ &\times \left[\frac{\mathbf{q}^2}{\mathbf{k}_1^2 \mathbf{k}_2^2 (\mathbf{k}_1 - \mathbf{q})^2 (\mathbf{k}_2 - \mathbf{q})^2} - \frac{1}{\mathbf{k}_2^2 (\mathbf{k}_1 - \mathbf{q})^2 (\mathbf{k}_1 - \mathbf{k}_2)^2} \right. \\ &\left. - \frac{1}{\mathbf{k}_1^2 (\mathbf{k}_2 - \mathbf{q})^2 (\mathbf{k}_1 - \mathbf{k}_2)^2} \right] \end{aligned}$$

Contribuição Imaginária

Contribuição Imaginária



Contribuição Imaginária



- Amplitude Imaginária:

$$\begin{aligned} \text{Im} \mathcal{A}_{\text{virtual}}^{(2)}(s, t) &= \frac{1}{2} \int d\Pi_2 \mathcal{A}^{(1)}(s, k_2^2) \mathcal{A}^{(0)\dagger}(s, [k_2 - q]^2) \\ &+ \frac{1}{2} \int d\Pi_2 \mathcal{A}^{(0)}(s, k_1^2) \mathcal{A}^{(1)\dagger}(s, [k_1 - q]^2) \end{aligned}$$

Diagrama Quadrado

Diagrama Quadrado

- Amplitude de Espalhamento:

$$\mathcal{A}^{(1)}(s, k_2^2) = 8\pi\alpha_s(t_{mj}^b t_{nl}^b) \left(\frac{s}{k_2^2} \right) \ln \left(\frac{s}{\mathbf{k}_2^2} \right) \epsilon(t)$$

$$\mathcal{A}^{(0)\dagger}(s, [k_2 - q]^2) = 8\pi\alpha_s(t_{mi}^{a*} t_{nk}^{a*}) \left[\frac{s}{(k_2 - q)^2} \right]$$

Diagrama Quadrado

- Amplitude de Espalhamento:

$$\mathcal{A}^{(1)}(s, k_2^2) = 8\pi\alpha_s(t_{mj}^b t_{nl}^b) \left(\frac{s}{k_2^2}\right) \ln\left(\frac{s}{\mathbf{k}_2^2}\right) \epsilon(t)$$

$$\mathcal{A}^{(0)\dagger}(s, [k_2 - q]^2) = 8\pi\alpha_s(t_{mi}^{a*} t_{nk}^{a*}) \left[\frac{s}{(k_2 - q)^2}\right]$$

- Amplitude do subprocesso: $\ln(s/k_2^2) \simeq \ln(s/|t|)$, $\mathcal{G}_{\text{virtual}} = (t^a t^b)_{ij} (t^a t^b)_{kl}$

$$\begin{aligned} \text{Im}\mathcal{A}_{\text{virtual},a}^{(2)}(s, t) &= -\left(\frac{N_c\alpha_s^3}{\pi^2}\right) \mathcal{G}_{\text{virtual}} s \ln\left(\frac{s}{|t|}\right) \\ &\times \int d^2\mathbf{k}_1 \int d^2\mathbf{k}_2 \left[\frac{1}{\mathbf{k}_1^2 (\mathbf{k}_2 - \mathbf{q})^2 (\mathbf{k}_1 - \mathbf{k}_2)^2}\right] \end{aligned}$$

Diagrama Cruzado e Amplitude Total

Diagrama Cruzado e Amplitude Total

- Amplitude do subprocesso:

$$\begin{aligned} \text{Im} \mathcal{A}_{\text{virtual,b}}^{(2)}(s, t) &= - \left(\frac{N_c \alpha_s^3}{\pi^2} \right) \mathcal{G}_{\text{virtual}} s \ln \left(\frac{s}{|t|} \right) \\ &\times \int d^2 \mathbf{k}_1 \int d^2 \mathbf{k}_2 \left[\frac{1}{\mathbf{k}_2^2 (\mathbf{k}_1 - \mathbf{q})^2 (\mathbf{k}_1 - \mathbf{k}_2)^2} \right] \end{aligned}$$

Diagrama Cruzado e Amplitude Total

- Amplitude do subprocesso:

$$\begin{aligned} \text{Im}\mathcal{A}_{\text{virtual,b}}^{(2)}(s, t) &= - \left(\frac{N_c \alpha_s^3}{\pi^2} \right) \mathcal{G}_{\text{virtual}} s \ln \left(\frac{s}{|t|} \right) \\ &\times \int d^2\mathbf{k}_1 \int d^2\mathbf{k}_2 \left[\frac{1}{\mathbf{k}_2^2 (\mathbf{k}_1 - \mathbf{q})^2 (\mathbf{k}_1 - \mathbf{k}_2)^2} \right] \end{aligned}$$

- Amplitude Total do Processo:

$$\begin{aligned} \text{Im}\mathcal{A}_{\text{virtual}}^{(2)}(s, t) &= - \left(\frac{N_c \alpha_s^3}{\pi^2} \right) \mathcal{G}_{\text{virtual}} s \ln \left(\frac{s}{|t|} \right) \int d^2\mathbf{k}_1 \int d^2\mathbf{k}_2 \\ &\times \left[\frac{1}{\mathbf{k}_1^2 (\mathbf{k}_2 - \mathbf{q})^2 (\mathbf{k}_1 - \mathbf{k}_2)^2} + \frac{1}{\mathbf{k}_2^2 (\mathbf{k}_1 - \mathbf{q})^2 (\mathbf{k}_1 - \mathbf{k}_2)^2} \right] \end{aligned}$$

Octeto de Cor

Octeto de Cor

⇒ Contribuição da emissão do glúon real:

$$\begin{aligned} \text{Im} \mathcal{A}_{\underline{8}, \text{real}}^{(2)}(s, t) &= \left(\frac{2\alpha_s^3}{\pi^2} \right) \left[2 \left(\frac{N_c^2}{8} \right) (t_{ij}^a t_{kl}^a) \right] s \ln \left(\frac{s}{|t|} \right) \int d^2 \mathbf{k}_1^2 \int d^2 \mathbf{k}_2^2 \\ &\times \left[\frac{\mathbf{q}^2}{\mathbf{k}_1^2 \mathbf{k}_2^2 (\mathbf{k}_1 - \mathbf{q})^2 (\mathbf{k}_2 - \mathbf{q})^2} - \frac{1}{\mathbf{k}_2^2 (\mathbf{k}_1 - \mathbf{q})^2 (\mathbf{k}_1 - \mathbf{k}_2)^2} \right. \\ &\left. - \frac{1}{\mathbf{k}_1^2 (\mathbf{k}_2 - \mathbf{q})^2 (\mathbf{k}_1 - \mathbf{k}_2)^2} \right] \end{aligned}$$

Octeto de Cor

⇒ Contribuição da emissão do glúon real:

$$\begin{aligned} \text{Im}\mathcal{A}_{\underline{8},\text{real}}^{(2)}(s, t) &= \left(\frac{2\alpha_s^3}{\pi^2}\right) \left[2\left(\frac{N_c^2}{8}\right) (t_{ij}^a t_{kl}^a)\right] s \ln\left(\frac{s}{|t|}\right) \int d^2\mathbf{k}_1^2 \int d^2\mathbf{k}_2^2 \\ &\times \left[\frac{\mathbf{q}^2}{\mathbf{k}_1^2 \mathbf{k}_2^2 (\mathbf{k}_1 - \mathbf{q})^2 (\mathbf{k}_2 - \mathbf{q})^2} - \frac{1}{\mathbf{k}_2^2 (\mathbf{k}_1 - \mathbf{q})^2 (\mathbf{k}_1 - \mathbf{k}_2)^2} \right. \\ &\left. - \frac{1}{\mathbf{k}_1^2 (\mathbf{k}_2 - \mathbf{q})^2 (\mathbf{k}_1 - \mathbf{k}_2)^2} \right] \end{aligned}$$

⇒ Contribuição do glúon virtual:

$$\begin{aligned} \text{Im}\mathcal{A}_{\underline{8},\text{virtual}}^{(2)}(s, t) &= -\left(\frac{N_c \alpha_s^3}{\pi^2}\right) \left[2\left(-\frac{N_c}{4}\right) (t_{ij}^a t_{kl}^a)\right] s \ln\left(\frac{s}{|t|}\right) \int d^2\mathbf{k}_1 \int d^2\mathbf{k}_2 \\ &\times \left[\frac{1}{\mathbf{k}_1^2 (\mathbf{k}_2 - \mathbf{q})^2 (\mathbf{k}_1 - \mathbf{k}_2)^2} + \frac{1}{\mathbf{k}_2^2 (\mathbf{k}_1 - \mathbf{q})^2 (\mathbf{k}_1 - \mathbf{k}_2)^2} \right] \end{aligned}$$

Contribuição Total

Contribuição Total

- Contribuição Total do **octeto de cor** \rightarrow **REAL!**

$$\begin{aligned}
 \text{Im}\mathcal{A}_{\underline{8}}^{(2)}(s, t) &= \text{Im}\mathcal{A}_{\underline{8},\text{real}}^{(2)}(s, t) + \text{Im}\mathcal{A}_{\underline{8},\text{virtual}}^{(2)}(s, t) = \\
 &= \left(\frac{N_c^2 \alpha_s^3}{2\pi^3} \right) (t_{ij}^a t_{kl}^a) s \ln \left(\frac{s}{|t|} \right) \\
 &\times \int d^2\mathbf{k}_1 \int d^2\mathbf{k}_2 \left[\frac{\mathbf{q}^2}{\mathbf{k}_1^2 \mathbf{k}_2^2 (\mathbf{k}_1 - \mathbf{q})^2 (\mathbf{k}_2 - \mathbf{q})^2} \right]
 \end{aligned}$$

Contribuição Total

- Contribuição Total do **octeto de cor** \rightarrow **REAL!**

$$\begin{aligned} \text{Im}\mathcal{A}_{\underline{8}}^{(2)}(s, t) &= \text{Im}\mathcal{A}_{\underline{8},\text{real}}^{(2)}(s, t) + \text{Im}\mathcal{A}_{\underline{8},\text{virtual}}^{(2)}(s, t) = \\ &= \left(\frac{N_c^2 \alpha_s^3}{2\pi^3} \right) (t_{ij}^a t_{kl}^a) s \ln \left(\frac{s}{|t|} \right) \\ &\times \int d^2\mathbf{k}_1 \int d^2\mathbf{k}_2 \left[\frac{\mathbf{q}^2}{\mathbf{k}_1^2 \mathbf{k}_2^2 (\mathbf{k}_1 - \mathbf{q})^2 (\mathbf{k}_2 - \mathbf{q})^2} \right] \end{aligned}$$

$$\text{Im}\mathcal{A}_{\underline{8}}^{(2)}(s, t) = 8\pi^2 \alpha_s (t_{ij}^a t_{kl}^a) \left(\frac{s}{|t|} \right) \ln \left(\frac{s}{|t|} \right) \epsilon^2(t)$$

Contribuição Total

- Contribuição Total do **octeto de cor** \rightarrow **REAL!**

$$\begin{aligned} \text{Im}\mathcal{A}_{\underline{8}}^{(2)}(s, t) &= \text{Im}\mathcal{A}_{\underline{8},\text{real}}^{(2)}(s, t) + \text{Im}\mathcal{A}_{\underline{8},\text{virtual}}^{(2)}(s, t) = \\ &= \left(\frac{N_c^2 \alpha_s^3}{2\pi^3} \right) (t_{ij}^a t_{kl}^a) s \ln \left(\frac{s}{|t|} \right) \\ &\times \int d^2\mathbf{k}_1 \int d^2\mathbf{k}_2 \left[\frac{\mathbf{q}^2}{\mathbf{k}_1^2 \mathbf{k}_2^2 (\mathbf{k}_1 - \mathbf{q})^2 (\mathbf{k}_2 - \mathbf{q})^2} \right] \end{aligned}$$

$$\text{Im}\mathcal{A}_{\underline{8}}^{(2)}(s, t) = 8\pi^2 \alpha_s (t_{ij}^a t_{kl}^a) \left(\frac{s}{|t|} \right) \ln \left(\frac{s}{|t|} \right) \epsilon^2(t)$$

- Amplitude Dominante:

$$\mathcal{A}_{\underline{8}}^{(2)}(s, t) = 4\pi \alpha_s (t_{ij}^a t_{kl}^a) \left(\frac{s}{t} \right) \ln^2 \left(\frac{s}{|t|} \right) \epsilon^2(t)$$

Amplitude em ordem $\mathcal{O}(\alpha_s^3)$

Amplitude em ordem $\mathcal{O}(\alpha_s^3)$

- Amplitude Total do **Octeto de cor** no **Limite LLA**:

$$\begin{aligned} \mathcal{A}_8(s, t) &= 8\pi\alpha_s \left(\frac{s}{t}\right) (t_{ij}^a t_{kl}^a) \\ &\times \left[1 + \epsilon(t) \ln\left(\frac{s}{|t|}\right) + \frac{1}{2}\epsilon^2(t) \ln^2\left(\frac{s}{|t|}\right) + \dots \right] \end{aligned}$$

Amplitude em ordem $\mathcal{O}(\alpha_s^3)$

- Amplitude Total do **Octeto de cor** no **Limite LLA**:

$$\begin{aligned} \mathcal{A}_{\underline{8}}(s, t) &= 8\pi\alpha_s \left(\frac{s}{t}\right) (t_{ij}^a t_{kl}^a) \\ &\times \left[1 + \epsilon(t) \ln\left(\frac{s}{|t|}\right) + \frac{1}{2}\epsilon^2(t) \ln^2\left(\frac{s}{|t|}\right) + \dots \right] \end{aligned}$$

- Expansão:

$$\begin{aligned} \mathcal{A}_{\underline{8}}(s, t) &= 8\pi\alpha_s (t_{ij}^a t_{kl}^a) \left(\frac{s}{t}\right) \left(\frac{s}{|t|}\right)^{\epsilon(t)} \\ &= 8\pi\alpha_s (t_{ij}^a t_{kl}^a) \left(\frac{s}{|t|}\right)^{\alpha_s(t)} \end{aligned}$$

Amplitude em ordem $\mathcal{O}(\alpha_s^3)$

- Amplitude Total do **Octeto de cor** no **Limite LLA**:

$$\begin{aligned} \mathcal{A}_{\underline{8}}(s, t) &= 8\pi\alpha_s \left(\frac{s}{t}\right) (t_{ij}^a t_{kl}^a) \\ &\times \left[1 + \epsilon(t) \ln\left(\frac{s}{|t|}\right) + \frac{1}{2}\epsilon^2(t) \ln^2\left(\frac{s}{|t|}\right) + \dots \right] \end{aligned}$$

- Expansão:

$$\begin{aligned} \mathcal{A}_{\underline{8}}(s, t) &= 8\pi\alpha_s (t_{ij}^a t_{kl}^a) \left(\frac{s}{t}\right) \left(\frac{s}{|t|}\right)^{\epsilon(t)} \\ &= 8\pi\alpha_s (t_{ij}^a t_{kl}^a) \left(\frac{s}{|t|}\right)^{\alpha_s(t)} \end{aligned}$$

$$\alpha_s(t) = 1 + \epsilon(t)$$

Conclusões

Conclusões

- ⇒ Concordância com os dados experimentais;
- ⇒ Análise de processos Difrativos;
- ⇒ Descrição necessária;
- ⇒ Dados de LHC.

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