Geometric scaling in high-energy QCD

nonzero momentum transfer and fluctuation effects

Gregory Soyez

Based on : C. Marquet, R. Peschanski, G.S., hep-ph/0502020, Nucl.Phys.A756:399-418,2005

C. Marquet, G.S., hep-ph/0504080, accepted to Nucl. Phys. A

G.S., hep-ph/0504129

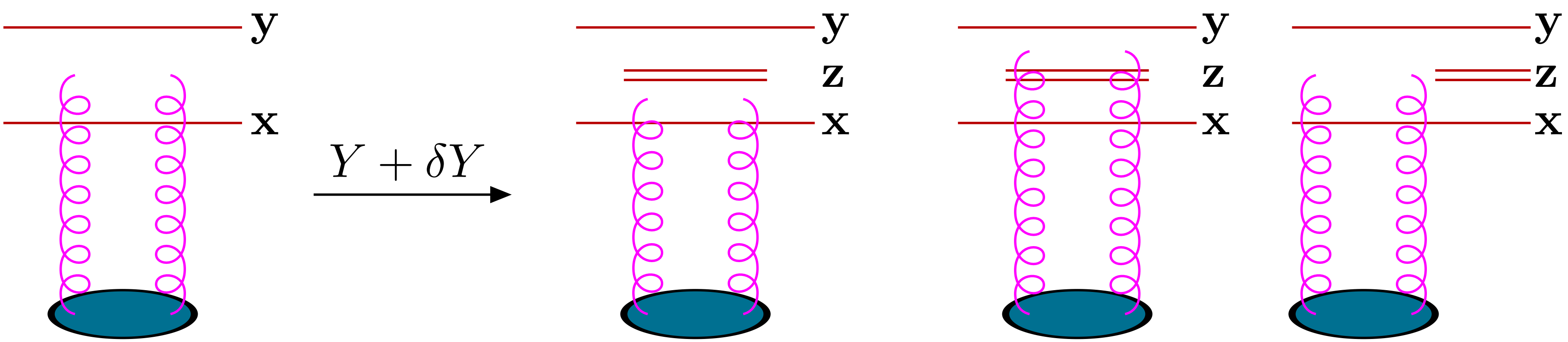
Outline

- Introduction: BFKL and BK equations
- Asymptotic solutions: the b-independent case
  - Diffusive approximation: F-KPP equation
  - Geometric scaling from statistical physics
- Asymptotic solutions: the *full* case
  - Coordinate vs. momentum space
  - BFKL dynamics and geometric scaling
  - Numerical solutions
- Effects of fluctuations
  - Evolution equation: JIMWLK & fluctuations
  - Hierarchy (master eq.) vs. Langevin equation
  - Noise term: probability and front compacity
  - Saturation scale and geometric scaling violations
- Conclusions and perspectives

BFKL and BK evolution

Consider a $q\bar{q}$ dipole at large rapidity $Y = \log(s)$

BFKL: Rapidity increase $\Rightarrow$ Splitting into 2 dipoles



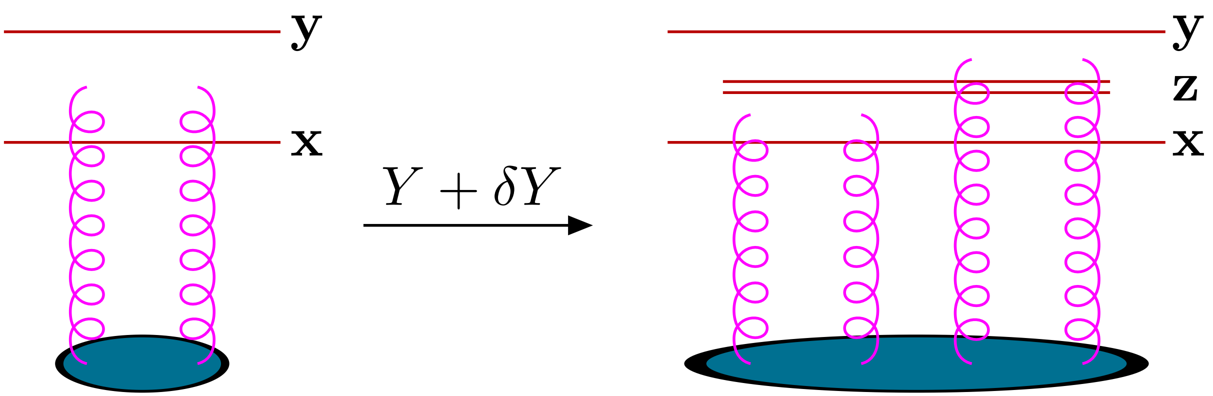
$$\partial_Y T(\mathbf{x},\mathbf{y}) = \bar{\alpha} \int d^2z \, \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2} \left[T(\mathbf{x},\mathbf{z}) + T(\mathbf{z},\mathbf{y}) - T(\mathbf{x},\mathbf{y})\right]$$

Solution: $T \propto e^{\omega Y}$ Violates unitarity $T(x,y) \leq 1$

BFKL and BK evolution

Consider a $q\bar{q}$ dipole at large rapidity $Y = \log(s)$

BK: $T^2 \approx T \approx 1 \ \Rightarrow$ multiple scattering



$$\partial_Y T(\mathbf{x},\mathbf{y}) = \bar{\alpha} \int d^2z \, \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2} \left[T(\mathbf{x},\mathbf{z}) + T(\mathbf{z},\mathbf{y}) - T(\mathbf{x},\mathbf{y}) - T(\mathbf{x},\mathbf{z})T(\mathbf{z},\mathbf{y})\right]$$

Evolution for $\langle T \rangle$:

$$\partial_Y \langle T(\mathbf{x},\mathbf{y})\rangle = \bar{\alpha} \int_z \mathcal{M}_{\mathbf{xyz}} \left[\langle T(\mathbf{x},\mathbf{z})\rangle + \langle T(\mathbf{z},\mathbf{y})\rangle - \langle T(\mathbf{x},\mathbf{y})\rangle - \langle T(\mathbf{x},\mathbf{z}) T(\mathbf{z},\mathbf{y})\rangle\right]$$

Higher-order correlations:

- $\langle T \rangle$, $\langle T^2 \rangle$, ...: JIMWLK/Balitsky equations
- Mean-field approximation: $\langle T^2 \rangle = \langle T \rangle^2$ (BK equation)

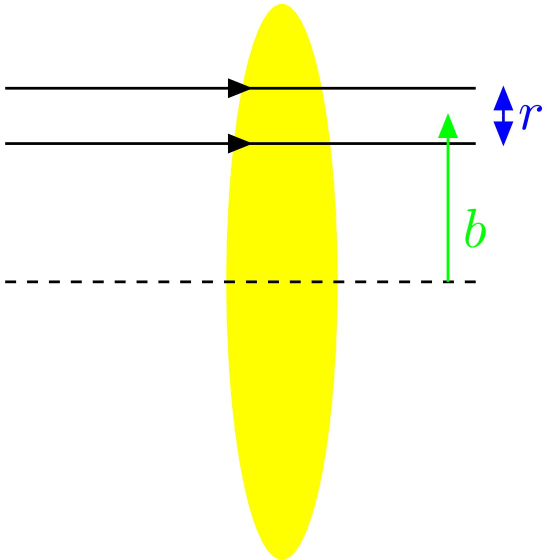
What are the solutions of this equation ?

Asymptotic solutions

The b independent case

The b independent case

We start to study the impact-parameter-independent situation:



$$T(\mathbf{x},\mathbf{y}) \equiv T(\mathbf{r};\mathbf{b}) \longrightarrow T(\mathbf{r})$$

$$T(k) = \int \frac{d^2r}{r^2}\, \mathrm{e}^{i\mathbf{k}.\mathbf{r}}\, T(r)$$

BK equation in momentum space:

$$\partial_Y T(k) = \underbrace{\frac{\bar{\alpha}}{\pi} \int \frac{dp^2}{p^2} \left[\frac{p^2 T(p) - k^2 T(k)}{|k^2 - p^2|} + \frac{k^2 T(k)}{\sqrt{4p^4 + k^4}} \right]}_{\bar{\alpha}\chi(-\partial_L)} - \bar{\alpha} T^2(k)$$

Link with statistical physics (1/2)

[S. Munier, R. Peschanski]

Saddle point/diffusive approximation:

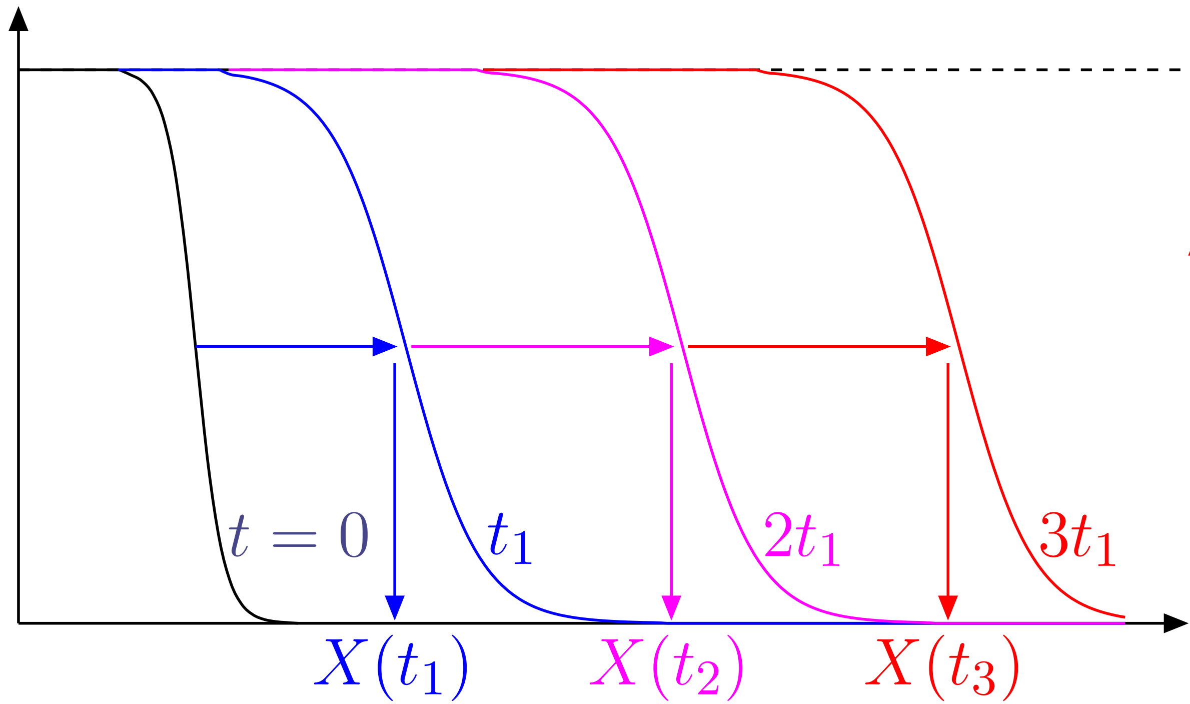
$$\chi(\gamma) = \chi\left(\frac{1}{2}\right) + \frac{1}{2}\chi''\left(\frac{1}{2}\right)\left(\gamma - \frac{1}{2}\right)^2$$

+ linear change of variable: $t = \bar{\alpha}Y$, $x \approx L = \log(k^2/k_0^2)$

$\rightarrow$ Fisher-Kolmogorov-Petrovsky-Piscounov equation

$$\partial_t u(x,t) = \partial_{xx} u(x,t) + u(x,t) - u^2(x,t)$$

Link with statistical physics (2/2)



Asymptotic solution:

traveling wave

$$u(x,t) = u(x - v_c t)$$

Position: $X(t) = X_0 + v_c t$

Asymptotic behaviour (1/3)

Traveling waves under more general conditions:

- 0 is an unstable solution, 1 is a stable solution
- The initial condition is steep enough
- The linear equation admits superposition of waves as solution

$$T_{\text{lin}} = \int_{c-i\infty}^{c+\infty} \frac{d\gamma}{2i\pi} a_0(\gamma) \exp\left[\omega(\gamma)Y - \gamma L\right]$$

Asymptotic behaviour (1/3)

Traveling waves under more general conditions: e.g. QCD

- 0 is an unstable solution, 1 is a stable solution
  BFKL growth, BK damping
- The initial condition is steep enough
  Colour transparency
- The linear equation admits superposition of waves as solution

$$T_{\text{lin}} = \int_{c-i\infty}^{c+\infty} \frac{d\gamma}{2i\pi} a_0(\gamma) \exp\left[\omega(\gamma)Y - \gamma L\right]$$

BFKL: $\omega(\gamma) = \bar{\alpha}\chi(\gamma) = \bar{\alpha}\ \left[2\psi(1) - \psi(\gamma) - \psi(1-\gamma)\right]$

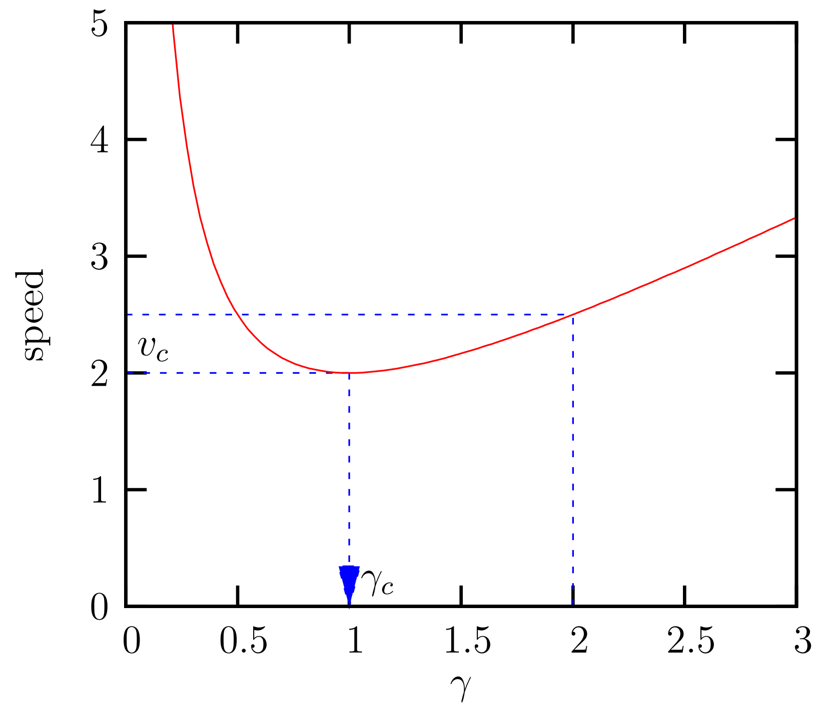
Asymptotic behaviour (2/3)

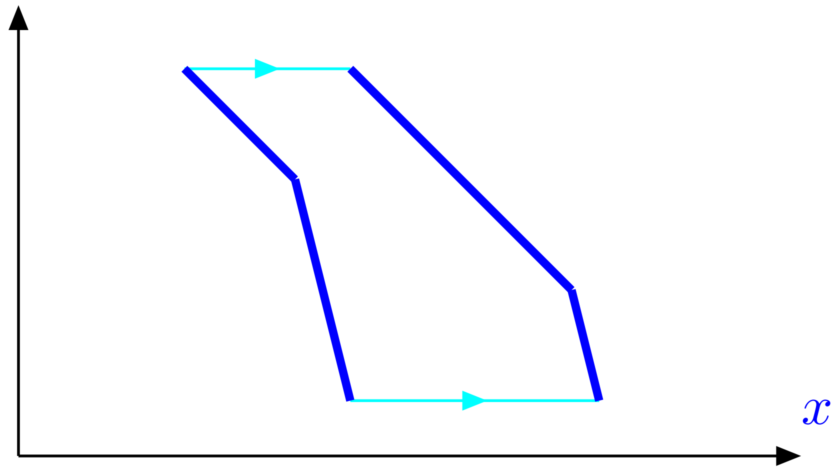
Wave $\sim e^{-\gamma L}$

$\rightarrow$ speed $v(\gamma) = \frac{\omega(\gamma)}{\gamma}$

γ_c, v_c:

$$\omega'(\gamma_c) = \frac{\omega(\gamma_c)}{\gamma_c} = \min \frac{\omega(\gamma)}{\gamma}$$





Asymptotic behaviour (3/3)

For the BK equation:

- non-linearities select γ_c
- Asymptotics from linear BFKL kernel only

  Tail of the front:

$$T(k,Y) = T\left(\frac{k^2}{Q_s^2(Y)}\right) \approx \log\left(\frac{k^2}{Q_s^2(Y)}\right)\left|\frac{k^2}{Q_s^2(Y)}\right|^{-\gamma_c}$$

Saturation Scale:

$$\log(Q_s^2(Y)) = v_c Y - \frac{3}{2\gamma_c}\log(Y)$$

$\Rightarrow$ geometric scaling in terms of $\frac{k^2}{Q_s^2(Y)}$

Asymptotic solutions

The full BK equation

Full BK equation

[C. Marquet, R. Peschanski, G.S]

Question: do we have the same properties for the full BK equation ?

$$\partial_Y T_{\mathbf{xy}} = \frac{\bar{\alpha}}{\pi} \int d^2z \, \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2} \left[T_{\mathbf{xz}} + T_{\mathbf{zy}} - T_{\mathbf{xy}} - T_{\mathbf{xz}}T_{\mathbf{zy}}\right]$$

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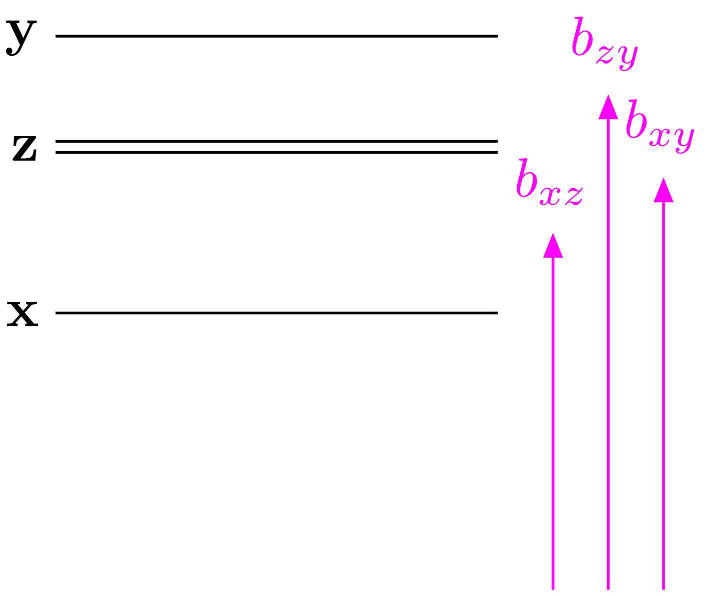
Problem:



Diffusion/non-locality in b

More BFKL dynamics

Solutions of the full BFKL kernel:

$$\tilde{T}_{\text{lin}}(\mathbf{k},\mathbf{q}) = \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2i\pi}\, e^{\bar{\alpha}\chi(\gamma)Y} f^{\gamma}(\mathbf{k},\mathbf{q})\phi_0(\gamma,\mathbf{q})$$

with

$$f^{\gamma}(\mathbf{k},\mathbf{q}) = \frac{\Gamma^2(\gamma)}{\Gamma^2\left(\frac{1}{2}+\gamma\right)}\frac{2}{|k|}\left|\frac{q}{4k}\right|^{2\gamma-1} \underbrace{{}_2\mathrm{F}_1\left(\gamma,\gamma;2\gamma;\frac{q}{k}\right){}_2\mathrm{F}_1\left(\gamma,\gamma;2\gamma;\frac{\bar{q}}{\bar{k}}\right)}_{\longrightarrow 1 \quad \text{when } k \gg q} - (\gamma \to 1-\gamma)$$

$$\Rightarrow \quad \tilde{T}_{\text{lin}}(\mathbf{k},\mathbf{q}) = \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2i\pi}\,\phi_0(\gamma,\mathbf{q})\,\exp\left[\bar{\alpha}\chi(\gamma)Y - \gamma\log\left(\frac{k^2}{q^2}\right)\right]$$

Recall the b-independent case:

$$\tilde{T}_{\text{lin}} = \int_{c-i\infty}^{c+\infty} \frac{d\gamma}{2i\pi} a_0(\gamma)\exp\left[\omega(\gamma)Y - \gamma L\right]$$

Geometric scaling (1/3)

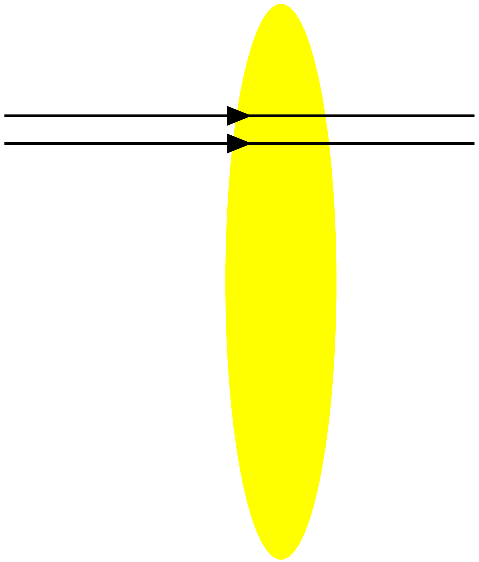
⇒ *geometric scaling for the full BK equation*

Saturation Scale: same Y dependence as previously

$$Q_s^2(Y) \sim q^2 \exp\left[v_c Y - \frac{3}{2\gamma_c}\log(Y) - \frac{3}{\gamma_c^2}\sqrt{\frac{2\pi}{\bar{\alpha}\chi''(\gamma_c)}}\frac{1}{\sqrt{Y}}\right]$$
$$\sim q^2\Omega_s^2(Y)$$

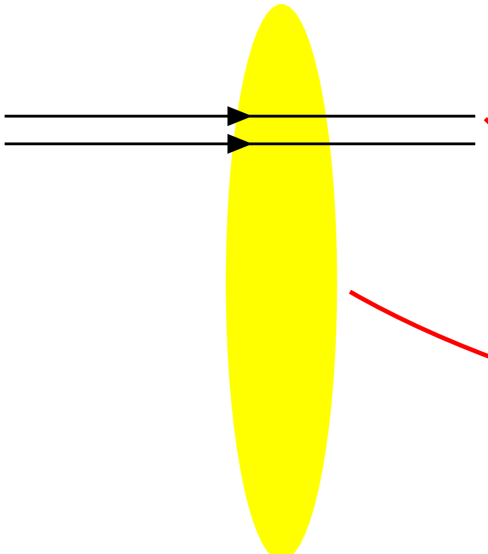
Tail of the front: same slope γ_c

$$T(k,Y) = T\left(\frac{k^2}{q^2\Omega_s^2(Y)}\right) \approx \log\left(\frac{k^2}{q^2\Omega_s^2(Y)}\right)\left|\frac{k^2}{q^2\Omega_s^2(Y)}\right|^{-\gamma_c}$$



$$\tilde{T}_{\text{lin}}(\mathbf{k},\mathbf{q}) = \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2i\pi}\, e^{\bar{\alpha}\chi(\gamma)Y} f^{\gamma}(\mathbf{k},\mathbf{q})\phi_0(\gamma,\mathbf{q})$$

Geometric scaling (2/3)



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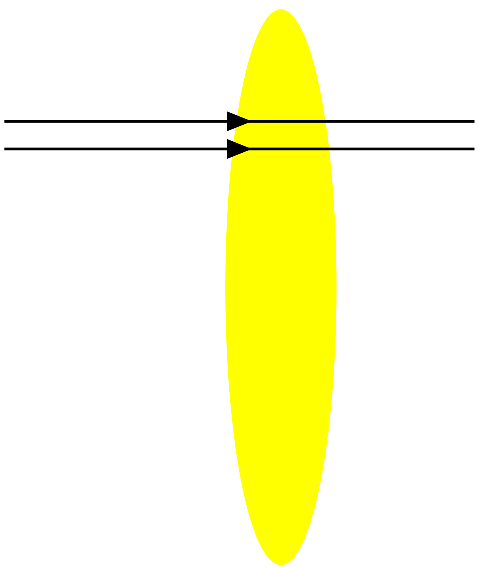
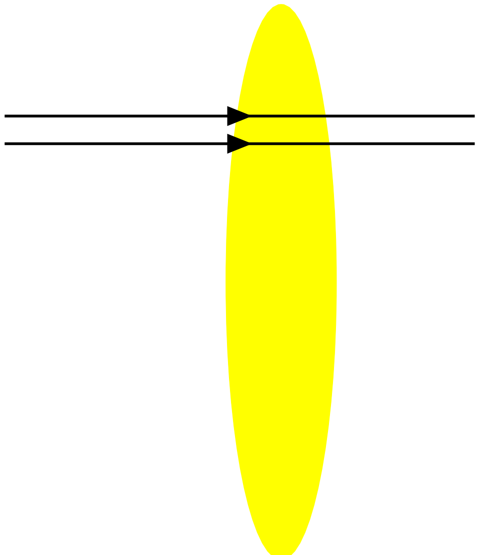


$$\tilde{T}_{\text{lin}}(\mathbf{k},\mathbf{q}) = \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2i\pi}\, e^{\bar{\alpha}\chi(\gamma)Y} f^{\gamma}(\mathbf{k},\mathbf{q})\phi_0(\gamma,\mathbf{q})$$

$$f^{\gamma}(\mathbf{k},\mathbf{q}) \sim \left|\frac{q^2}{k^2}\right|^{\gamma} {}_2\mathrm{F}_1\left(\frac{q}{k}\right) {}_2\mathrm{F}_1\left(\frac{\bar{q}}{\bar{k}}\right) \underset{k\gg q}{\longrightarrow} \left|\frac{q^2}{k^2}\right|^{\gamma}$$

Geometric scaling (2/3)



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$$f^{\gamma}(\mathbf{k},\mathbf{q}) \sim \left|\frac{q^2}{k^2}\right|^{\gamma} {}_2\mathrm{F}_1\left(\frac{q}{k}\right) {}_2\mathrm{F}_1\left(\frac{\bar{q}}{\bar{k}}\right) \underset{k\gg q}{\longrightarrow} \left|\frac{q^2}{k^2}\right|^{\gamma}$$

$$\phi_0(\gamma,\mathbf{q}) \sim \left|\frac{q^2}{Q_T^2}\right|^{1-\gamma} {}_2\mathrm{F}_1\left(\frac{q}{Q_T}\right) {}_2\mathrm{F}_1\left(\frac{\bar{q}}{\bar{Q}_T}\right) \quad \underset{q\gg Q_T}{\longrightarrow} \quad 1$$

$$\underset{Q_T\gg q}{\longrightarrow} \quad \left|\frac{q^2}{Q_T^2}\right|^{1-\gamma}$$

$Q_T \equiv$ caracteristic scale of the target

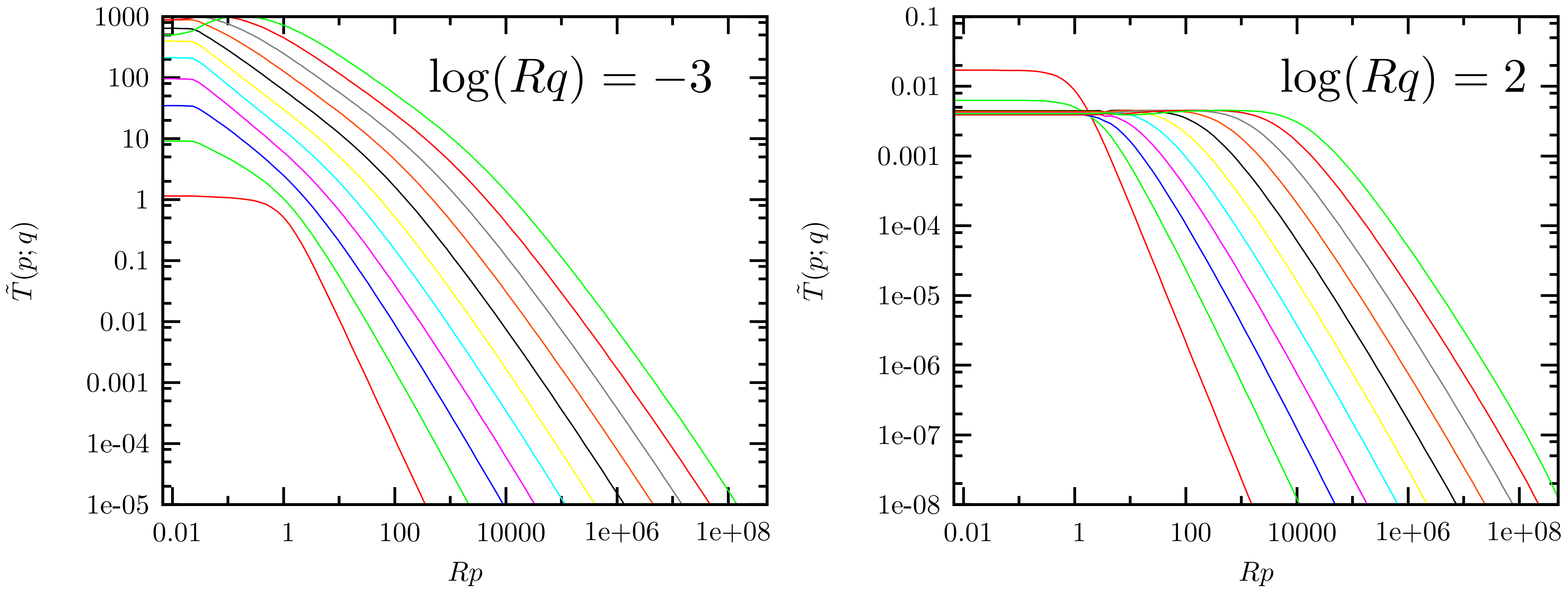
Geometric scaling (3/3)

$Q_T \ll q \ll k$	$q \ll Q_T \ll k$
$f^\gamma(\mathbf{k},\mathbf{q})\phi_0(\gamma,\mathbf{q}) \sim \left\|\frac{k^2}{q^2}\right\|^{-\gamma}$	$f^\gamma(\mathbf{k},\mathbf{q})\phi_0(\gamma,\mathbf{q}) \sim \left\|\frac{k^2}{Q_T^2}\right\|^{-\gamma}$
$Q_s^2 \sim q^2\Omega^2(Y)$	$Q_s^2 \sim Q_T^2\Omega^2(Y)$

- same rapidity dependence
- smooth transition from Q_T to q
- scaling for $k \gg q, Q_T$

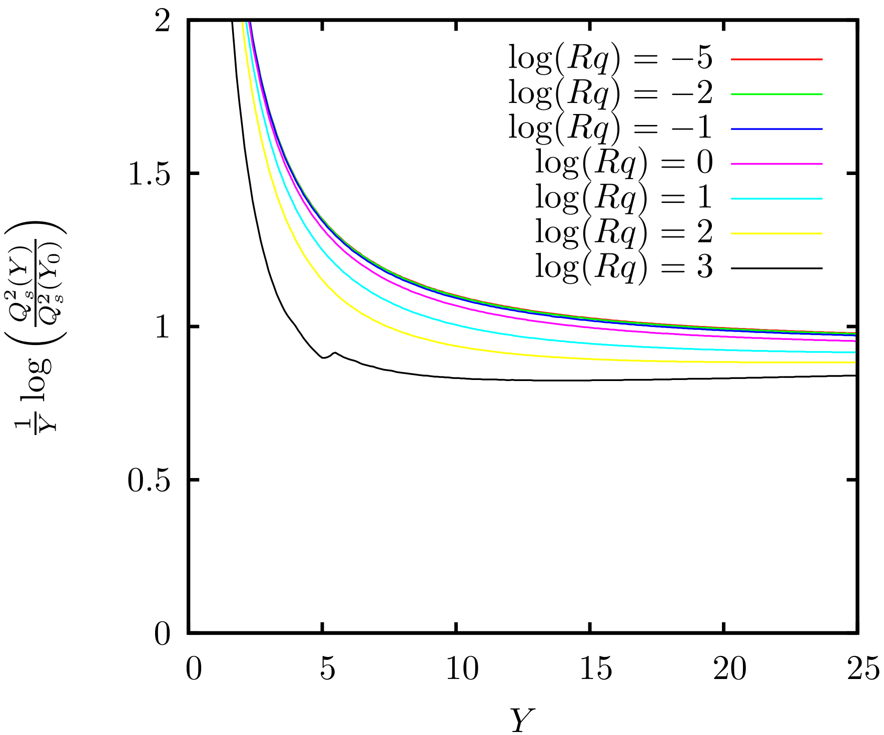
Numerical simulations (1/2)

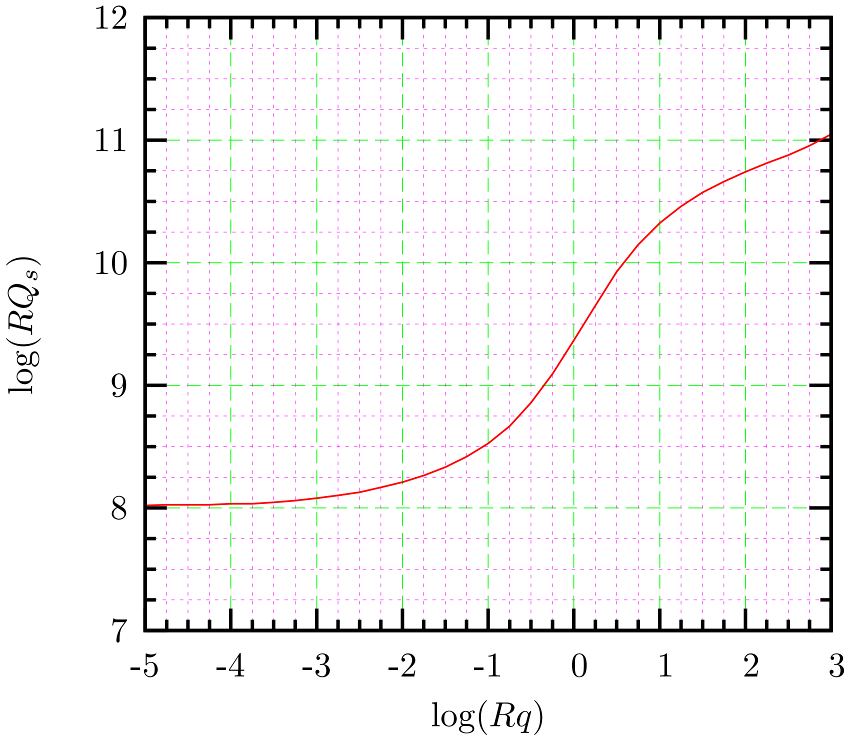
Dependence on momentum transfer k: traveling waves [C. Marquet, G.S]



- formation of a traveling wave at large $p = k - \frac{q}{2}$ (or k)
- cut-off effect in the infrared region

Saturation scale





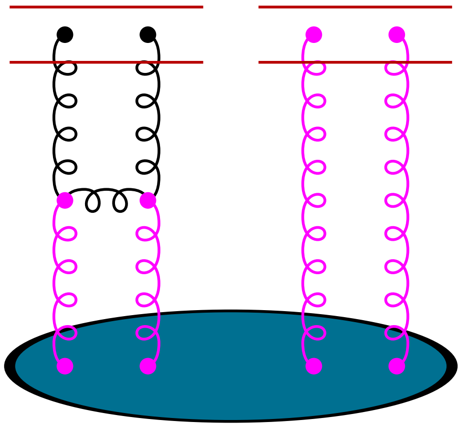
- Y dependence: converges to v_c
- q dependence: scales like a constant or linearly ($Y = 25$)

Effects of fluctuations

Fluctuations

[E. Iancu, D. Triantafyllopoulos]
Also A. Mueller, S. Munier, A. Shoshi,
W. van Saarloos, S. Wong

Consider correlations $\langle T^{(k)} \rangle$



- Usual BFKL ladder

$T^{(k)} \to T^{(k)}$

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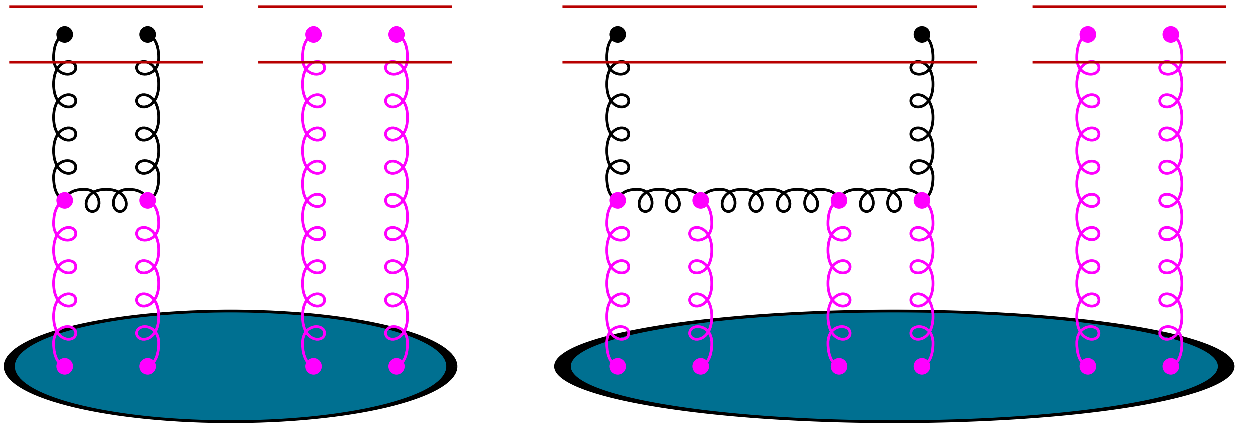

- Usual BFKL ladder $T^{(k)} \to T^{(k)}$
- fan diagram $\longrightarrow$ saturation effects $T^{(k+1)} \to T^{(k)}$

Fluctuations

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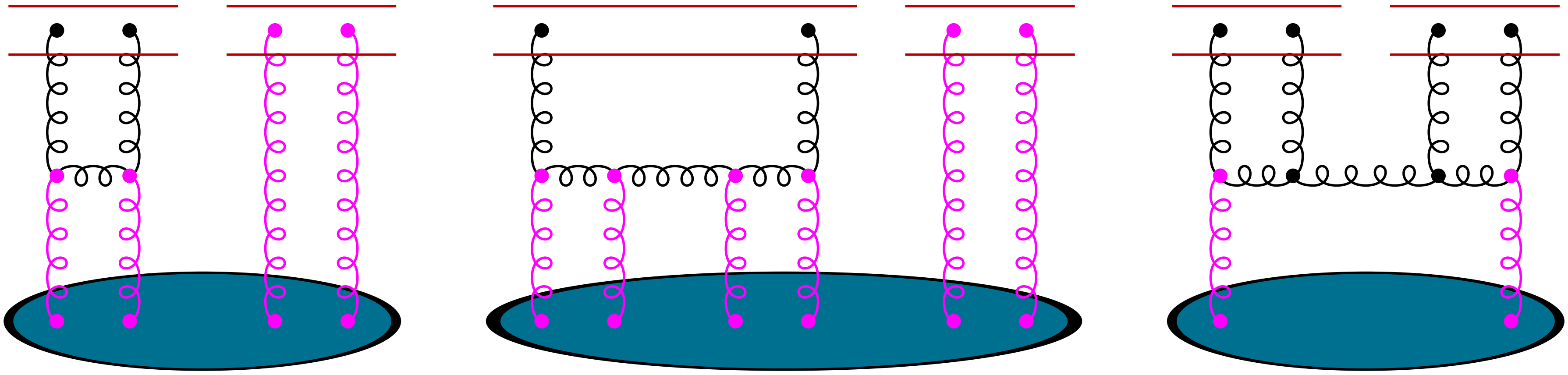


- Usual BFKL ladder $T^{(k)} \to T^{(k)}$
- fan diagram $\longrightarrow$ saturation effects $T^{(k+1)} \to T^{(k)}$
- splitting $\longrightarrow$ fluctuations, pomeron loops $T^{(k-1)} \to T^{(k)}$

Evolution hierarchy

$\Rightarrow$ complicated hierarchy

$$\begin{aligned}
&\partial_Y T^{(2)}(\mathbf{x}_1,\mathbf{y}_1;\mathbf{x}_2,\mathbf{y}_2;Y)\\
&\quad= \bar\alpha \int d^2z\, \frac{(\mathbf{x}_1-\mathbf{y}_1)^2}{(\mathbf{x}_1-\mathbf{z})^2(\mathbf{z}-\mathbf{y}_1)^2}\Big[T^{(2)}(\mathbf{x}_1,\mathbf{y}_1;\mathbf{x}_2,\mathbf{z};Y)+T^{(2)}(\mathbf{x}_1,\mathbf{y}_1;\mathbf{z},\mathbf{y}_2;Y)\\
&\qquad -T^{(2)}(\mathbf{x}_1,\mathbf{y}_1;\mathbf{x}_2,\mathbf{y}_2;Y)-T^{(3)}(\mathbf{x}_1,\mathbf{y}_1;\mathbf{x}_2,\mathbf{z};\mathbf{z},\mathbf{y}_2;Y)+(1\leftrightarrow 2)\Big]\\
&\quad+ \bar\alpha\alpha_s^2\kappa\,\frac{(\mathbf{x}_1-\mathbf{y}_1)^2(\mathbf{x}_2-\mathbf{y}_2)^2}{(\mathbf{x}_1-\mathbf{y}_2)^2}\,T^{(1)}(\mathbf{x}_1,\mathbf{y}_2;Y)\,\delta^{(2)}(\mathbf{y}_1-\mathbf{x}_2).
\end{aligned}$$

- Merging term: important when $T^{(2)}\sim T^{(1)}\sim 1$ i.e. at saturation
- Splitting term: important when $T^{(2)}\sim \bar\alpha^2 T^{(1)}$ or $T\sim\bar\alpha^2$ i.e. in the dilute regime

Event evolution

Hierarchy $\equiv$ master equation

$\Rightarrow$ without b-dependence, equivalent to a Langevin equation

$$\partial_Y T(k,Y) = \bar{\alpha} K_{\text{BFKL}} \otimes T(k,Y) - \bar{\alpha} T^2(k,Y) + \bar{\alpha}\sqrt{\kappa \alpha_s^2 T(k,Y)}\,\nu(k,Y)$$

with

$$\langle \nu(k,Y) \rangle = 0 \qquad \langle \nu(k,Y)\nu(k',Y') \rangle = \frac{1}{\bar{\alpha}}\delta(Y-Y')\, k\delta(k-k')$$

Note: diffusive approximation $\rightarrow$ stochastic F-KPP equation

$$\partial_t u(x,t) = \partial_x^2 u + u - u^2 + \sqrt{2\kappa u}\,\nu(x,t)$$

Probability (1/2)

- From noise term to fluctuations (local noise $\leftrightarrow$ no space dependence)

$$du = \sqrt{2\kappa u}\nu(t) \quad \overset{\text{Ito}}{\Rightarrow} \quad u_{j+1} = u_j + \delta t\sqrt{2\kappa u_j}\nu_j \qquad \text{with } \langle \nu_i \nu_j \rangle = \frac{1}{\delta t}\delta_{ij}$$

$$\Rightarrow \quad F(u_{j+1}) = F(u_j) + \delta t\sqrt{2\kappa u_j}\nu_j F'(u_j) + \delta t^2 \kappa u_j \nu_j^2 F''(u_j)$$

$$\Rightarrow \quad \partial_t \langle F(u) \rangle = \kappa \langle u F''(u) \rangle$$

Note: $F(u) = u^n$ gives the hierarchy

- Associated probability

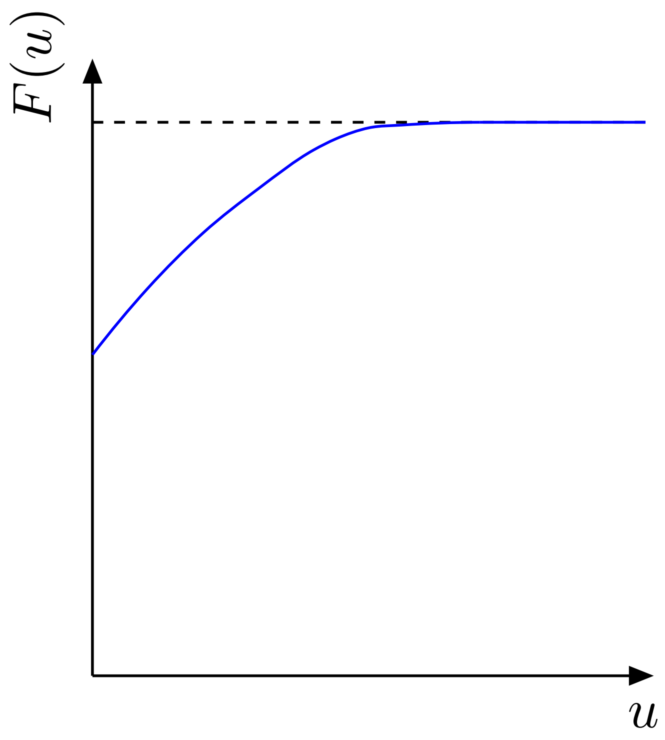
$$\langle F(u) \rangle = \int du\, F(u)\, P(u,t) \qquad \overset{\partial_t}{\Rightarrow} \qquad \partial_t P(u,t) = \kappa \partial_u^2 \left[u P(u,t) \right]$$

Including the initial contition $u(t=0) = u_0$, we get

$P_t(u_0 \to u) \equiv$ probability to go from u_0 to u in a time t.

Define the cumulative probability

$$F_{u_0,t}(u) = \int_{0^-}^{u} dv P_t(u_0 \to v).$$



Probability (2/2)

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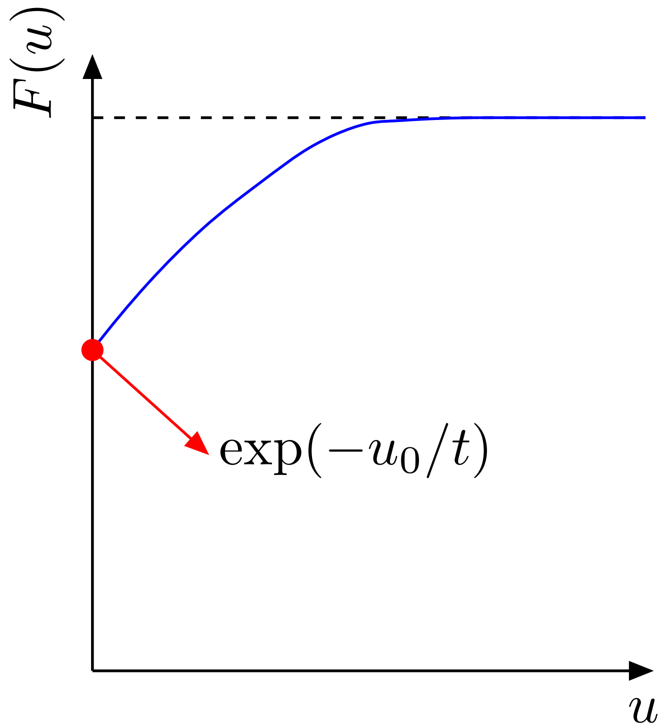

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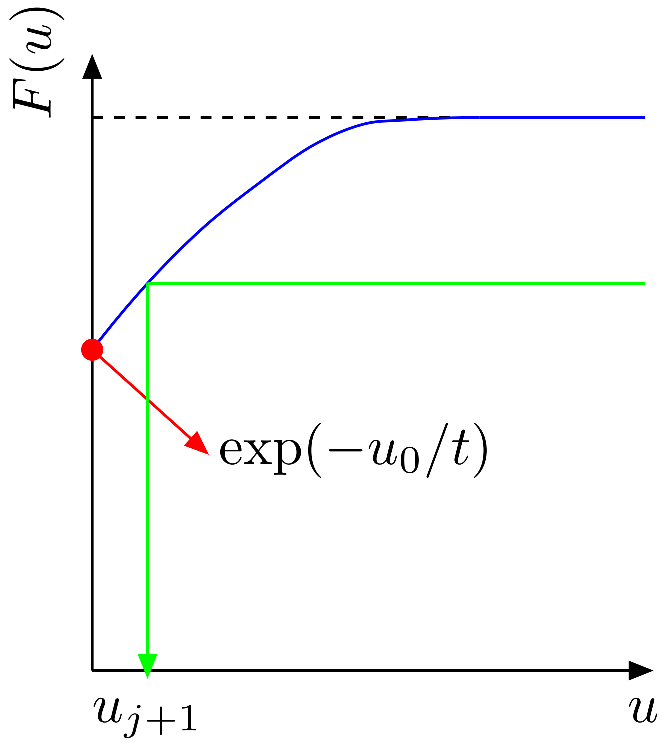

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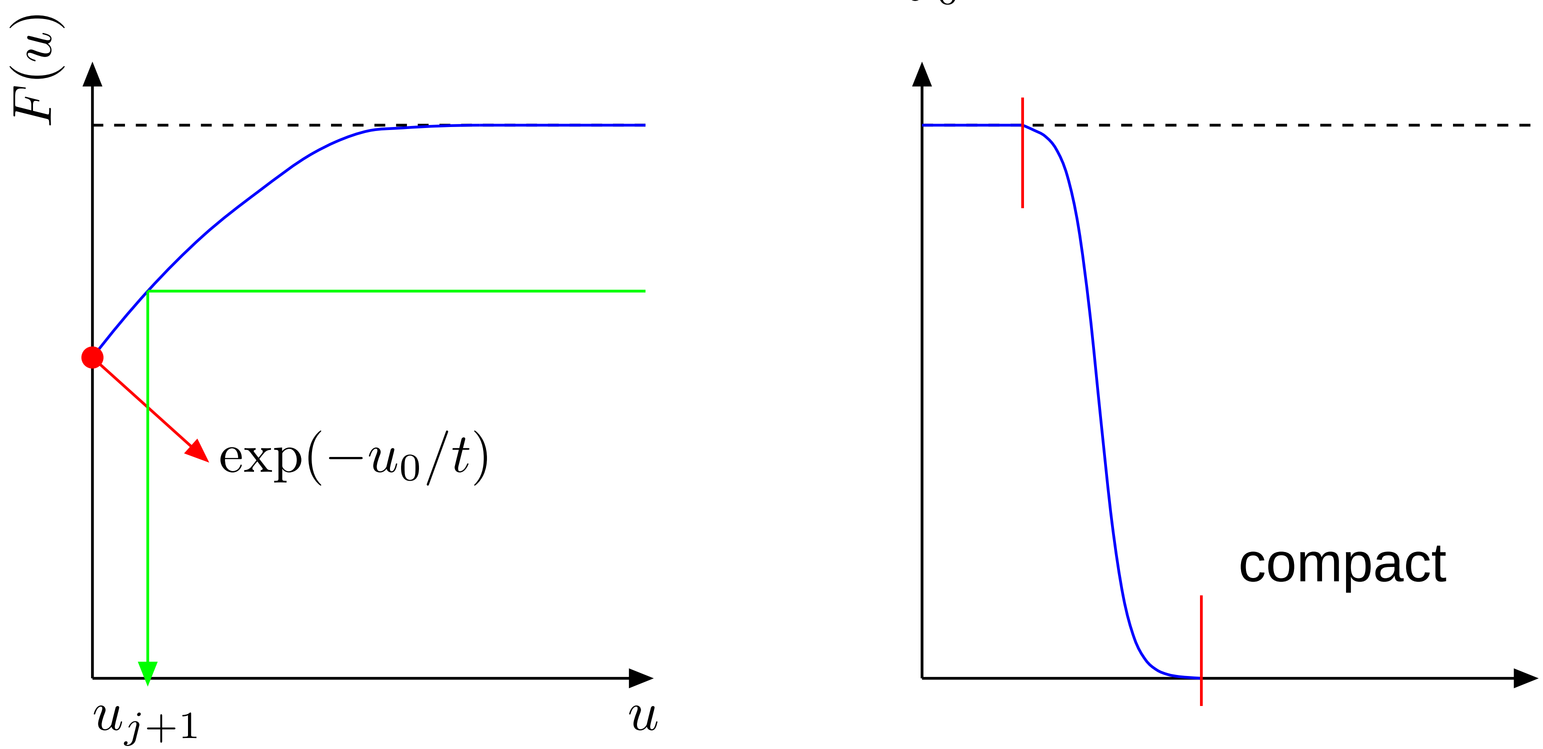


$$\partial_Y T(k,Y) = \bar\alpha K_{\text{BFKL}} \otimes T(k,Y) - \bar\alpha T^2(k,Y) + \bar\alpha\sqrt{\kappa\alpha_s^2 T(k,Y)}\nu(k,Y)$$

Rapidity step δY:

- Step 1: Use probability: $0 < y < 1$ uniform random variable

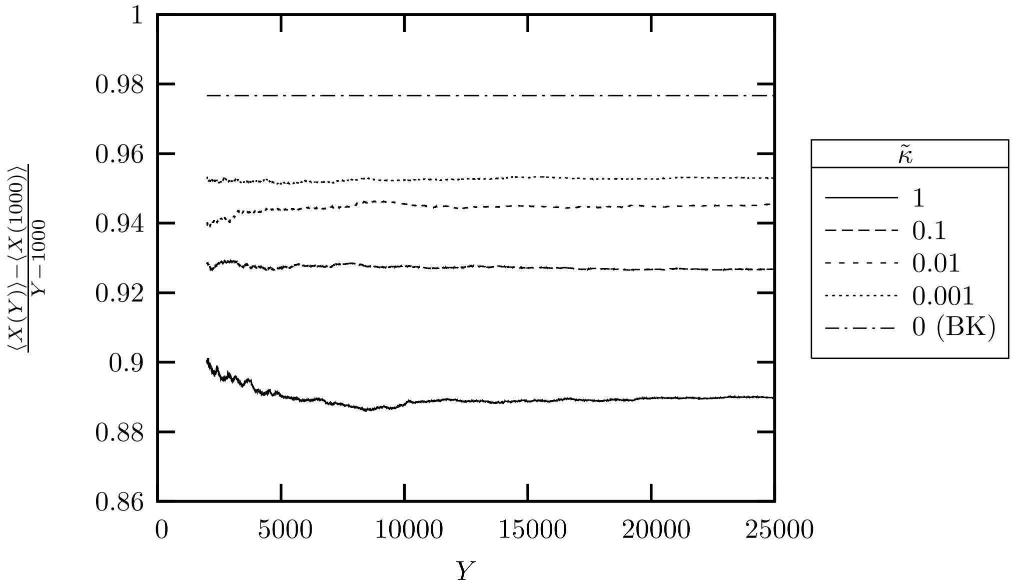
$$T_{\text{noise}}(k,Y) = F^{-1}_{T(k,Y),\delta Y}(y)$$

- Step 2: Apply the remaining equation

$$T(k,Y+\delta Y) = T_{\text{noise}}(k,Y) + \delta Y\left[\bar\alpha K_{\text{BFKL}} \otimes T_{\text{noise}}(k,Y) - \bar\alpha T^2_{\text{noise}}(k,Y)\right]$$

Event properties (1/2)

[G.S.]



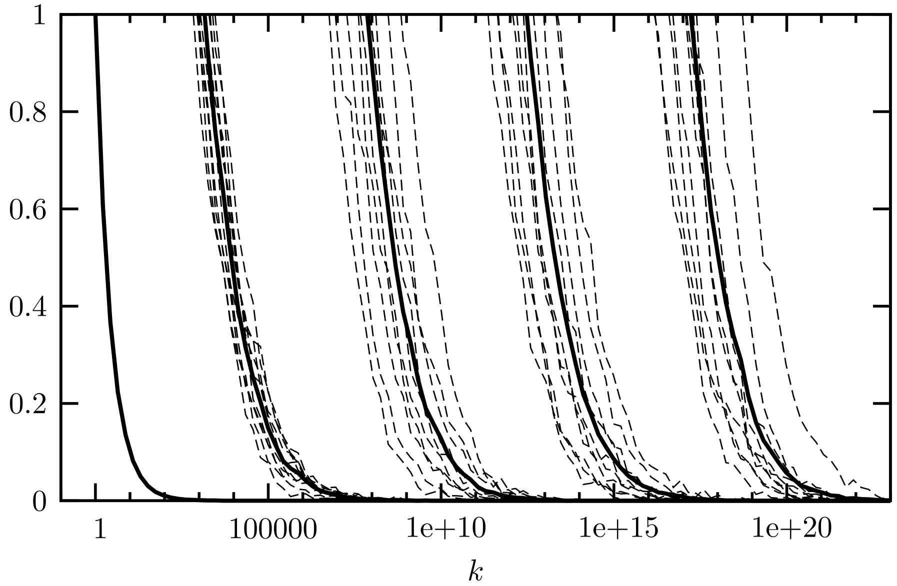
Decrease of the asymptotic velocity

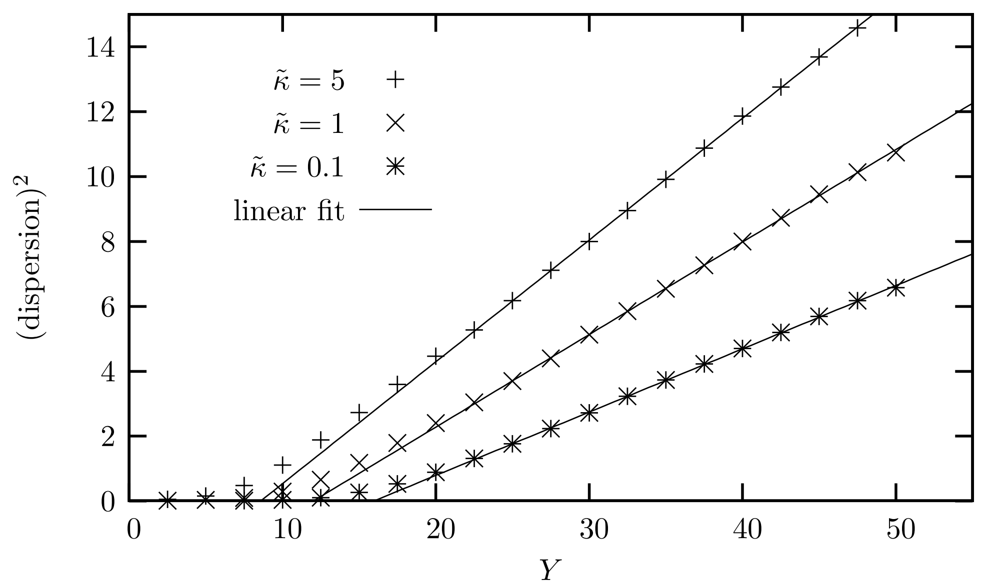
For asymptotically small α_s (not true here)

$$v^* \underset{\alpha_s^2\kappa \to 0}{\longrightarrow} v_c - \frac{\bar{\alpha}\pi^2\gamma_c\chi''(\gamma_c)}{2\log^2(\alpha_s^2\kappa)}$$

Event properties (2/2)

[G.S.]





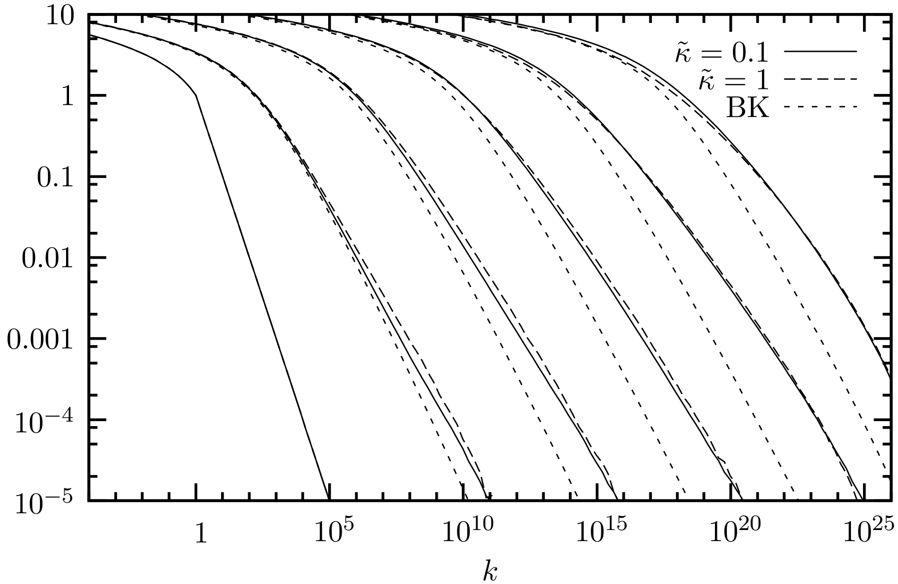
- Dispersion of the events

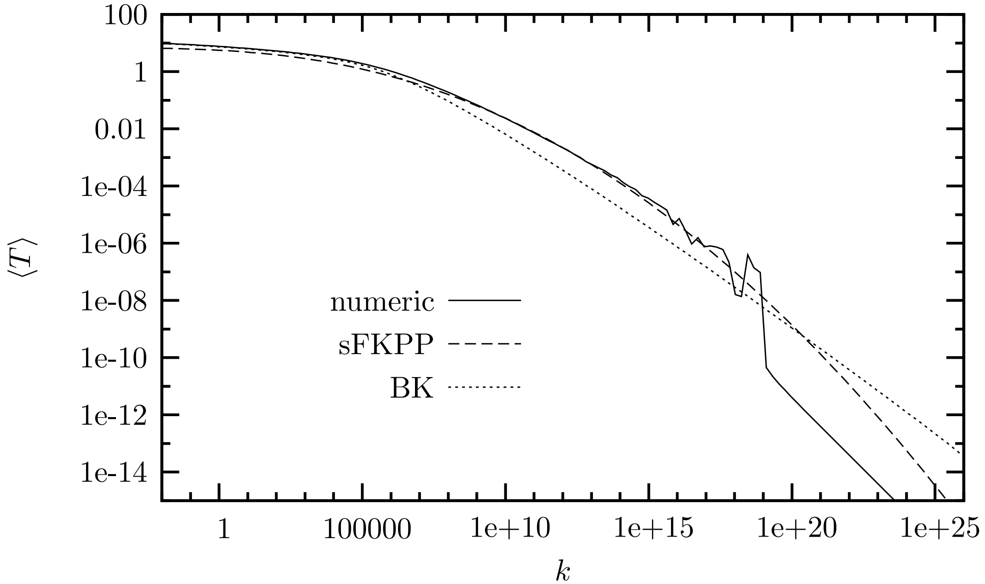
$$\Delta \log[Q_s^2(Y)] \approx \sqrt{D_{\text{diff}}\bar{\alpha}Y} \quad \text{with} \quad D_{\text{diff}} \underset{\alpha_s^2\kappa\to 0}{\sim} \frac{1}{|\log^3(\alpha_s^2\kappa)|}.$$

- No important dispersion in early stages of the evolution !

Averaged amplitude

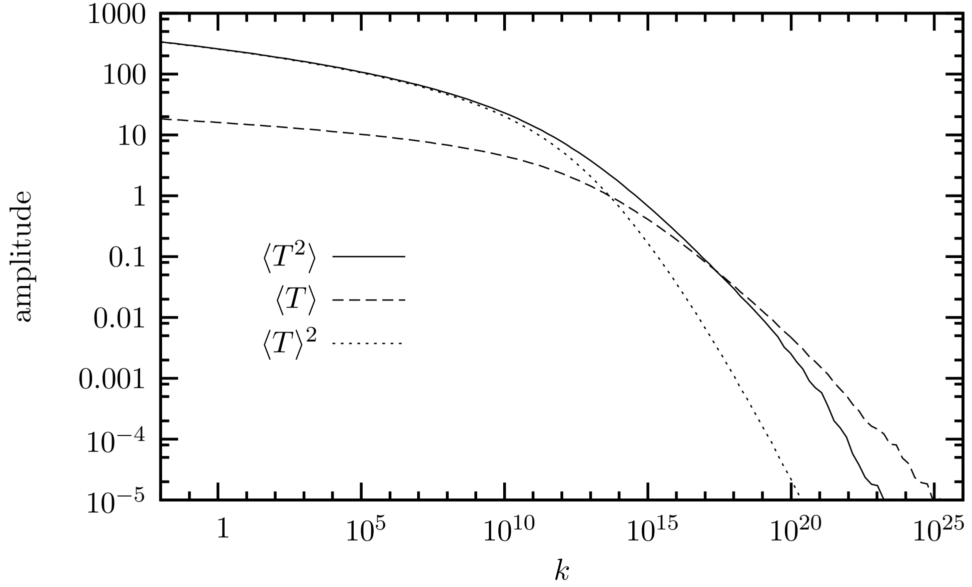
[G.S.]





- Clear effect of fluctuations
- Violations of geometric scaling (not in early stages)
- Agrees with predictions

[G.S.]



- Dense regime: $\left\langle T^2 \right\rangle \approx \left\langle T \right\rangle^2$
- Dilute regime: $\left\langle T^2 \right\rangle \approx \left\langle T \right\rangle$

Conclusion

- New form of the BK equation in momentum space (locality)
- Geometric scaling for $k \gg q$
  - same critical speed and slope as in the b-independent case
  - saturation scale proportional to momentum transfer

$$Q_s^2(Y) \sim \text{max}(q^2, 1/R^2)\Omega_s^2(Y) \sim \text{max}(q^2, 1/R^2)e^{v_c Y}$$

  - Confirmed by numerical simulations
- Fluctuation effects: first numerical studies
  - slower speed, dispersion
  - violations of geometric scaling (maybe not so important!)

Perspectives

- phenomenological tests:
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  - do we observe geometric scaling at nonzero momentum transfer ?
  - numerical test: BK vs. data

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