

# Parton Saturation and Dipole Picture

M.V.T. Machado and M.B. Gay Ducati

*Instituto de Física, Universidade Federal do Rio Grande do Sul*

*Av. Bento Gonçalves, 9500. C.P.15051, CEP 91510-970, Porto Alegre RS, Brazil.*

## Abstract

We have studied the color dipole picture for the description of deep inelastic process, mainly the structure functions which are driven directly by the gluon distribution. Estimates are obtained using the Glauber-Mueller dipole cross section in QCD encoding the corrections due to the unitarity effects which are associated with the saturation phenomenon. Frame invariance is verified in the calculations when analysing the experimental data.

## 1 Introduction

In the kinematical region of small proton momentum fraction  $x$ , the gluon is the main parton driving the behavior of the deep inelastic quantities. The standard QCD evolution [1] furnishes a powerlike growth for the gluon distribution and related observables. This result leads, at first glance, to the unitarity violation at asymptotic energies, requiring a sort of control. In the partonic language, at the infinite momentum frame, the small  $x$  region corresponds to the high parton density wisdom. The latter is connected with the black disk limit of the proton target and with the parton recombination phenomenon. These issues can be addressed through a non-linear dynamics beyond the usual DGLAP formalism. The complete knowledge about the non-linear dynamical regime plays an important role in the theoretical description of the reactions in the forthcoming experiments RHIC and the LHC, where these effects are enhanced by the high energies reached or by nuclear probes.

The description of DIS in the color dipole picture is somewhat intuitive, providing with a simple representation in contrast to the involved one from the Breit (infinite momentum) frame. Considering small values of the Bjorken variable  $x$ , the virtual photon fluctuates into a  $q\bar{q}$  pair (dipole) with fixed transverse separation  $r$  at large distances upstream of the target and interacts in a short time with the proton. More complicated configurations should be considered for larger transverse size systems, for instance the photon Fock state  $q\bar{q} + \text{gluon}$ . An immediate consequence from the lifetime of the pair ( $l_c = 1/2m_p x$ ) to be bigger than the interaction one is the factorization between the photon wavefunction and the cross section dipole-proton in the  $\gamma^* p$  total cross section. The

wavefunctions are perturbatively calculable, namely through QED for the  $q\bar{q}$  configuration [2] and from QCD for the  $q\bar{q}G$  one [3]. The effective dipole cross section should be modeled and it includes perturbative and non-perturbative content. However, since the interaction strength relies only on the configuration of the interacting system the dipole cross section turns out to be universal and may be employed in a wide variety of small  $x$  processes.

We have taken into account a sound formalism providing the unitarity corrections to the DIS at small  $x$ , namely the Glauber-Mueller approach in QCD. It was introduced by A. Mueller [4], who developed the Glauber formalism to study saturation effects in the quark and gluon distributions in the nucleus considering the heavy onium scattering. Later developments obtained an evolution equation taking into account the unitarity corrections (perturbative shadowing), generating a non-linear dynamics which is connected with higher twist contributions. Its main characteristic is to provide a theoretical framework for the saturation effects, lying on the multiscattering of the pQCD Pomeron. In this contribution we report our studies considering the parton saturation formalism to describe the observables driven by the gluonic content of the proton at the color dipole picture [5]. The inclusive structure function  $F_2$  is calculated, disregarding the fairly approximations commonly considered in previous calculations [6]. The structure functions  $F_L$  and  $F_2^{\gamma^*e}$  are also presented using the Glauber-Mueller approach and rest frame in comparison with the experimental data.

## 2 The DIS at the Rest Frame and Glauber-Mueller Approach

The rest frame physical picture is advantageous since the lifetime of the photon fluctuation and the interaction process are well defined [7]. The more simple case is the quark-antiquark state (color dipole), which is the leading configuration for small transverse size systems. The well known coherence length is expressed as  $l_c = 1/(2xm_p)$ , where  $x$  is the Bjorken variable and  $m_p$  the proton mass. An important consequence of this formulation is that the photoabsorption cross section can be derived from the expectation value of the interaction cross section for the multiparticle Fock states of the virtual photon weighted by the light-cone wave functions of these states [2]. That cross section can be cast in the quantum mechanical factorized form,

$$\sigma_{T,L}^{\gamma^*p}(x, Q^2) = \int d^2\mathbf{r} \int_0^1 dz |\Psi_{T,L}(z, \mathbf{r})|^2 \sigma^{\text{dipole}}(x, z, \mathbf{r}) , \quad (1)$$

The formulation above is valid even beyond perturbation theory, since it is determined from the space-time structure of the process. The  $\Psi_{T,L}(z, \mathbf{r})$  are the photon wavefunctions (for transverse  $T$ , and longitudinal  $L$ , polarizations) describing the pair configuration;  $z$  and  $1 - z$  are the fractions of the photon's light-cone momentum carried by the quark and antiquark of the pair, respectively. The transverse separation of the pair is  $\mathbf{r}$ . The explicit expressions for

the wavefunctions are well known,

$$\begin{aligned}
|\Psi_T(z, \mathbf{r})|^2 &= \frac{6\alpha_{\text{em}}}{4\pi^2} \sum_i^{n_f} e_i^2 \left\{ [z^2 + (1-z)^2] \varepsilon^2 K_1^2(\varepsilon r) + m_q^2 K_0^2(\varepsilon r) \right\} \\
|\Psi_L(z, \mathbf{r})|^2 &= \frac{6\alpha_{\text{em}}}{4\pi^2} \sum_i^{n_f} e_i^2 \left\{ 4Q^2 z^2 (1-z)^2 K_0^2(\varepsilon r) \right\}.
\end{aligned} \tag{3}$$

Lets define the auxiliary variable  $\varepsilon^2 = z(1-z)Q^2 + m_q^2$ , with  $m_q$  the light quark mass, and  $K_0$  and  $K_1$  are the Mc Donald functions of rank zero and one, respectively. The quantity  $\sigma^{\text{dipole}}$  is interpreted as the cross section of the scattering of the effective dipole with fixed tranverse separation  $\mathbf{r}$  [2]. The most important feature of the dipole cross section is its universal character, namely it depends only on the transverse separation  $\mathbf{r}$  of the color dipole. The dependence on the external probe particle, i.e., the photon virtuality, relies in the wavefunctions. In general, an ansatz for the effective dipole cross section is obtained and the process is analyzed in the impact parameter space. The main feature of the current models in the literature is to interpolate the physical regions of small transverse separations (perturbative QCD picture) and the large ones (Regge-soft picture). Here we have used the Glauber-Mueller approach to determine the dipole cross section, with the advantage of providing the corrections required by unitarity in an eikonal expansion. For the large  $r$  region, we choose to follow a similar procedure from the saturation model (GBW) [8], namely saturating ( $r$ -independent constant value) the dipole cross section at this region.

Now, we shortly show the main results from the Glauber-Mueller approach. Considering the scattering amplitude dependent on the usual Mandelstan variables  $s$  and  $t$ , now written in the impact parameter representation  $\mathbf{b}$ ,

$$a(s, \mathbf{b}) \equiv \frac{1}{2\pi} \int d^2\mathbf{q} e^{-i\mathbf{q}\cdot\mathbf{b}} \mathcal{A}(s, t = -q^2). \tag{4}$$

the corresponding total and elastic cross sections (from Optical theorem) are rewritten in the impact parameter representation ( $\mathbf{b}$ ) as

$$\sigma_{tot} = 2 \int d^2\mathbf{b} \text{Im} a(s, \mathbf{b}); \quad \sigma_{el} = \int d^2\mathbf{b} |a(s, \mathbf{b})|^2, \tag{5}$$

The most important property when treating the scattering in the impact parameter space is the simple definition for the unitarity constraint [6]. If the real part of the scattering amplitude vanishes at the high energy limit, corresponding to small  $x$  values, the solution to the that constraint is

$$a(s, \mathbf{b}) = i \left[ 1 - e^{-\frac{1}{2}\Omega(s, \mathbf{b})} \right]; \quad \sigma_{tot} = 2 \int d^2\mathbf{b} \left[ 1 - e^{-\frac{1}{2}\Omega(s, \mathbf{b})} \right], \tag{6}$$

where the opacity  $\Omega$  is an arbitrary real function and it should be determined by a detailed model for the interaction. The opacity function has a simple

physical interpretation, namely  $e^{-\Omega}$  corresponds to the probability that no inelastic scatterings with the target occur. To realize the connection with the Glauber formalism, the opacity function can be written in the factorized form  $\Omega(s, \mathbf{b}) = \Omega(s) S(\mathbf{b})$ , considering  $S(\mathbf{b})$  normalized as  $\int d^2\mathbf{b} S(\mathbf{b}) = 1$  (for a detailed discussion, see i.e. [9]).

We identify the opacity  $\Omega(s \approx Q^2/x; \mathbf{r}) = \sigma^{\text{nucleon}}(x, \mathbf{r})$ . The  $(q\bar{q})$  pair dipole-proton cross section is well known [6, 9] and in double logarithmic approximation (DLA) has the following form

$$\sigma_{\text{nucleon}}^{q\bar{q}}(x, r) = \frac{\pi^2 \alpha_s(\tilde{Q}^2)}{3} r^2 x G(x, \tilde{Q}^2) \quad (7)$$

with the  $r$ -dependent scale  $\tilde{Q}^2 = r_0^2/r^2$ . Considering Eq. (7) one can connect directly the dipole picture with the usual parton distributions (gluon), since they are solutions of the DGLAP equations. In our case, we follow the calculations in Ref. [6, 9] and consider the effective scale  $\tilde{Q}^2 = 4/r^2$ . From the above expression, one obtains a dipole cross section satisfying the unitarity constraint and a framework to study the unitarity effects (saturation) in the gluon DGLAP distribution function. Hence, hereafter we use the Glauber-Mueller dipole cross section given by

$$\sigma_{\text{dipole}}^{GM} = 2 \int d^2\mathbf{b} \left( 1 - e^{-\frac{1}{2} \sigma_{\text{nucleon}}^{q\bar{q}}(x, \mathbf{r}) S(\mathbf{b})} \right). \quad (8)$$

In order to perform numerical estimates one needs to define the profile function  $S(b)$ . This function contains information about the angular distribution in the scattering. We have chosen a simple gaussian shape in the impact parameter space,  $S(b) = \frac{A}{\pi R_A^2} e^{-b^2/R_A^2}$ , where  $A$  is the atomic number and  $R_A$  is the target radius. We will keep this notation although we are only concerned with the nucleon case. The  $R_A^2$  value should be determined from the data, ranging between 5 – 10 GeV<sup>-2</sup> for the proton case [6]. Here, we have used the value ( $R_A^2 = 5$  GeV<sup>-2</sup>) obtained from a good description of both inclusive structure function and its derivative [10]. Such a value corresponds to significant unitarity corrections to the standard DGLAP input even in the current HERA kinematics.

In the calculations we have used the GRV94 parametrization [11]: bearing in mind that  $Q^2 = 4/r^2$ , its evolution initial scale  $Q_0^2 = 0.4$  GeV<sup>2</sup> allows to scan dipole sizes up to  $r_{\text{cut}} = \frac{2}{Q_0} \text{ GeV}^{-1}$  ( $= 0.62$  fm). For recent parametrizations, where  $Q_0^2 \sim 1$  GeV<sup>2</sup> ( $r_{\text{cut}} \approx 0.4$  fm), the uncertainty due to nonperturbative content in the calculations would increase. An additional advantage is that GRV94 does not include non-linear effects to the DGLAP evolution since it was obtained from rather large  $x$  values, i.e. this ensures that GRV94 does not include unitarity corrections in the initial scale. To proceed, for the large  $r$  region, we choose the following ansatz: the gluon distribution is frozen at scale  $r_{\text{cut}}$ , namely  $xG(x, \tilde{Q}_{\text{cut}}^2)$ . Then, for the large distance contribution  $r \leq r_{\text{cut}}$

the gluon distribution reads as

$$x G(x, Q^2 \leq Q_0^2) = \frac{Q^2}{Q_0^2} x G(x, Q^2 = Q_0^2), \quad (9)$$

leading to the correct behavior  $x G(x, Q^2) \sim Q^2$  as  $Q^2 \rightarrow 0$ .

### 3 Obtaining the Structure Functions

This section is devoted to the study and estimate of the gluon driven observables measured at HERA kinematical domain in the rest frame. The first one is the inclusive structure function  $F_2(x, Q^2)$ , the main quantity testing the small  $x$  physics. The unitarity corrections are well established for this observable considering Glauber-Mueller approach [13] as well as its high energy asymptotics, namely the black disk limit. We review these issues considering the dipole picture (rest frame), using a more complete analysis similar to [8], but mostly, discuss in detail the role played by the nonperturbative physics needed to describe the structure function, and where in the transverse separation  $r$  range it starts to be important.

The longitudinal structure function  $F_L(x, Q^2)$  is also addressed, verifying the frame invariance in comparison with previous laboratory frame calculations. The longitudinal wavefunction strongly suppress large  $r$  contributions, thus selecting smaller nonperturbative contribution in comparison with the  $F_2$  case. Moreover,  $F_L$  is one of the main observables scanning possible higher twist corrections in the standard Operator Product Expansion (OPE). Therefore, a reasonable description of this quantity suggests that the Glauber-Mueller formalism (or similar eikonal-like approaches) take into account the most important contributions to the complete higher-twist corrections at current kinematical regime.

The structure function  $F_2^{c\bar{c}}(x, Q^2)$  gives the charm quark content on the proton and is directly driven by the gluon distribution. Therefore it is a powerful observable to scan saturation effects in the small  $x$  region. However the current experimental status requires more dedicated measurements and a better statistics. We verify a consistent description in the rest frame corroborating the similar analysis in the dipole models and in those ones considering unitarity corrections in the laboratory frame.

#### 3.1 The structure function $F_2$

First, we perform estimates for the structure function  $F_2$  at the rest frame considering the Glauber-Mueller dipole cross section [5]. The expression, with the explicit integration limits on photon momentum fraction  $z$  and transverse separation  $r$  is,

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{\text{em}}} \int_0^\infty d^2\mathbf{r} \int_0^1 dz (|\Psi_T(z, \mathbf{r})|^2 + |\Psi_L(z, \mathbf{r})|^2) \sigma_{\text{dipole}}^{GM}(x, \mathbf{r}^2). \quad (10)$$

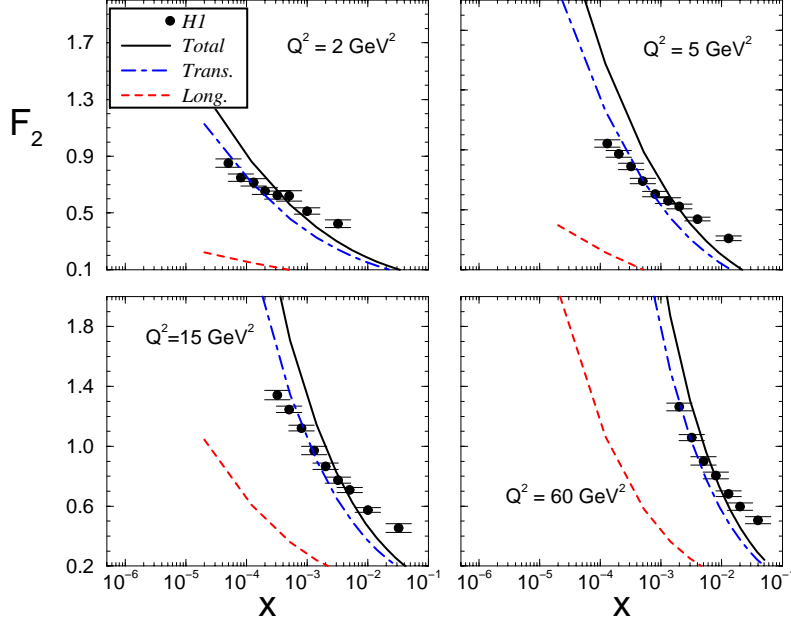


Figure 1: The Glauber-Mueller (GM) result for the  $F_2(x, Q^2)$  structure function. It is shown the transverse contribution (dot-dashed), the longitudinal one (dashed) and total one (solid line).

In the Fig. (1) one shows  $F_2$  for representative virtualities  $Q^2$  from the latest H1 Collaboration measurements [12]. The longitudinal and transverse contributions are shown separately. An effective light quark mass ( $u, d, s$  quarks) was taken, with the value  $m_q = 300$  MeV, and the target radius is considered  $R_A^2 = 5$  GeV $^{-2}$ . It should be stressed that this value leads to larger saturation corrections rather than using radius ranging over  $R_A^2 \sim 8 - 15$  GeV $^{-2}$ . The soft contribution comes from the freezing of the gluon distribution at large transverse separation as discussed at the previous section.

From the plots we verify a good agreement in the normalization, however the slope seems quite steep. This fact is due to the modeling for the soft contribution and it suggests that a more suitable nonperturbative input should be taken. To clarify the role played by the soft nonperturbative contribution to  $F_2$ , in the Fig. (2) we plot separately the perturbative contribution and parametrize the soft contribution introducing the nonperturbative structure function  $F_2^{\text{soft}} = C_{\text{soft}} x^{-0.08} (1-x)^{10}$  [13], which is added to the perturbative one. The soft piece normalization is  $C_{\text{soft}} = 0.22$ . Accordingly, we have used just shadowing corrections for the quark sector, taking into account only the transverse photon wavefunction and zero quark mass. The integration on the transverse separation is taken over  $1/Q^2 \leq r^2 \leq 1/Q_0^2$ , with  $Q_0^2 = 0.4$  GeV $^2$  for leading order GRV94 gluon distribution. This leads to a residual contribution to the soft piece which

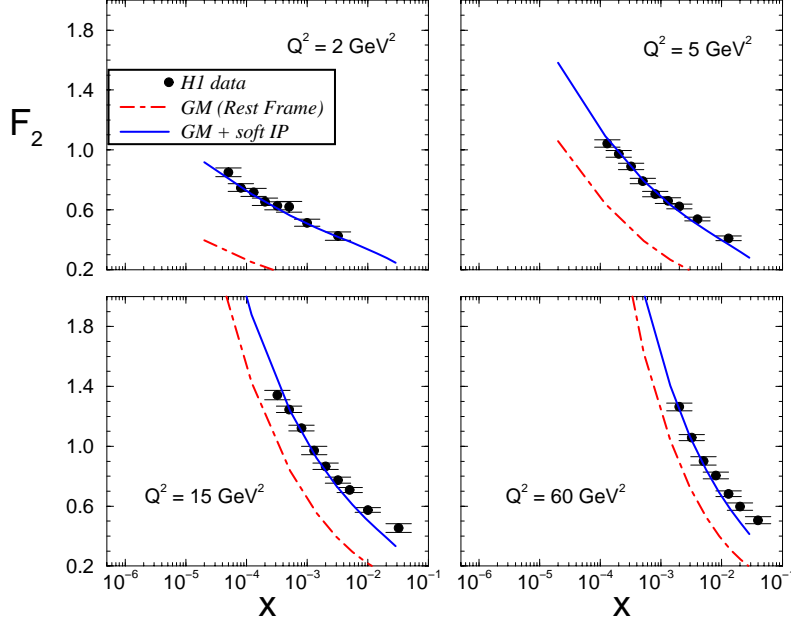


Figure 2: The Glauber-Mueller prediction for the  $F_2$  structure function in the rest frame. For sake of comparison, one uses quark sector ( $R_A^2 = 5 \text{ GeV}^{-2}$ ,  $m_q = 0$ ) and only transverse wavefunction. Radius integration  $1/Q^2 < r^2 < 1/Q_0^2$  and soft Pomeron added ( $F_2^{\text{soft}} = C_{\text{soft}} x^{-0.08} (1-x)^{10}$ ).

would come from the transverse separations  $r^2 < 1/Q^2$ . It is again verified that the soft contribution is important at small virtualities and decreasing as it gets larger. The data description is quite successful.

Concluding, we have a theoretical estimate, i.e. no fitting procedure, of the inclusive structure function  $F_2(x, Q^2)$  through the Glauber-Mueller approach for the dipole cross section, detecting a non negligible importance of a suitable input for the large dipole size region.

### 3.2 The structure function $F_L$

From QCD theory, the structure function  $F_L$  has a non-zero value due to the gluon radiation, as is encoded in the Altarelli-Martinelli equation (see [14]), considering the Breit frame. Experimentally, the determination of the  $F_L$  is quite limited, providing few data points. Most recently, the H1 Collaboration has determined the longitudinal structure function through the reduced double differential cross section, where the data points were obtained consistently with the previous measurements, however being more precise and lying into a broader kinematical range [12].

In Fig. (3) we present the estimates for the  $F_L$  structure function, in repre-

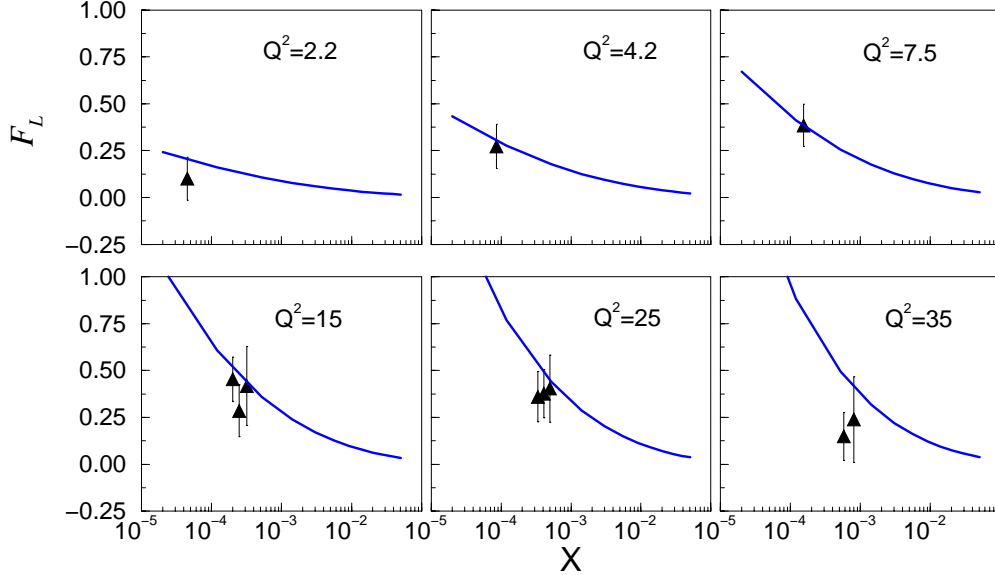


Figure 3: The Glauber-Mueller estimates for the  $F_L$  structure function. One uses light quarks ( $m_q = 300$  MeV), target size  $R_A^2 = 5$  GeV $^{-2}$  and frozen gluon distribution at large  $r$ . Data from H1 Collaboration [12].

sentative virtualities as a function of  $x$  [5]. For the calculations, it was considered light quarks ( $u, d, s$ ) with effective mass  $m_q = 300$  MeV and the target radius  $R_A^2 = 5$  GeV $^{-2}$ . The large  $r$  region is considered by the freezing of the gluon distribution at this region. Our expression for the observable is then,

$$F_L(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{\text{em}}} \int_0^\infty d^2\mathbf{r} \int_0^1 dz |\Psi_L(z, \mathbf{r})|^2 \sigma_{\text{dipole}}^{GM}(x, \mathbf{r}^2). \quad (11)$$

The behavior is quite consistent with the experimental result, either in shape as in normalization. The quantity is less sensitive to the non-perturbative content than  $F_2$ . A better description can be obtained by fine tuning the target size or the considered gluon distribution function, however it should be stressed that the present prediction is parameter-free and determined using the dipole picture taking into account unitarity (saturation) effects in the effective dipole cross section [5]. We verify that the rest frame calculation, taking into account the dipole degrees of freedom and unitarity effects produces similar conclusions to those ones using the Breit system. For instance, in a previous work [14], the



unitarity corrections to the longitudinal structure function were estimated in the laboratory frame considering the Altarelli-Martinelli equation, with unitarized expressions for  $F_2$  and  $xG(x, Q^2)$ , obtaining that the expected corrections reach up to 70 % as  $\ln(1/x) = 15$ , namely on the kinematical corner of the upcoming THERA project.

The higher twist corrections to the longitudinal structure function have been pointed out. For instance, Bartels et al. [15] have calculated numerically the twist-four correction founding they are large for  $F_T$  and  $F_L$ , however having opposite signs. This fact leads to remaining small effects to the inclusive structure function by almost complete cancellation between those contributions. The higher twist content is analyzed considering the model [8] as initial condition. Concerning  $F_L$ , it was found that the twist-four correction is large and has negative signal, concluding that a leading twist analysis of  $F_L$  is unreliable for high  $Q^2$  and not too small  $x$ . The results are in agreement with the simple parametrization for higher twist (HT) studied by the MRST group in Ref. [16], where  $F_2^{HT}(x, Q^2) = F_2^{LT}(x, Q^2)(1 + \frac{D_2^{HT}(x)}{Q^2})$ . The second term would parametrize the higher twist content. In our case, the unitarity corrections provide an important amount of higher twist content, namely it takes into account some of the several graphs determining the twist expansion.

### 3.3 The structure function $F_2^{c\bar{c}}$

In perturbative QCD, the heavy quark production in electron-proton interaction occurs basically through photon-gluon fusion, in which the emitted photon interacts with a gluon from the proton generating a quark-antiquark pair. Therefore, the heavy quark production allows to determine the gluon distribution and the amount of unitarity (saturation) effects for the observable. In particular, charmed mesons have been measured at deep-inelastic at HERA and the corresponding structure function  $F_2^{c\bar{c}}(x, Q^2)$  is defined from the differential cross section for the  $c\bar{c}$  pair production.

Experimentally, the measurements of the charm structure function are obtained by measuring mesons  $D^{*\pm}$  production [17]. The function  $F_2^{c\bar{c}}(x, Q^2)$  shows an increase with decreasing  $x$  at constant values of  $Q^2$ , whereas the rise becomes sharper at higher virtualities. The data are consistent with the NLO DGLAP calculations. Concerning the ratio  $R^{c\bar{c}} = F_2^{c\bar{c}}/F_2$ , the charm contribution to  $F_2$  grows steeply as  $x$  diminishes. It contributes less than 10% at low  $Q^2$  and reaches to about 30 % for  $Q^2 > 120 \text{ GeV}^2$  [17].

Once more the color dipole picture will provide a quite simple description for the charm structure function in a factorized way. Now, the Glauber-Mueller dipole cross section is weighted by the photon wavefunction constituted by a  $c\bar{c}$  pair with mass  $m_c$ . Our expression for the charmed contribution in deep inelastic is thus written as

$$F_2^{c\bar{c}}(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{\text{em}}} \int_0^\infty d^2\mathbf{r} \int_0^1 dz (|\Psi_T^{c\bar{c}}(z, \mathbf{r})|^2 + |\Psi_L^{c\bar{c}}(z, \mathbf{r})|^2) \sigma_{\text{dipole}}^{GM}(x, \mathbf{r}^2) \quad (12)$$

where  $|\Psi_{T,L}^{c\bar{c}}(z, \mathbf{r})|^2$  is the probability to find in the photon the  $c\bar{c}$  color dipole

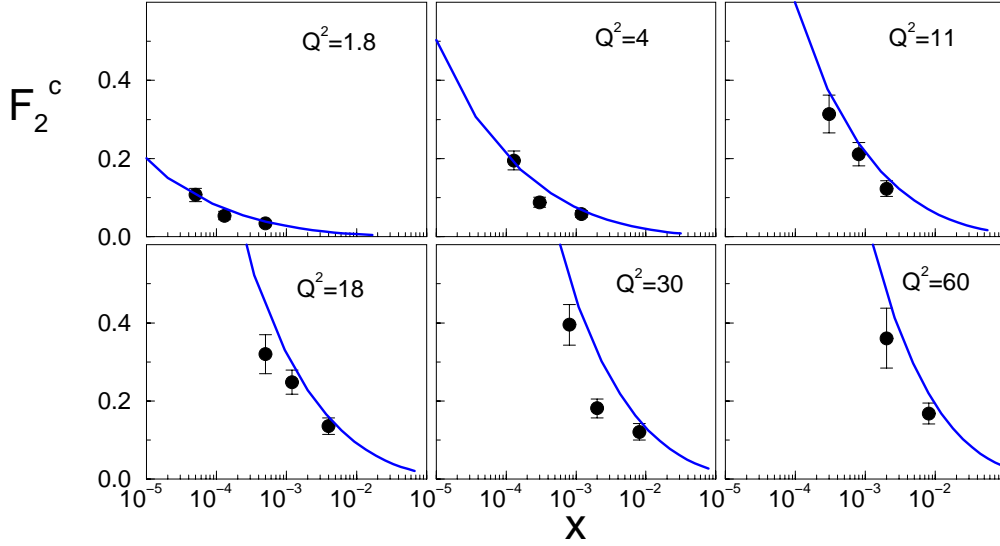


Figure 4: The Glauber-Mueller result for the  $F_2^{c\bar{c}}$  structure function as a function of Bjorken variable  $x$  at fixed virtualities (in  $\text{GeV}^2$ ). One uses charm mass  $m_c = 1.5 \text{ GeV}$ , target size  $R_A^2 = 5 \text{ GeV}^{-2}$  and frozen gluon distribution at large  $r$ . Data from ZEUS Collaboration [17] (statistical errors only).

with the charmed quark carrying fraction  $z$  of the photon's light-cone momentum with  $T, L$  polarizations. For the correspondent wavefunctions, the quark mass in Eqs. (2,3) should be substituted by the charm quark mass  $m_c$ . Here, we should take care of the connection between the Regge parameter  $x = (W^2 + Q^2)/(Q^2 + 4m_q^2)$  and the Bjorken variable  $x_{\text{Bj}}$ . For calculations with the light quarks these variables are equivalent, however for heavier quarks the correct relation is [18]:  $x_{\text{Bj}} = x(Q^2/Q^2 + 4m_c^2)$ .

In Fig. (4) we show the estimates for the charm structure function as a function of  $x_{\text{Bj}}$  at representative virtualities [5]. In our calculations, it was used charm mass  $m_c = 1.5 \text{ GeV}$ , target size  $R_A^2 = 5 \text{ GeV}^{-2}$  and frozen gluon distribution at large  $r$ . We have verified small soft contribution, decreasing as the virtuality rises. There is a slight sensitivity to the value for the charm mass, increasing the overall normalization as  $m_c$  diminishes. Such a feature suggests that the charm mass is a hard scale suppressing the non-perturbative contribution to the corresponding cross section. This conclusion is in agreement with the recent BFKL color dipole calculations of Nikolaev-Zoller [18] and those from Donnachie-Dosch [19].

Regarding the Breit system description, in Ref. [14] it was found strong corrections to the charm structure function, which are larger than those of the  $F_2$  ones. Considering the ratio  $R_2^c = F_2^{c\text{GM}}(x, Q^2)/F_2^{c\text{DGLAP}}(x, Q^2)$ , the

corrections predicted by the Glauber-Mueller approach would reach up to 62 % at values of  $\ln(1/x) \approx 15$  (THERA region). Then, an important result is a large deviation of the standard DGLAP expectations at small  $x$  for the ratio  $R^{c\bar{c}} = F_2^{c\bar{c}}/F_2$  due to the saturation phenomena (unitarization). With our calculation [5] one verifies that it is obtained a good description of data in both reference systems, suggesting a consistent estimation of the unitarity effects for that quantity.

## 4 Conclusions

We study the dipole picture for the description of deep inelastic scattering, focusing on observables driven directly by the gluon distribution. Starting from the dipole cross section provided by the Glauber-Mueller approach in QCD, we perform estimates for the inclusive structure function  $F_2$ , the longitudinal function  $F_L$  and the charm structure function on the proton  $F_2^{c\bar{c}}$ .

For each of the observables discussed, we obtain theoretical estimates, at the rest frame, without further fitting procedure, in good agreement with the updated data from HERA. The resulting calculations corroborate a quite consistent picture for the unitarity corrections from Glauber-Mueller approach in both Breit and rest reference systems. In the laboratory frame the unitarity effects are connected with the gluon distribution function, whereas in the color dipole framework the basic block is the dipole cross section which is corrected by saturation effects.

The small transverse separation  $r$  region is dominated by the leading log DGLAP formalism, with the additional ingredient of unitarization phenomenon as the momentum fraction acquires quite small values. Such corrections are associated with the taming of the gluon distribution in the very small  $x$  region, in general named saturation regime. However, it should be stressed that Glauber-Mueller approach and similar eikonal-like models provide a logarithmic  $\ln(1/x)$  asymptotic behavior for the inclusive structure function and gluon distribution, instead of a constant value at asymptotic energies.

The large transverse separation is described by non-perturbative aspects of QCD. Since this domain is not well determined at the moment, some modeling of the soft region is needed. In this work we choose the ansatz in which the gluon distribution is frozen for virtualities above a cut radius  $r^2 > r_{\text{cut}}^2$ , which corresponds to the region  $Q^2 < Q_0^2$ . A convenient choice for the gluon pdf in order to cover the widest possible kinematical window diminishes the uncertainty coming from the soft sector. The most appropriated input is the GRV94 parametrization, where  $r_{\text{cut}} = 0.6$  fm is found, whereas it can take values  $r_{\text{cut}} = 0.4 - 0.5$  fm for the more recent pdf's. Throughout the paper we used the target size  $R_A^2 = 5 \text{ GeV}^{-2}$ , which corresponds to strong unitarity corrections.

When considering the structure function  $F_2$ , we have found that it is dominated by small transverse distances contributions. However, a non-negligible content from the soft sector is present. Moreover, the photon wavefunctions enhances the dipole cross section into smaller dipole sizes, since the weight

function selects smaller  $r$  as the virtuality  $Q^2$  diminishes. Our estimates here are parameter-free, however a fine tuning of the parameters can improve the data description. Furthermore, we notice that in calculations from [6], only the aligned jet dipole configuration  $z$ ,  $(1 - z) \approx 0$  (and only transverse contribution) is considered, whereas we take into account all configurations, including the symmetric ones. Thus, all dipole sizes, even those from non-perturbative region are included in our results.

Concerning  $F_L$ , the estimates are consistent with the previous calculations in the Breit system and are in good agreement with data. A remarkable feature is that the Glauber-Mueller approach in the color dipole framework gives important higher twist contributions to the leading twist calculation in a simple way. As is well known,  $F_L$  is the main quantity to study the expected higher twist effects in low virtualities.

The function  $F_2^{c\bar{c}}$  is directly dependent of the gluon distribution and important unitarity correction had been found when considering the Breit frame. Here, we verify consistent results in the rest frame in comparison with the previous ones in the fast proton system. We verified that the charm mass suppressed soft contributions in comparison with the  $F_2$  case, and the results present a slight dependence with the specific value of  $m_c$ .

In conclusion, the Glauber-Mueller approach provides a well established formalism to take into account the unitarity effects. It allows to estimate the higher twist contributions to the quantities in a simplified way. Moreover, it matches DGLAP evolution equation at Born level and includes the impact parameter dependence properly.

## Acknowledgments

This work was supported by CNPq, Brazil.

## References

- [1] Yu.L. Dokshitzer. *Sov. Phys. JETP* **46**, 641 (1977);  
G. Altarelli and G. Parisi. *Nucl. Phys.* **B126**, 298 (1977);  
V.N. Gribov and L.N. Lipatov. *Sov. J. Nucl. Phys* **28**, 822 (1978).
- [2] N.N. Nikolaev, B.G. Zakharov, *Z. Phys.* **C49**, 607 (1991); *Phys. Lett.* **B260**, 414 (1991); *Z. Phys.* **C53**, 331 (1992).
- [3] M. Wüsthoff, *Phys. Rev.* **D56**, 4311 (1997).
- [4] A. H. Mueller, *Nucl. Phys.* **B335**, 115 (1990).
- [5] M.B. Gay Ducati, M.V.T. Machado, Los Alamos preprint [hep-ph/0111093].
- [6] A.L. Ayala, M.B. Gay Ducati and E.M. Levin, *Nucl. Phys.* **B493**, 305 (1997); *Nucl. Phys.* **B511**, 355 (1998).

- [7] V. Del Duca, S.J. Brodsky, P. Hoyer, *Phys. Rev.* **D46**, 931 (1992).
- [8] K. Golec-Biernat, M. Wüsthoff. *Phys. Rev.* **D59**, 014017 (1999); *Phys. Rev.* **D60**, 114023 (1999).
- [9] E.Gotsman, E.Levin, U. Maor, *Nucl. Phys.* **B493**, 354 (1997).
- [10] M.B. Gay Ducati, V.P. Gonçalves, *Phys. Lett.* **B487**, 110 (2000);
- [11] M. Gluck, E. Reya, A. Vogt, *Z. Phys.* **C67**, 433 (1995).
- [12] H1 Collaboration, *Eur. Phys. J.* **C21**, 33 (2001).
- [13] A.L. Ayala, M.B. Gay Ducati, E.M. Levin, *Eur. Phys. J* **C8**, 115 (1999).
- [14] A.L. Ayala, M.B. Gay Ducati, V.P. Gonçalves, *Phys. Rev.* **D59**, 054010 (1999).
- [15] J. Bartels, K.Golec-Biernat, K. Peters. *Eur. Phys. J.* **C17**, 121 (2000).
- [16] A.D. Martin, R.G. Roberts, W.J. Stirling, R.S. Thorne, *Phys. Lett.* **B443**, 301 (1998);
- [17] ZEUS Collaboration, *Eur. Phys. J.* **C12**, 35 (2000).
- [18] N.N. Nikolaev, V.R. Zoller, *Phys. Lett.* **B509**, 283 (2001).
- [19] A. Donnachie, H.G. Dosch, *Phys. Lett.* **B502**, 74 (2001); *Phys. Rev.* **D65**, 014019 (2002).