Fluctuations in DIS data: a momentum space analysis*

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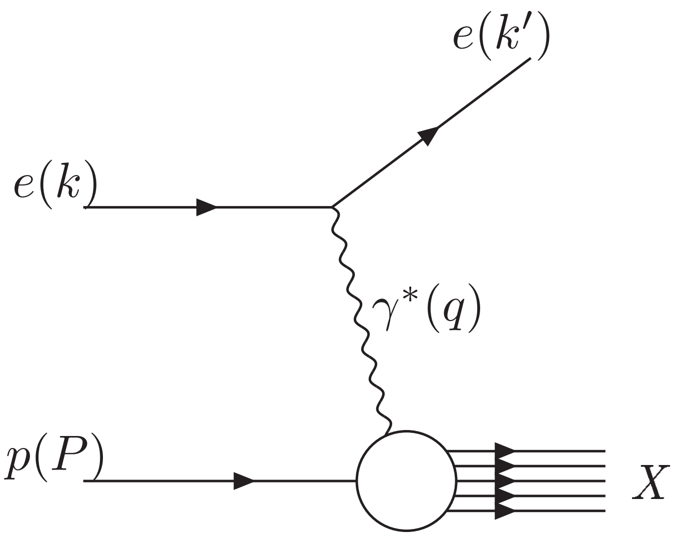
Deep Inelastic Scattering (DIS)

Kinematics and variables



- The total energy squared of the photon-nucleon system

$$s = (P+q)^2$$

- Photon virtuality

$$q^2 = (k-k') = -Q^2 < 0$$

- The Bjorken variable

$$x \equiv x_{Bj} = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{Q^2+s}$$

- The rapidity variable

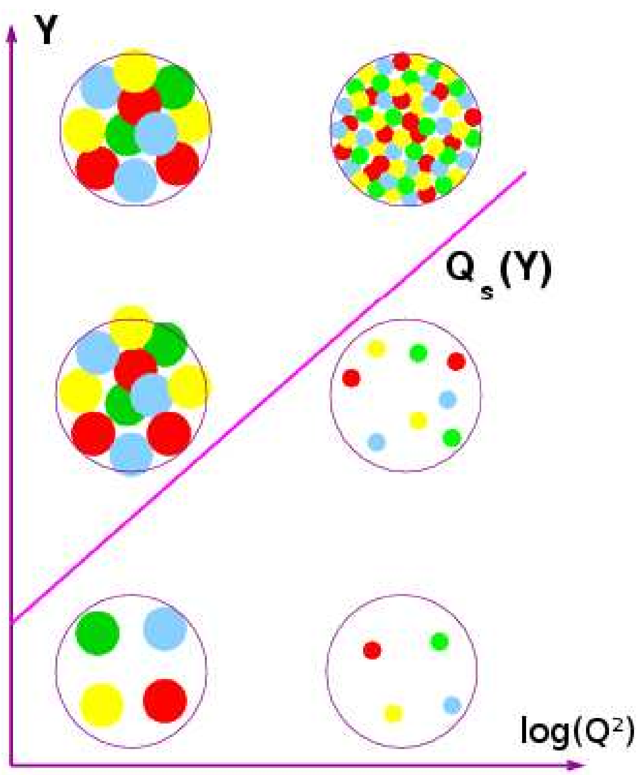
$$Y \equiv \ln(1/x)$$

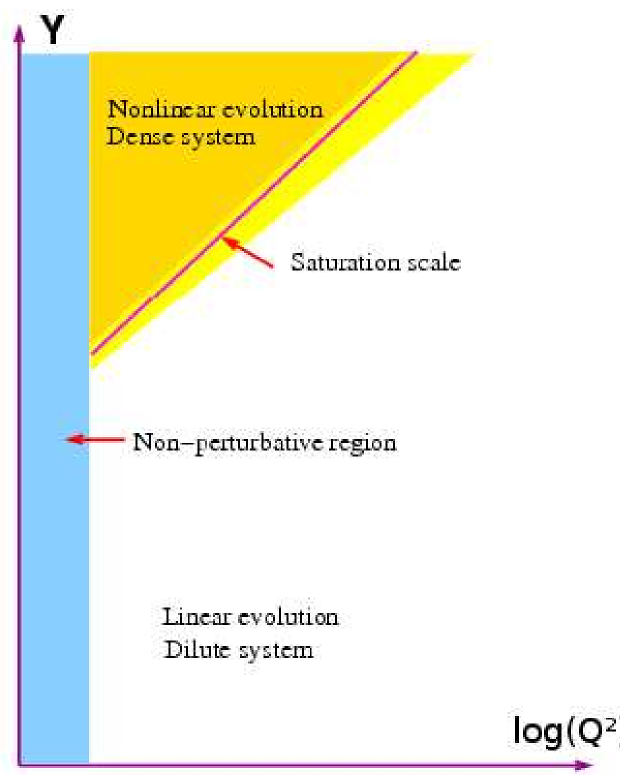
- The high energy limit:

$$s \to \infty, \quad x \approx \frac{Q^2}{s} \to 0$$

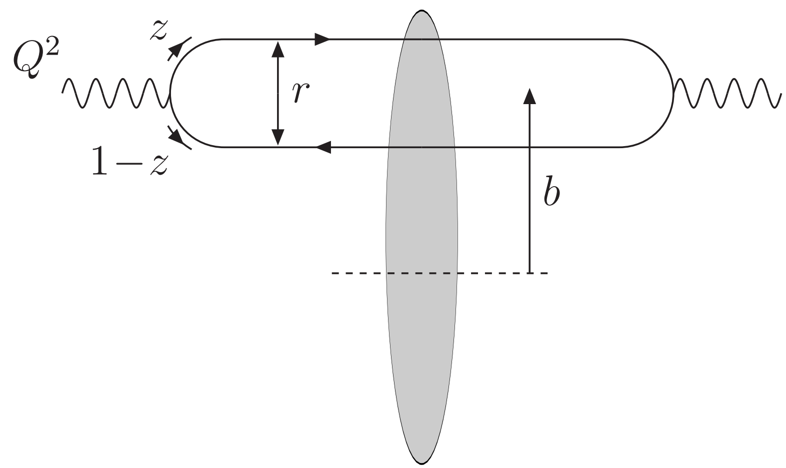
QCD at high energies

- One of the most intriguing problems in Quantum Chromodynamics is the growth with energy (and Q fixed) of the gluon density, and consequently of the cross sections, for hadronic interactions
- At very high energies gluon recombination and multiple scattering might be important to restore unitarity: **nonlinear evolution equations**





$\sigma^{\gamma^* p}$ cross section: dipole frame



- In this frame, the virtual foton (which travels fast) fluctuates into a $q\bar{q}$ pair of size r which then interacts with the proton
- The cross section is factorized like

$$\sigma_{T,L}^{\gamma^* p}(Y, Q) = \int d^2r \int_0^1 dz \, \left|\Psi_{T,L}(r, z; Q^2)\right|^2 \sigma_{dip}(r, Y), \tag{1}$$

where $\sigma_{dip}^{\gamma^* p}(Y, r)$ is the dipole-proton cross section, z is the fraction of photon's momentum carried by the quark, r is the size of the dipole and b is the impact parameter and $\Psi_{T,L}(r, z; Q^2)$ stands for the transverse and longitudinal photon wavefunctions

F_2 structure function

- Assuming an independence on the impact paremeter, the dipole-proton cross section is proportional to the dipole-proton forward scattering amplitude $T(r, Y)$ through the relation

$$\sigma_{dip}(r, Y) = 2\pi R_p^2 T(r, Y)$$

where $R_p$is the proton radius.

- The proton structure function F_2 can be obtained from the $\gamma^* p$ cross section through the relation

$$\begin{aligned} F_2(x, Q^2) &= \frac{Q^2}{4\pi^2 \alpha_{em}} \left[\sigma_T^{\gamma^* p}(x, Q^2) + \sigma_L^{\gamma^* p}(x, Q^2)\right] \\ &= \frac{Q^2}{4\pi^2 \alpha_{em}} \sigma^{\gamma^* p}(x, Q^2) \end{aligned} \quad (2)$$

- It is possible to express the $\gamma^* p$ cross section in terms of the scattering amplitude in momentum space, $\tilde{T}(k, Y)$, through the Fourier transform

$$\tilde{T}(k, Y) = \int_0^\infty \frac{dr}{r} J_0(kr)\, T(r, Y) \quad (3)$$

F_2 in momentum space

- The proton structure function is related to $\tilde{T}(k,Y)$ through

$$F_2(x,Q^2) = \frac{Q^2 R_p^2 N_c}{4\pi^2} \int_0^\infty \frac{dk}{k} \int_0^1 dz\, |\tilde{\Psi}(k,z;Q^2)|^2 \tilde{T}(k,Y) \qquad (4)$$

  where the photon wavefunction is now expressed in momentum space [1]

- The scattering amplitude $\tilde{T}(k,Y)$ obeys the The Balitsky-Kovchegov (BK) nonlinear equation in momentum space ($\bar{\alpha} = \alpha_s N_c/\pi$)

$$\partial_Y \tilde{T} = \bar{\alpha}\chi(-\partial_L)\tilde{T} - \bar{\alpha}\tilde{T}^2 \qquad (5)$$

  where

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma) \qquad (6)$$

  is the characteristic function of the leading-order (LO) Balitsky-Fadin-Kuraev-Lipatov (BFKL) kernel and $L = \log(k^2/k_0^2)$

- The BK equation lies [2] in the equivalence class of the Fisher-Kolmogorov-Petrovsky-Piscounov (F-KPP) equation, which admits the the ***traveling wave solutions***

Scattering Amplitude $\tilde{T}(k)$

- The asymptotic behaviours of the solutions to BK equation are naturally expressed in momentum space
- At asymptotic rapidities, the amplitude $\tilde{T}(k, Y)$, instead of depending separately on k and Y, depends only on the scaling variable $k^2/Q_s^2(Y)$, where we have introduced the *saturation scale* $Q_s^2(Y) = k_0^2 \exp(\lambda Y)$, measuring the position of the wavefront
- The expression for the tail of the scattering amplitude [1]

$$\tilde{T}\,(k, Y) \overset{k \gg Q_s}{\approx} \left(\frac{k^2}{Q_s^2(Y)}\right)^{-\gamma_c} \log\left(\frac{k^2}{Q_s^2(Y)}\right) \exp\left[-\frac{\log^2\left(k^2/Q_s^2(Y)\right)}{2\bar{\alpha}\chi''(\gamma_c)Y}\right] \tag{7}$$

- In the infrared domain, one can show that the amplitude behaves like

$$\tilde{T}\left(\frac{k}{Q_s(Y)}, Y\right) \overset{k \ll Q_s}{=} c - \log\left(\frac{k}{Q_s(Y)}\right) \tag{8}$$

where c is an unfixed constant

The AGBS model

- The description of the transition to the saturation region is performed by an analytic interpolation between both asymptotic behaviours [2]
- The expression for the amplitude is unitarised – up to a logarithmic factor – by an eikonal *i.e.* $T_{\text{unit}} = 1 - \exp(-T_{\text{dil}})$
- The following choice gives good results:

$$\tilde{T}(k,Y) = \left[\log\left(\frac{k}{Q_s} + \frac{Q_s}{k}\right) + 1\right]\left(1 - e^{-T_{\text{dil}}}\right) \tag{9}$$

where

$$T_{\text{dil}} = \exp\left[-\gamma_c \log\left(\frac{k^2}{Q_s^2(Y)}\right) - \frac{L_{\text{red}}^2 - \log^2(2)}{2\bar{\alpha}\chi''(\gamma_c)Y}\right] \tag{10}$$

and

$$L_{\text{red}} = \log\left[1 + \frac{k^2}{Q_s^2(Y)}\right] \qquad \text{and} \qquad Q_s^2(Y) = k_0^2\, e^{\lambda Y} \tag{11}$$

- The equations above determine the model for the scattering amplitude, to be inserted into the expression for the F_2 structure function, resulting in a good fit to the HERA data [1]

Evolution and fluctuations

- BK equation is a “mean field” approximation of the Balitsky hierachy $\left(\mathcal{M}_{\mathbf{xyz}} = \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2}\right)$

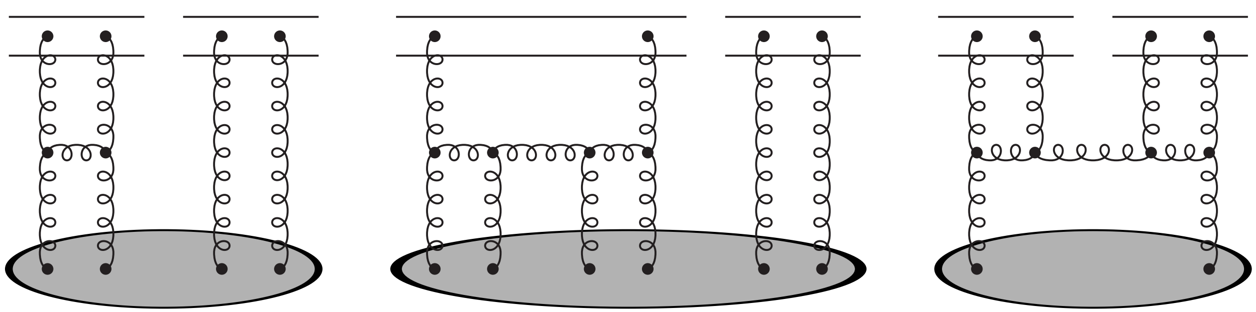
$$\partial_Y \langle T_{\mathbf{xy}} \rangle = \frac{\bar{\alpha}}{2\pi} \int_z \mathcal{M}_{\mathbf{xyz}} \left(\langle T_{\mathbf{xz}} \rangle + \langle T_{\mathbf{zy}} \rangle - \langle T_{\mathbf{xy}} \rangle - \langle T_{\mathbf{xz}} T_{\mathbf{zy}} \rangle \right) \tag{12}$$

$$\vdots$$

- For a fixed color field in the target we have $\langle T_{\mathbf{xz}} T_{\mathbf{zy}} \rangle = \langle T_{\mathbf{xz}} \rangle \langle T_{\mathbf{zy}} \rangle$, wich yields the BK equation

$$\partial_Y \langle T_{\mathbf{xy}} \rangle = \frac{\bar{\alpha}}{2\pi} \int_z \mathcal{M}_{\mathbf{xyz}} \left(\langle T_{\mathbf{xz}} \rangle + \langle T_{\mathbf{zy}} \rangle - \langle T_{\mathbf{xy}} \rangle - \langle T_{\mathbf{xz}} \rangle \langle T_{\mathbf{zy}} \rangle \right) \tag{13}$$

- However, the Balitsky equations don’t take into account the possibility of multiple scattering in the projectile, or the fluctuations in the particle (dipole or gluon) number in the target [3]



Evolution with fluctuations

- In the diagrams for $\langle T_{\mathbf{xz}} T_{\mathbf{zy}} \rangle \equiv \langle T^2 \rangle$ we identify:
  a BFKL term: $\partial_Y \langle T^2 \rangle \propto \langle T^2 \rangle$
  a saturation term (pomerons merging): $\partial_Y \langle T^2 \rangle \propto \langle T^3 \rangle$
  a fluctuation term (pomeron splitting): $\partial_Y \langle T^2 \rangle \propto \langle T \rangle$
- pomeron merging + pomerons splitting $\Rightarrow$ pomeron loop equations
- We consider the fluctuations to be local $\rightarrow$ $\langle T_{\mathbf{xy}} \rangle = \kappa \alpha_s^2 |\mathbf{x} - \mathbf{y}|^4 \langle n_{\mathbf{xy}} \rangle$, with $\kappa \approx \mathcal{O}(1)$
- After Fourier transform ($r = |\mathbf{x} - \mathbf{y}| \rightarrow k$) and an coarse-graining approximation to get rid of the impact parameter dependence of the fluctuation term, we have

$$\begin{aligned}
\partial_Y \langle T_k \rangle &= \bar{\alpha} \chi(-\partial_L) \langle T_k \rangle - \langle T_{k,k} \rangle \\
\partial_Y \left\langle T^2_{k_1,k_2} \right\rangle &= \bar{\alpha} \chi(-\partial_{L_1}) \left\langle T^2_{k_1,k_2} \right\rangle - \left\langle T^3_{k_1,k_1,k_2} \right\rangle + (1 \leftrightarrow 2) \\
&\quad + \bar{\alpha} \kappa \alpha_s^2 k_1^2 \delta(k_1^2 - k_2^2) \langle T_{k_1} \rangle \\
&\vdots
\end{aligned} \tag{14}$$

where $L_i = \log(k_i^2 / k_0^2)$.

Langevin equation

- The hierarchy of pomeron loop equations can be rewriten under a form of a Langevin equation for the event–by–event amplitude

$$\partial_Y T(L,Y) = \bar{\alpha}\left[\chi(-\partial_L)T(L,Y) - T^2(L,Y) + \sqrt{\kappa\alpha_s^2 T(L,Y)}\eta(L,Y)\right] \quad (15)$$

where η is a Gaussian white noise satisfying:

$$\begin{aligned}\langle\eta(L,Y)\rangle &= 0 \\ \langle\eta(L_1,Y_1)\eta(L_2,Y_2)\rangle &= \frac{4}{\bar{\alpha}}\delta(L_1-L_2)\delta(Y_1-Y_2)\end{aligned} \quad (16)$$

- Langevin equation $\equiv$ BK $+$ noise term
- In the diffusive approximation for $\chi(-\partial_L)$, this equation is equivalent to the sFKPP $\equiv$ FKPP $+$ noise
- For each realization of the noise (single event), the evolved amplitude shows a travelling-wave patern, *i.e.*, the geometric scaling is preserved

Some consequences

- Different realizations of the target lead to a dispersion of the solutions, and then in the saturation momentum $\rho_s \equiv \ln(Q_s^2/k_0^2)$ which define the position of the wavefront
- The saturation scale is now a random variable whose average value is given by

$$\langle Q_s^2(Y) \rangle = \exp[\lambda^* Y] \tag{17}$$

- The dispersion in the position of the individual fronts is given by

$$\sigma^2 = \langle \rho_s^2 \rangle - \langle \rho_s \rangle^2 = D\bar{\alpha}Y \tag{18}$$

where D is the diffusion coefficient
- The probability distribution of ρ_s is, to a good approximation, a Gaussian

$$P_Y(\rho_s) \simeq \frac{1}{\sqrt{\pi\sigma^2}} \exp\left[-\frac{(\rho_s - \langle\rho_s\rangle)^2}{\sigma^2}\right] \tag{19}$$

Physical amplitude

- For each single event the geometric scaling is preserved, but the speed of the wave is smaller than the speed predicted by BK equation. This speed, or the (average) saturation exponent, has been found to be [3]

$$\lambda^* \simeq \lambda - \frac{\pi^2 \gamma_c \chi''(\gamma_c)}{\ln(1/\alpha_s^2)}. \tag{20}$$

- The average (physical) amplitude is determined by ($X \equiv \ln(1/r^2 k_0^2)$)

$$\langle T(X, \rho_s) \rangle = \int_{-\infty}^{+\infty} d\rho_s \, P_Y(\rho_s) T(X, \rho_s). \tag{21}$$

- For sufficiently high energies the physical amplitudes do not show geometric scaling
- Additional dependencies upon Y, through the front dispersion σ, lead to the replacement of the geometric scaling by the so-called diffusive scaling [3,4,5]

$$\langle T(X, \rho_s) \rangle = \mathcal{T} \left(\frac{X - \langle \rho_s \rangle}{\sqrt{\bar{\alpha} D Y}} \right). \tag{22}$$

AGBS model with fluctuations

- The scattering amplitude given by AGBS model act as a single event one to be inserted into the expression for the average scattering amplitude ($\rho = \ln(k^2/k_0^2)$)

$$\left\langle T^{AGBS}(\rho, \rho_s) \right\rangle = \int_{-\infty}^{+\infty} d\rho_s \, P_Y(\rho_s) T^{AGBS}(\rho, \rho_s). \tag{23}$$

  which is now used in the F_2 expression to correctly describe the last HERA data with the fluctuations included [6]

- D and λ^* are analytically known only in the asymptotic limit $\alpha_s \to 0$, and then are treated as free parameters

  Free parameters: R_p, χ_c'', k_0^2, λ e D

  Fixed parameters: $\gamma_c = 0.6275$ e $\bar{\alpha} = 0.2$

- Kinematical range:

$$\begin{cases} x \leq 0.01, \\ 0.045 \leq Q^2 \leq 150 \text{ GeV}^2 \end{cases}$$

- We consider only light quarks in two cases: $m_{u,d,s} = 50$ MeV e $m_{u,d,s} = 140$ MeV
- As usual, we rescaled the H1 data by a factor 1.05, within the normalization uncertainty

Results

- $m_{u,d,s} = 50$ MeV

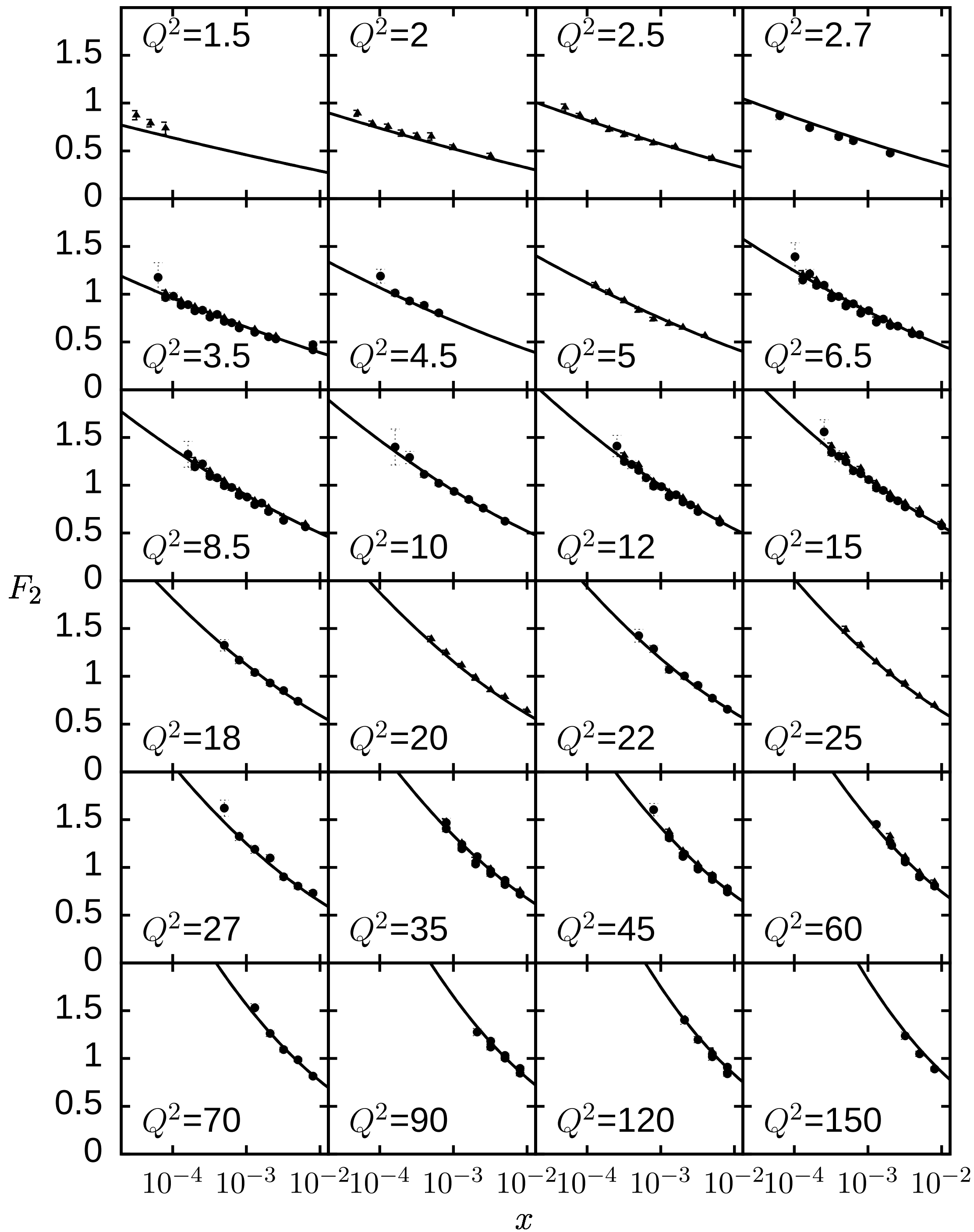
	χ^2/n.o.p	k_0^2 ($\times 10^{-3}$)	λ	R(GeV^{-1})	$\chi''(\gamma_c)$	D ($\times 10^{-2}$)
$\tilde{T}_Y^{\rm AGBS}$	0.949	3.79 ± 0.30	0.213 ± 0.003	3.576 ± 0.059	4.69 ± 0.23	0
$\left\langle \tilde{T}_Y^{\rm AGBS} \right\rangle$	0.949	3.79 ± 0.30	0.213 ± 0.003	3.576 ± 0.059	4.69 ± 0.23	0.0 ± 1.1

- $m_{u,d,s} = 140$ MeV

	χ^2/n.o.p	k_0^2 ($\times 10^{-3}$)	λ	R(GeV^{-1})	$\chi''(\gamma_c)$	D ($\times 10^{-3}$)
$\tilde{T}_Y^{\rm AGBS}$	0.942	1.69 ± 0.16	0.176 ± 0.004	4.83 ± 0.12	6.43 ± 0.29	0
$\left\langle \tilde{T}_Y^{\rm AGBS} \right\rangle$	0.942	1.69 ± 0.16	0.176 ± 0.004	4.83 ± 0.12	6.43 ± 0.29	0.0 ± 9.6

- Only ZEUS with $Q^2 < 50$ GeV^2, $m_{u,d,s} = 140$ MeV

	χ^2/n.o.p	k_0^2 ($\times 10^{-3}$)	λ	R(GeV^{-1})	$\chi''(\gamma_c)$	D
$\tilde{T}_Y^{\rm AGBS}$	0.778	1.97 ± 0.22	0.177 ± 0.006	4.68 ± 0.14	5.95 ± 0.94	0
$\left\langle \tilde{T}_Y^{\rm AGBS} \right\rangle$	0.768	1.38 ± 0.12	0.120 ± 0.010	5.46 ± 0.04	5.46 ± 0.55	1.78 ± 0.38



Q^2=1.5
Q^2=2
Q^2=2.5
Q^2=2.7
Q^2=3.5
Q^2=4.5
Q^2=5
Q^2=6.5
Q^2=8.5
Q^2=10
Q^2=12
Q^2=15
Q^2=18
Q^2=20
Q^2=22
Q^2=25
Q^2=27
Q^2=35
Q^2=45
Q^2=60
Q^2=70
Q^2=90
Q^2=120
Q^2=150
F_2
0
0.5
1
1.5
10^{-4} 10^{-3} 10^{-2}
x

Conclusions

- We conclude that, in the framework of the AGBS model, there is no evidence of fluctuations in DIS at HERA energies, because:
  We obtained a good fit to the F_2 structure function, shown by the good $\chi^2/$n.o.p
  For the whole data set (H1 + ZEUS), the diffusion coefficient $D \to 0$
  This indicate that a mean field treatment (BK), with fixed α_s, is enough to investigate high energy QCD phenomenology, at least at HERA energies
- For large energies (LHC) will be possible to see if the fluctuations are really important to the evolution at high energy, or if they are suppressed by the running of the coupling constant α_s

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