$\gamma^* - p$ cross section at high energies in momentum space

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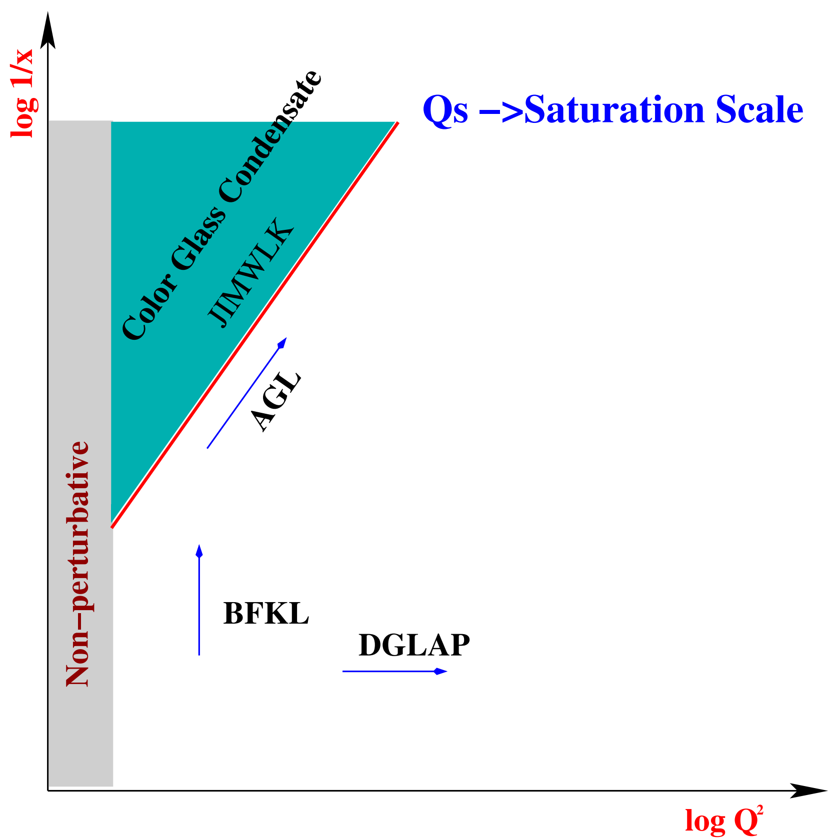
Introduction

- One of the most intriguing problems in Quantum Chromodynamics is the growth of the cross sections for hadronic interactions with energy; the increase of energy causes a fast growth of the gluon density and consequently of the cross section

- It is believed that at this regime gluon recombination might be important and it would decrease the growth of the parton density; this is called saturation

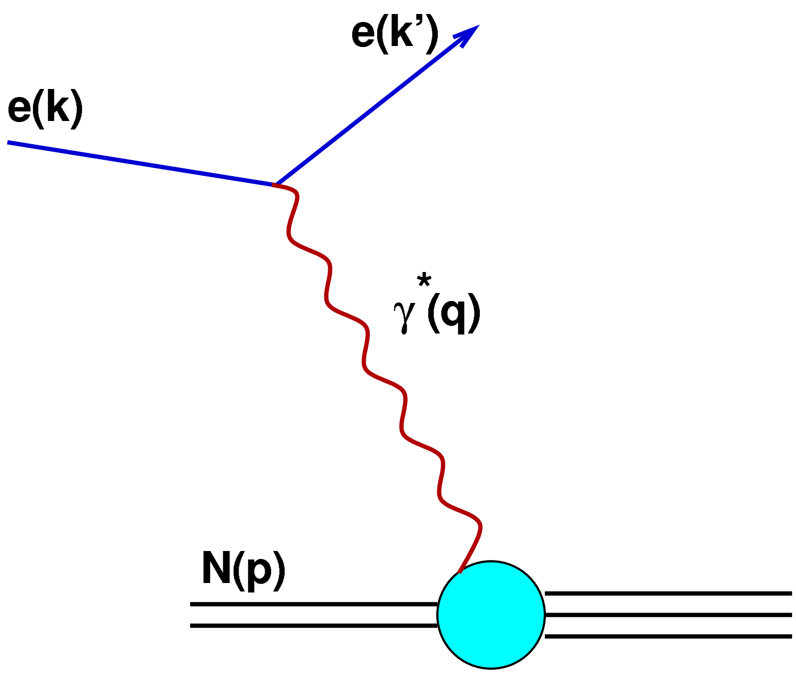


- $Q_s(Y)$ is the so called saturation scale

- The saturarion effects are important for all $Q \lesssim Q_S(Y)$, which is known as saturation region

Deep Inelastic Scattering (DIS)

Kinematics and variables



- The total energy squared of the photon-nucleon system

$$s = (p+q)^2$$

- Photon virtuality

$$q^2 = (k - k') = -Q^2 < 0$$

- The Bjorken variable

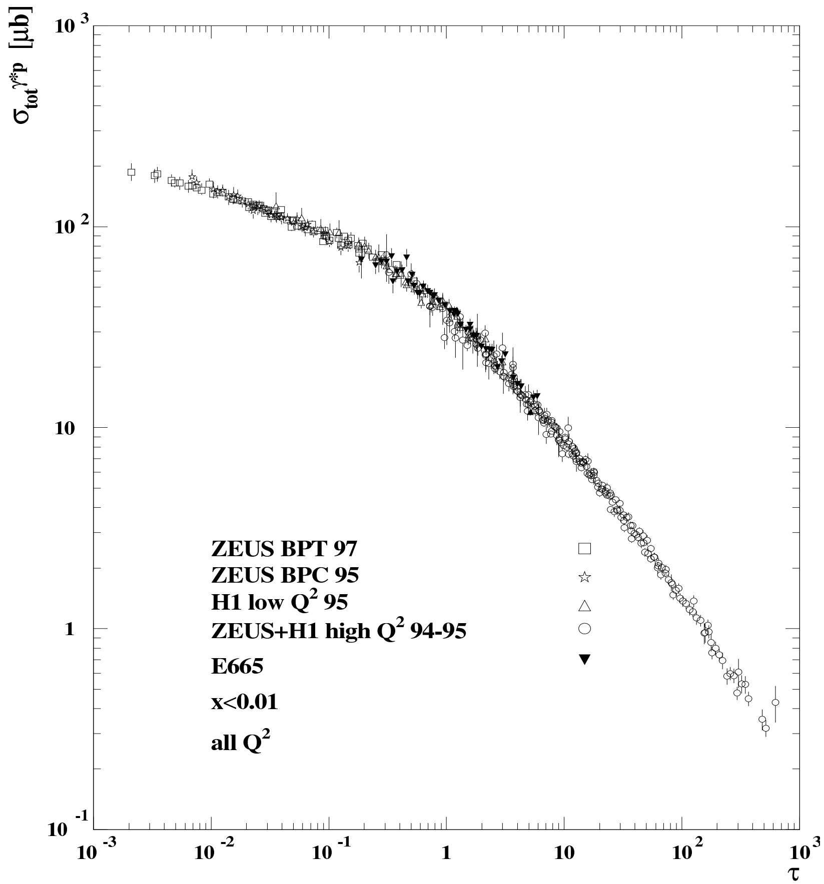
$$x \equiv x_{Bj} = \frac{Q^2}{2p\cdot q} = \frac{Q^2}{Q^2 + s}$$

- The high energy limit:

$$s \to \infty, \quad x \approx \frac{Q^2}{s} \to 0$$

Geometric Scaling

- Geometric scaling is a phenomenological feature of high energy deep inelastic scattering (DIS) which has been observed in the HERA data on inclusive $\gamma^* - p$ scattering, which is expressed as a scaling property of the virtual photon-proton cross section

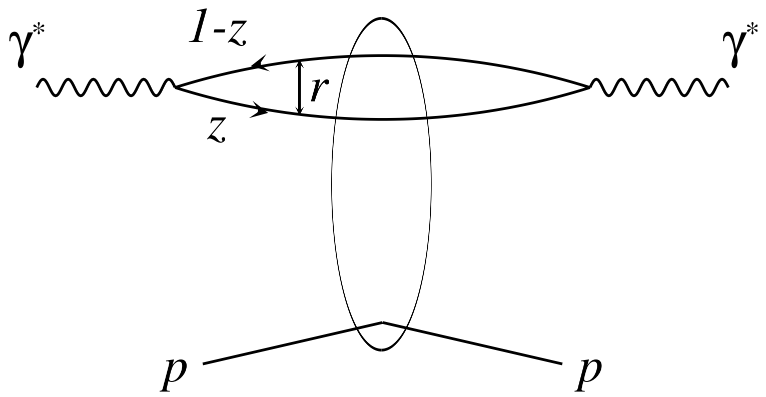


$$\sigma^{\gamma^* p}(Y,Q) = \sigma^{\gamma^* p}(\tau)\,, \quad \tau = \frac{Q^2}{Q_s^2(Y)}$$

where Q is the virtuality of the photon, $Y = \log 1/x$ is the total rapidity and $Q_s(Y)$ is the saturation scaleand is an increasing function of Y.

Dipole model

- It is convenient to work within the QCD dipole frame of DIS



- In the LLA of perturbative QCD (pQCD), the cross section factorizes as

$$\sigma_{T,L}^{\gamma^* p}(Y,Q) = \int d^2r \int_0^1 dz\, \left|\Psi_{T,L}(r,z;Q^2)\right|^2 \sigma_{dip}^{\gamma^* p}(r,Y),$$

$\sigma_{dip}^{\gamma^* p}(Y,\mathbf{r})$ is the dipole-proton cross section, z is the fraction of photon's momentum carried by the quark and $\mathbf{r}$ is the transverse separation of the quark-anti-quark pair

Dipole model

- The wavefunctions are obtained from QED

$$|\Psi_T(r,z;Q^2)|^2 = \frac{2N_c\alpha_{em}}{4\pi^2}\sum_q e_q^2\left\{\left[z^2+(1-z)^2\right]\bar{Q}_q^2K_1^2(\bar{Q}_qr)+m_q^2K_0^2(\bar{Q}_qr)\right\}$$

and

$$|\Psi_L(r,z;Q^2)|^2 = \frac{2N_c\alpha_{em}}{4\pi^2}\sum_q e_q^2\left\{4Q^2z^2(1-z)^2K_0^2(\bar{Q}_qr)\right\}$$

where $\bar{Q}_q = z(1-z)Q^2+m_q^2$, m_q the light quark mass and $K_{0,1}$ are the Mc Donald functions of rank zero and one, respectively.

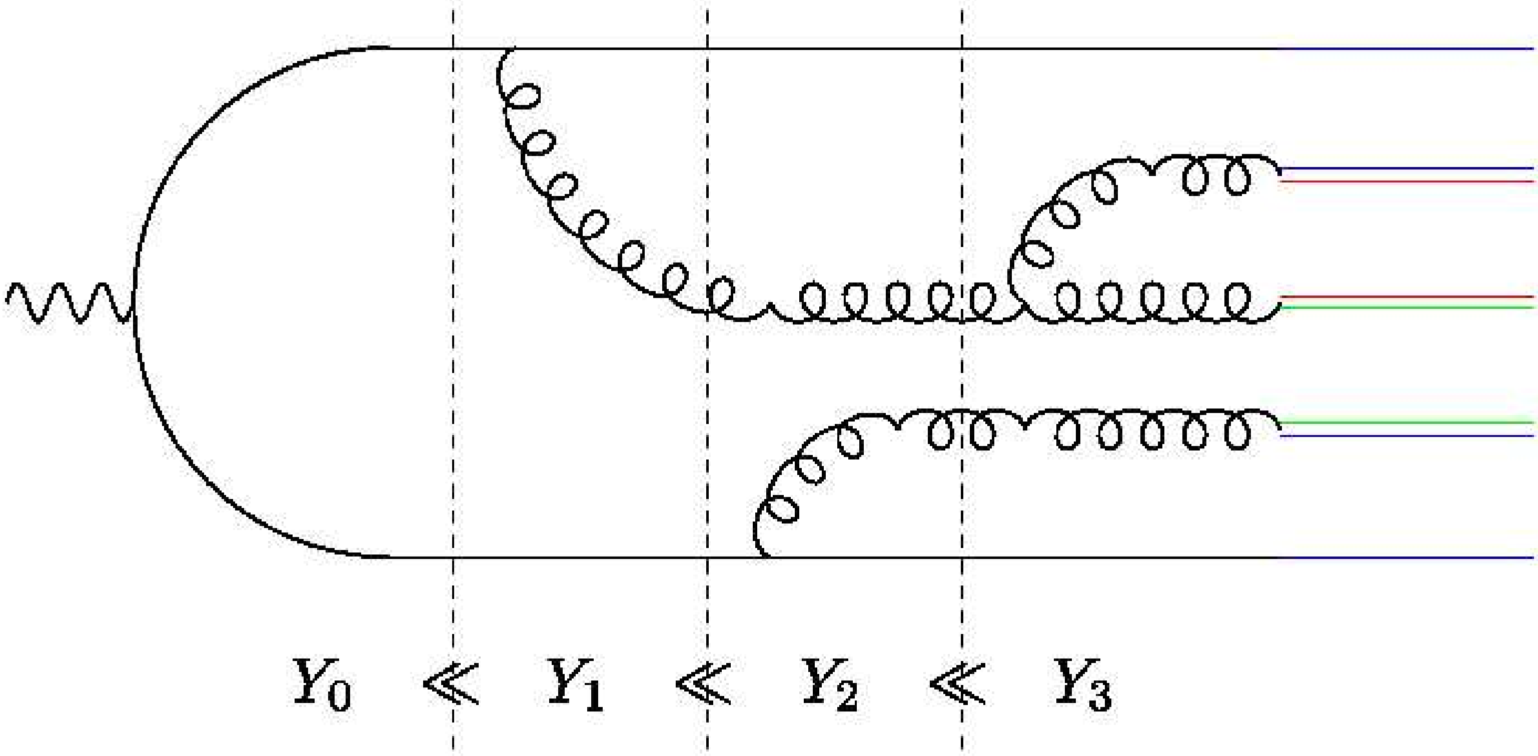
- If one treats the proton as a homogeneous disk of radius R_p

$$\sigma_{dip}^{\gamma^* p}(r,Y) = 2\pi R_p^2 T(r,Y)$$

- Where $T(r,Y)$ is the dipole scattering amplitude

The evolution

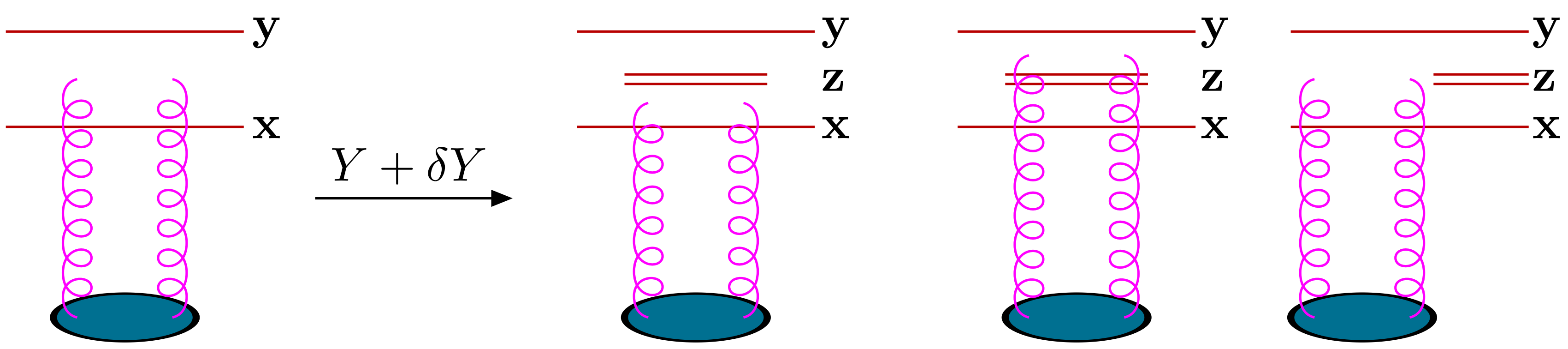
- Consider a fast-moving $q\bar{q}$



- In the large N_c limit the gluons emitted can be replaced by quark-anti-quark pairs, which interact with the target via two gluon exchanges

BFKL equation

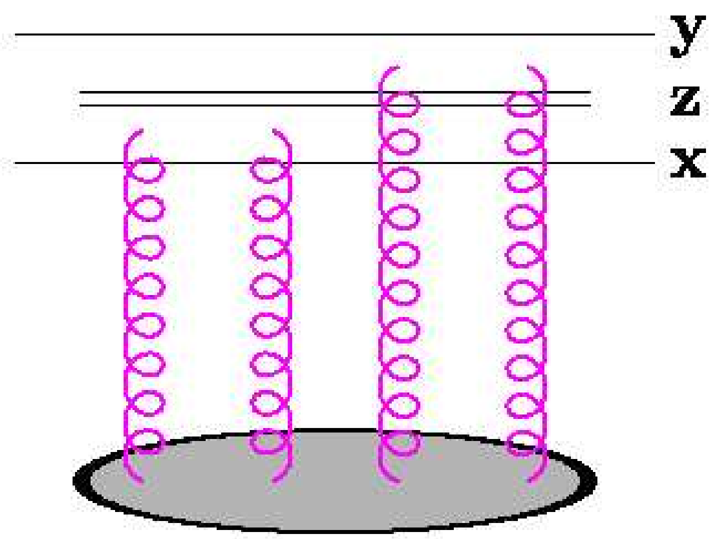
- Consider a $q\bar{q}$ dipole at large rapidities



$$\partial_Y T(\mathbf{x},\mathbf{y}) = \bar{\alpha} \int d^2 z \, \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2} \left[T(\mathbf{x},\mathbf{z}) + T(\mathbf{z},\mathbf{y}) - T(\mathbf{x},\mathbf{y})\right]$$

- BFKL equation
- $\bar{\alpha} = \frac{\alpha_s N_c}{\pi}$
- The solution $\Rightarrow T \propto e^{\omega Y} \Rightarrow$ violates unitarity $T(x,y) \leq 1$

Balitsky-Kovchegov equation



- If one considers multiple scatterings
- In the evolution, these multiple scatterings appear as a term proportional to T^2

$$\partial_Y T(\mathbf{x},\mathbf{y},Y) = \bar{\alpha}\int d^2\mathbf{z}\frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2}\left[T(\mathbf{x},\mathbf{z},Y)+T(\mathbf{z},\mathbf{y},Y)-T(\mathbf{x},\mathbf{y},Y)\right.$$
$$\left.-T(\mathbf{x},\mathbf{z},Y)T(\mathbf{z},\mathbf{y},Y)\right]$$

- BK equation
- The quadratic term is important when $T \approx 1$

BK equation in momentum space

- Let us consider that the amplitude T is independent of the impact parameter $\mathbf{b} = \frac{\mathbf{x}+\mathbf{y}}{2}$ and of the direction of $\mathbf{r} = \mathbf{x} - \mathbf{y}$

$$T(\mathbf{x}, \mathbf{y}) \to T(\mathbf{r}, \mathbf{b}) \to T(r)$$

- We define the forward scattering amplitude in momentum space $T(k, Y)$

$$T(k, Y) = \int_0^\infty \frac{dr}{r} J_0(kr) T(r, Y)$$

- The BK equation then reads

$$\partial_Y T(k) = \underbrace{\frac{\bar{\alpha}}{\pi} \int \frac{dp^2}{p^2} \left[\frac{p^2 T(p) - k^2 T(k)}{|k^2 - p^2|} + \frac{k^2 T(k)}{\sqrt{4p^4 + k^4}} \right]}_{\bar{\alpha}\chi(-\partial_L)} - \bar{\alpha} T^2(k)$$

$$\partial_Y T = \bar{\alpha}\chi(-\partial_L) T - \bar{\alpha} T^2,$$

- In this equation

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1 - \gamma)$$

  is the characteristic function of the well known Balitsky-Fadin-Kuraev-Lipatov (BFKL) kernel, and $L = \log\left(k^2/k_0^2\right)$, where k_0 is some fixed low momentum scale

- The kernel χ is an integro-differential operator which may be defined with the help of the formal series expansion

BK and FKPP equations

- Considering the kernel χ by the first three terms of the expansion (saddle point or difusse approximation).
- $\chi \Rightarrow$, second order differential operator.
- The change of variables

$$\bar{\alpha} Y \sim t, \quad \log(k^2/k_0^2) \sim x, \quad T \sim u$$

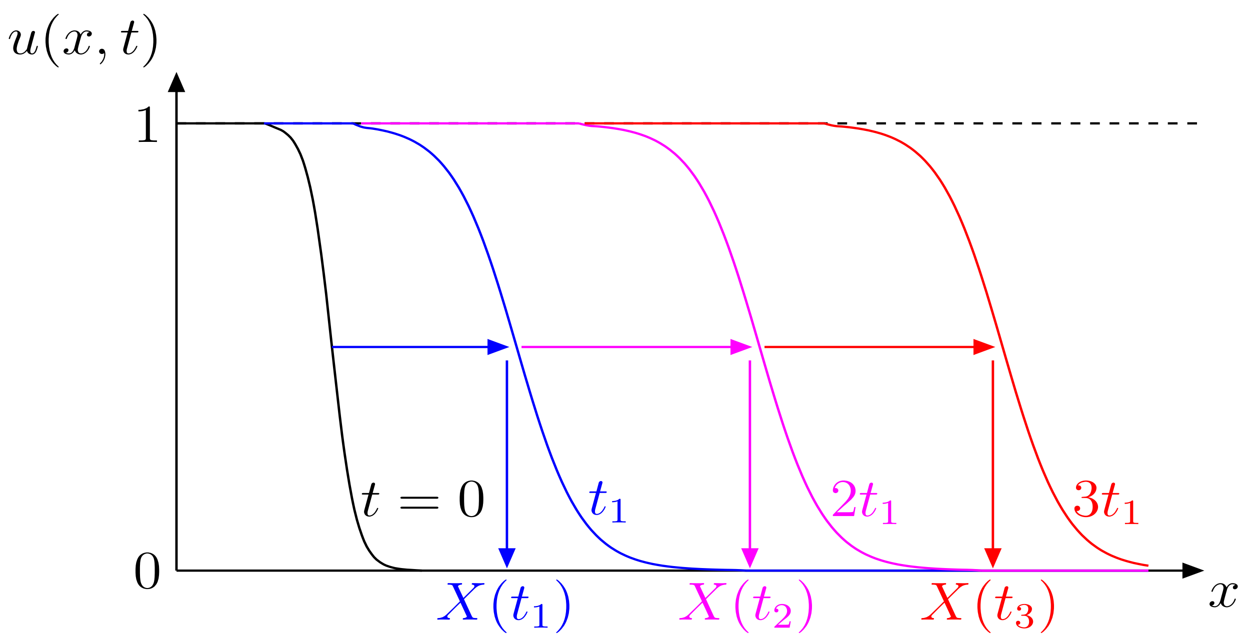
  BK equation reduces to FKPP equation.
- The Fisher and Kolmogorov-Petrovsky-Piscounov (FKPP) equation is a known equation in non-equilibrium statistical physics, whose dynamics is called reaction-diffusion dynamics,

$$\partial_t u(x,t) = \partial_x^2 u(x,t) + u - u^2,$$

  where t is time and x is the coordinate.

Traveling wave solutions

- The FKPP evolution equation admits the so-called traveling wave solutions
  - This implies that the solution $u(x,t)$ takes the form $u(x - v_c t)$, for large values of x at a critical speed v_c.
  - For a traveling wave solution one can define the position of a wave front $x(t) = v_c t$, irrespective of the details of the nonlinear effects
  - At larges times, the shape of a traveling wave is preserved during its propagation, and the solution becomes only a function of the scaling variable $x - v_c t$



Traveling waves and saturation

- Translation:

$$
\begin{array}{rcl}
\text{Time } t & \longrightarrow & Y \\
\text{Space } x & \longrightarrow & L = \log(k^2/k_0^2) \\
\text{Wave front } u(x - vt) & \longrightarrow & T(L - vY) \\
\text{Traveling Waves} & \longrightarrow & \text{Geometric Scaling}
\end{array}
$$

- In the language of saturation physics the position of the wave front is nothing but the saturation scale

$$\text{(position)} \quad x(t) = v_c t \Rightarrow Q_s^2(Y) = k_0^2 e^{v_c Y}$$

and the scaling corresponds to the geometric scaling

$$\text{(front)} \quad u(t) = u(x - v_c t) = e^{(-\gamma_c(x - v_c t))} \Rightarrow T = T(k^2/Q_s^2) = |k^2/Q_s^2|^{-\gamma_c}$$

with

$$v_c = \min \bar{\alpha} \frac{\chi(\gamma)}{\gamma} = \bar{\alpha} \frac{\chi(\gamma_c)}{\gamma_c} = \bar{\alpha} \chi'(\gamma_c)$$

where γ_c is the saddle point of the exponential phase factor. This fixes, for the BFKL kernel, $\gamma_c = 0.6275...$, $v_c = 4.88\bar{\alpha}$

Transition to saturation

- The goal of this work is to study the connection between the traveling wave solution and the saturation region

- These different domains can be parametrized as

$$T(\tau, Y)_{k \ll Q_s} = c - \log\left(\frac{k}{Q_s(Y)}\right)$$

and

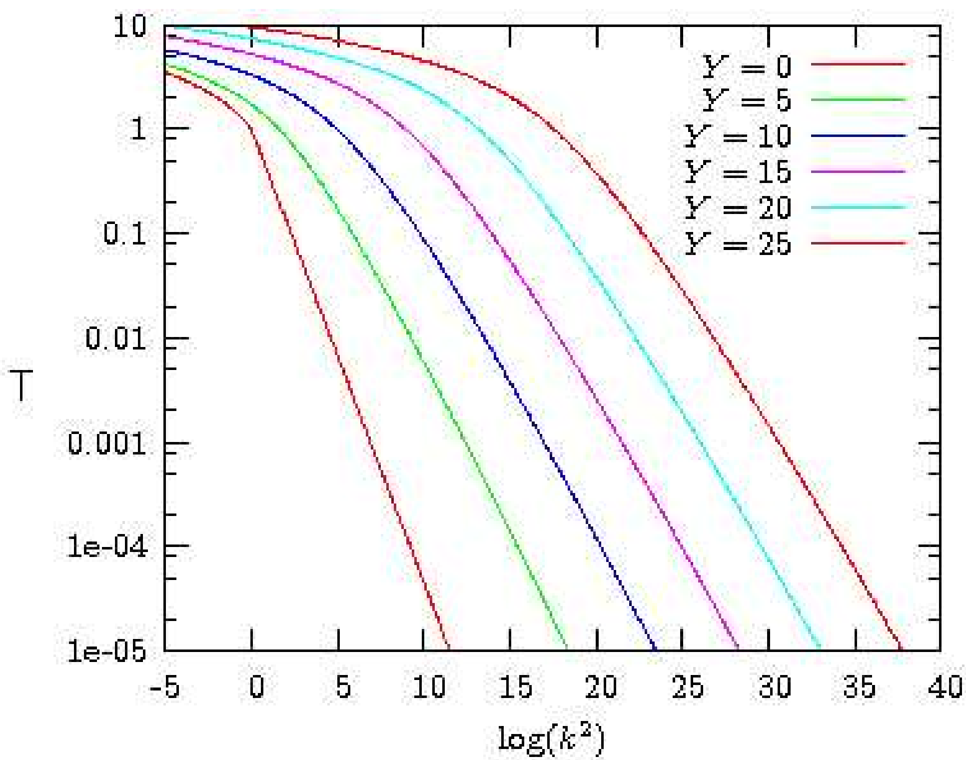
$$T(\tau, Y)_{k \gg Q_s} \approx \left(\frac{k^2}{Q_s^2(Y)}\right)^{-\gamma_c} \log\left(\frac{k^2}{Q_s^2(Y)}\right) \exp\left(-\frac{\log^2\left(\frac{k^2}{Q_s^2(Y)}\right)}{2\bar{\alpha}\chi''(\gamma_c)Y}\right).$$

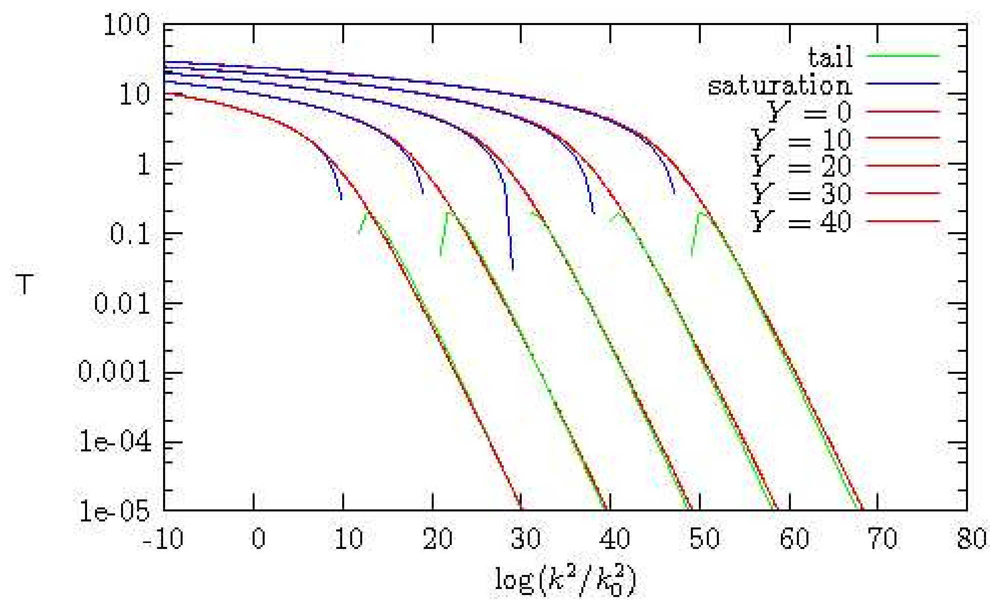
with

$$Q_s^2(Y) = k_0^2 \exp\left(\bar{\alpha}vY - \frac{3}{2\gamma_c}\log Y - \frac{3}{\gamma_c^2}\sqrt{\frac{2\pi}{\bar{\alpha}\chi''(\gamma_c)}\frac{1}{\sqrt{Y}}}\right).$$

Matching saturation and traveling waves

- The first attempt was to perform a matching between the two regions by imposing continuity of T and its first derivative at $k/Q_s(Y) = 1$
- However, this procedure does not necessarily imply a positive Fourier transform. Then, a better way to obtain the connection between the two regions is an interpolation through one expression only





The model

- The idea is to build the saturarion domain from the dilute one, in such a way that the scattering amplitude satisfies the correct asymptotic behaviour, that is,

$$T_{dil} \ll 1 \Rightarrow T \sim T_{dil}, \qquad T_{dil} \gg 1 \Rightarrow T \sim 1$$

- We do not want to use numerical solutions of BK in the momentum space, but rather use the properties of the solutions to build an analytical expression fot $T(k, Y)$.
- Our starting point is an expression which is monotically decreasing with L and reproduce (up to a logarithmic factor) the amplitude for diffusive scalling

$$T_{dil} = A \exp\left[-\gamma_c \log\left(\frac{k^2}{Q_s^2(Y)}\right) - \frac{L_{red}^2 - \log^2(2)}{2\bar{\alpha}\chi''(\gamma_c)Y}\right].$$

- with

$$L_{red} = \log\left[1 + \frac{k^2}{Q_s^2(Y)}\right] \quad \text{and} \quad Q_s^2(Y) = k_0^2 e^{\bar{\alpha}v_c Y}$$

- Based on the requirements above, and interpolation model was suggested by the authors and this reads

$$T(k, Y) = \left[\log\left(\frac{k}{Q_s} + \frac{Q_s}{k}\right) + 1\right](1 - \exp(-T_{dil}))$$

- **Phenomenology**$\Rightarrow$ Deep Inelastic Scattering: F_2 in the momentum space.

Observable: F_2 structure function

- The proton structure function F_2 can be obtained from the $\gamma^* p$ cross section through the relation

$$F_2(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}}\left[\sigma_T^{\gamma^* p}(x,Q^2) + \sigma_L^{\gamma^* p}(x,Q^2)\right].$$

- We can express the $\gamma^* p$ cross section in terms of $T(k,Y)$. After a bit of algebra one obtains

$$F_2(x,Q^2) = \frac{Q^2 R_p^2 N_c}{4\pi^2}\int_0^\infty \frac{dk}{k}\int_0^1 dz\,|\tilde{\Psi}(k^2,z;Q^2)|^2 T(k,Y),$$

where the wavefunction is now expressed in momentum space

$$|\tilde{\Psi}(k^2,z;Q^2)|^2 = \sum_q \left(\frac{4\bar{Q}_q^2}{k^2+4\bar{Q}_q^2}\right)^2 e_q^2 \left\{ \left[z^2+(1-z)^2\right]\left[\frac{4(k^2+\bar{Q}_q^2)}{\sqrt{k^2(k^2+4\bar{Q}_q^2)}}\text{arcsinh}\left(\frac{k}{2\bar{Q}_q}\right)\right.\right.$$
$$\left. + \frac{k^2-2\bar{Q}_q^2}{2\bar{Q}_q^2}\right] + \frac{4Q^2z^2(1-z)^2+m_q^2}{\bar{Q}_q^2}\left.\left[\frac{k^2+\bar{Q}_q^2}{\bar{Q}_q^2} - \frac{4\bar{Q}_q^4+2\bar{Q}_q^2k^2+k^4}{\bar{Q}_q^2\sqrt{k^2(k^2+4\bar{Q}_q^2)}}\text{arcsinh}\left(\frac{k}{2\bar{Q}_q}\right)\right]\right\}$$

Fitting procedure

- We fit the HERA (H1 and ZEUS) data in the kinematic range

$$0.045 \leq Q^2 \leq 150\text{GeV}^2, \quad x \leq 0.01$$

and the fixed parameters are $\gamma_c = 0.6275$, $\bar{\alpha} = 0.2$ and $A = 1.0$

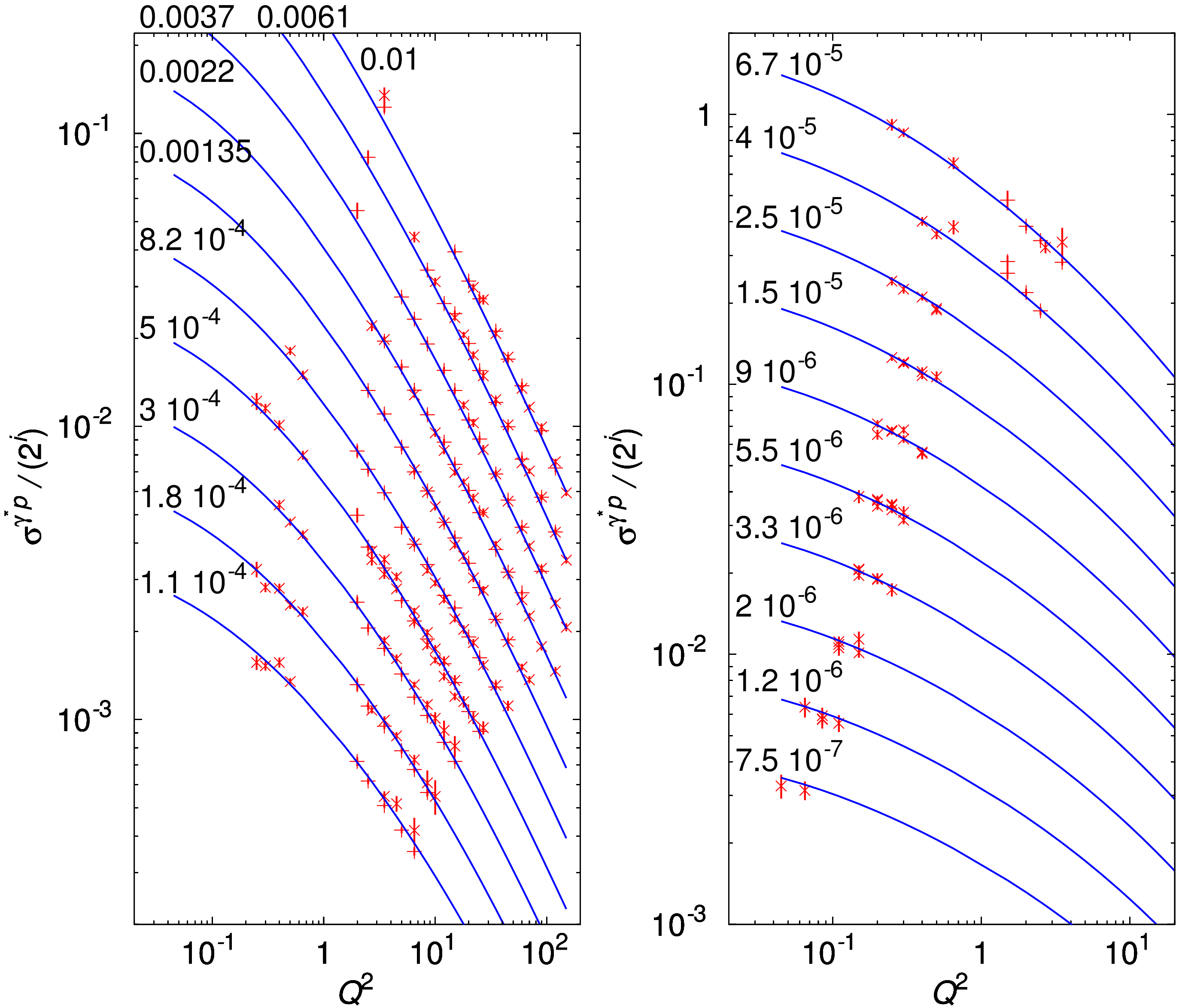
- The values of light quark masses used were $m_q = 50, 140$ MeV
- The charm mass value used here $m_c = 50$ MeV and $m_c = 1.3$ GeV.
- 279 datapoints.
- We have allowed for a 5% renormalisation uncertainty on the H1 data.

Results:

Masses	k_0^2 $(10^{-3}$ GeV$^2)$	v_c	χ_c''	R_p (GeV^{-2})	χ^2/nop
$m_q = 50$ MeV, $m_c = 50$ MeV	3.782 ± 0.293	1.065 ± 0.018	4.691 ± 0.221	2.770 ± 0.045	0.960
$m_q = 50$ MeV, $m_c = 1.3$ GeV	7.155 ± 0.624	0.965 ± 0.017	2.196 ± 0.161	3.215 ± 0.065	0.988
$m_q = 140$ MeV, $m_c = 1.3$ GeV	3.917 ± 0.577	0.807 ± 0.025	2.960 ± 0.279	4.142 ± 0.167	1.071

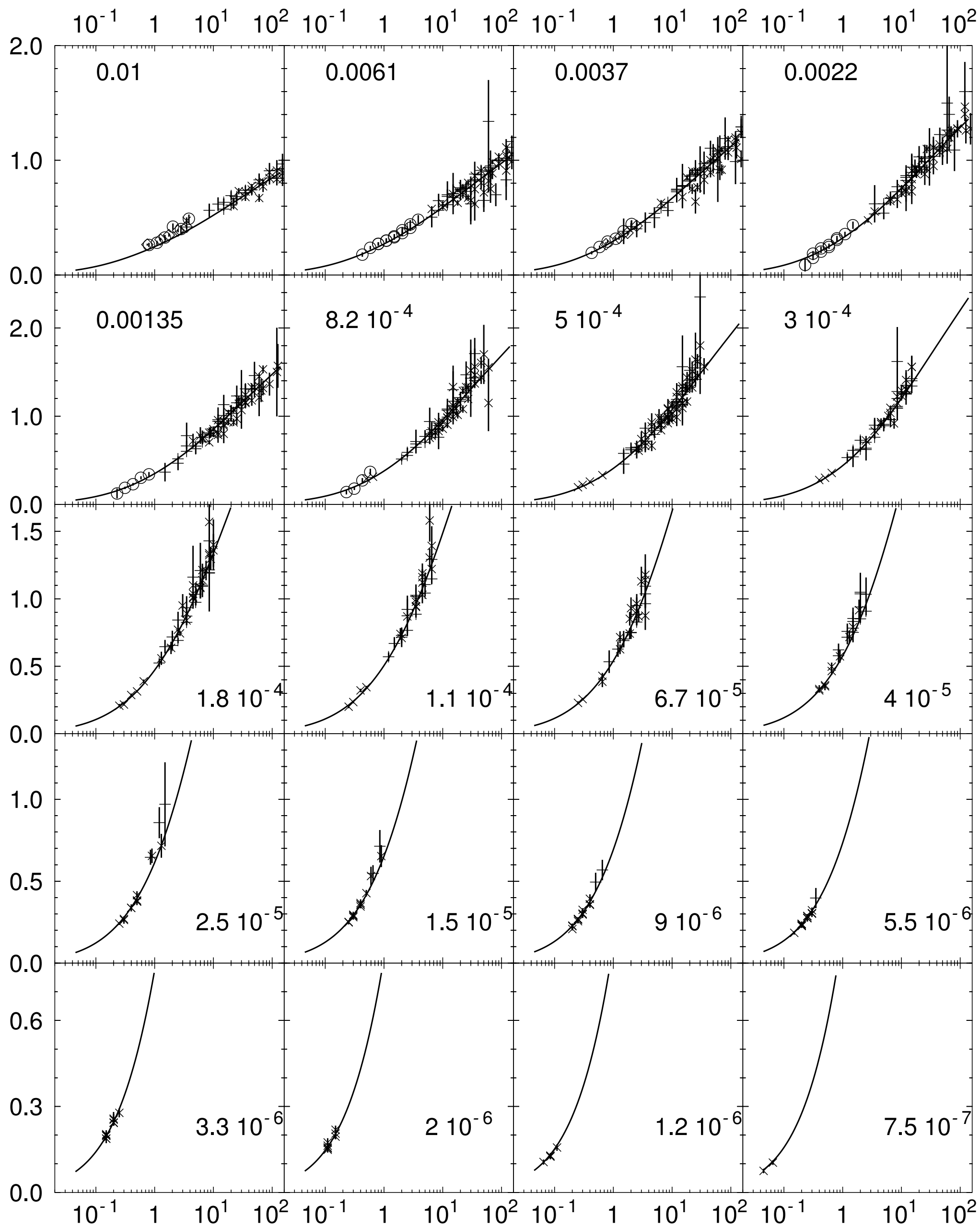
Results

$$\sigma^{\gamma^* p} = \frac{4\pi\alpha_{em}}{Q^2} F_2(x, Q^2)$$



ZEUS, H1

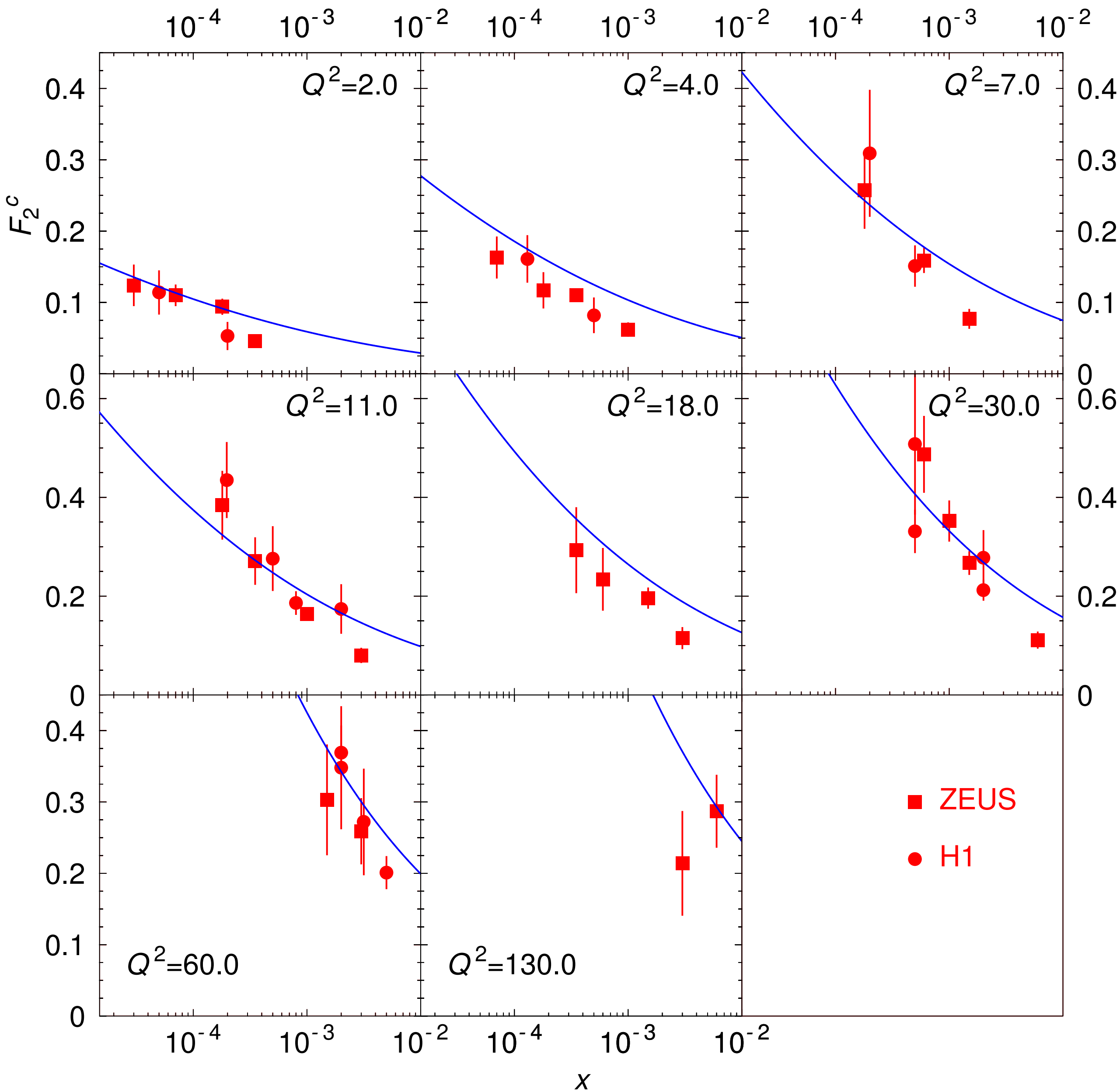
Results



F_2 as a function of Q^2
fixed Bjorken x

ZEUS, H1 and NMC data

Predictions to F_2^C



Conclusions...

- We have proposed a parametrization for the dipole cross section, based on the solutions of BK equation in the context of the traveling waves solutions of the FKPP equation.
- We have obtained a parametrization in the momentum space, with a positive Fourier transform.
- The χ^2/nop reach very good results (including charm mass).
- Good predictions for the F_2^C.
- Perform some investigations with other observables.