Dipole scattering amplitude in momentum space: investigating fluctuations at HERA

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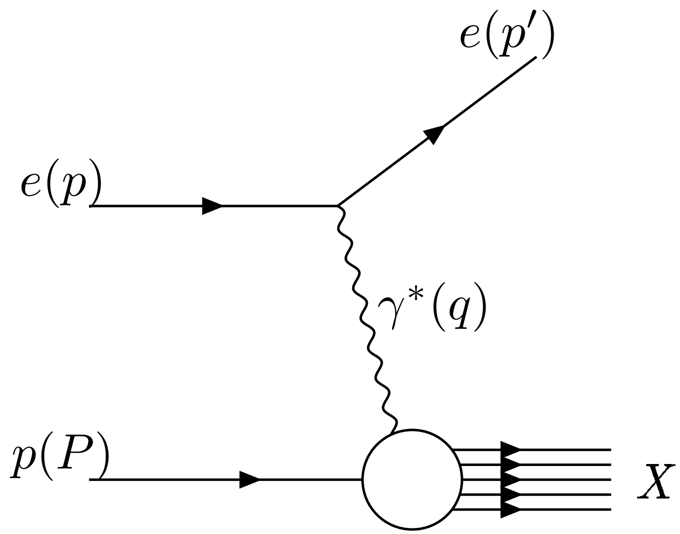
Based on work done with M. B. Gay Ducati, E. Basso and E. G. Oliveira

Outline

- Deep Inelastic Scattering and QCD at high energies
- The Dipole Frame
- Evolution of Scattering Amplitudes
- Stochasticity in High energy QCD
- Description of DIS data: possible effects of fluctuations
- Analysis in Momentum Space

Deep Inelastic Scattering (DIS)



- The total energy squared of the photon-nucleon system

$$s = (P+q)^2$$

- Photon virtuality

$$q^2 = (p-p')^2 = -Q^2 < 0$$

- The Bjorken variable

$$x \equiv x_{Bj} = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{Q^2+s}$$

- The high energy limit:

$$s \to \infty, \quad x \approx \frac{Q^2}{s} \to 0$$

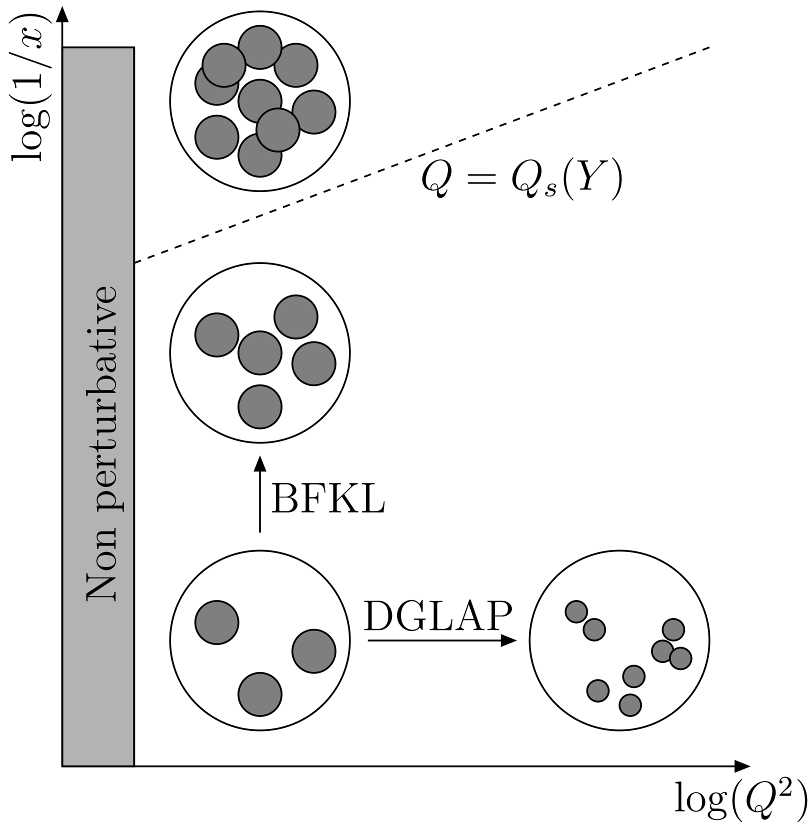
- The rapidity variable

$$Y \equiv \ln(1/x)$$

QCD at high energies

- As energy increases (with Q fixed) the gluon density grows fast and so does the cross sections for hadronic interactions
  - This is still a challenge in Quantum Chromodynamics

- At this regime gluon recombination and multiple scattering might be important to restore unitarity

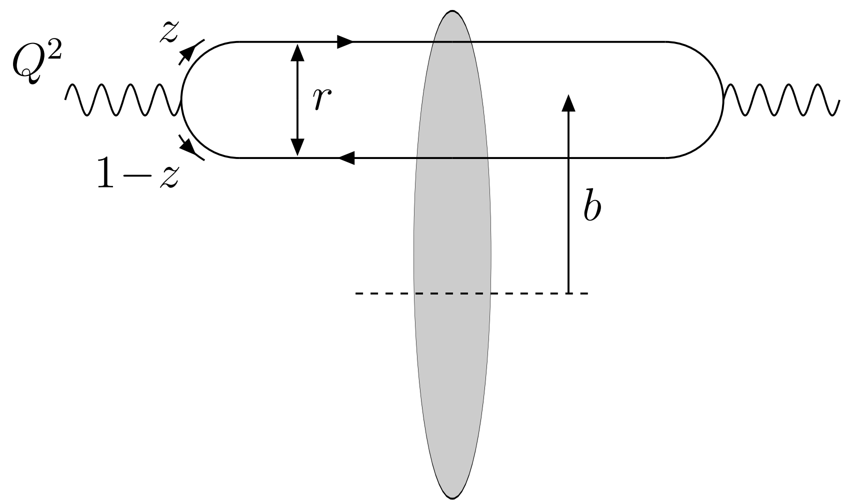


- $Q_s(Y)$ is the so called saturation scale

- The nonlinear saturation effects are important for all $Q \lesssim Q_S(Y)$, which is known as saturation region

Dipole frame

- In a frame where the proton carries most of the total energy one can consider that the photon fluctuates into a $q\bar{q}$ pair



- The cross section

$$\sigma_{T,L}^{\gamma^* p}(Y,Q) = \int d^2r \int_0^1 dz \, \left|\Psi_{T,L}(\mathbf{r}, z; Q^2)\right|^2 \sigma_{dip}^{\gamma^* p}(\mathbf{r}, Y), \tag{1}$$

$\sigma_{dip}^{\gamma^* p}(Y, \mathbf{r})$ is the dipole-proton cross section, z is the fraction of photon's momentum carried by the quark, $\mathbf{r}$ is the transverse size of the dipole and $\mathbf{b}$ is the impact parameter

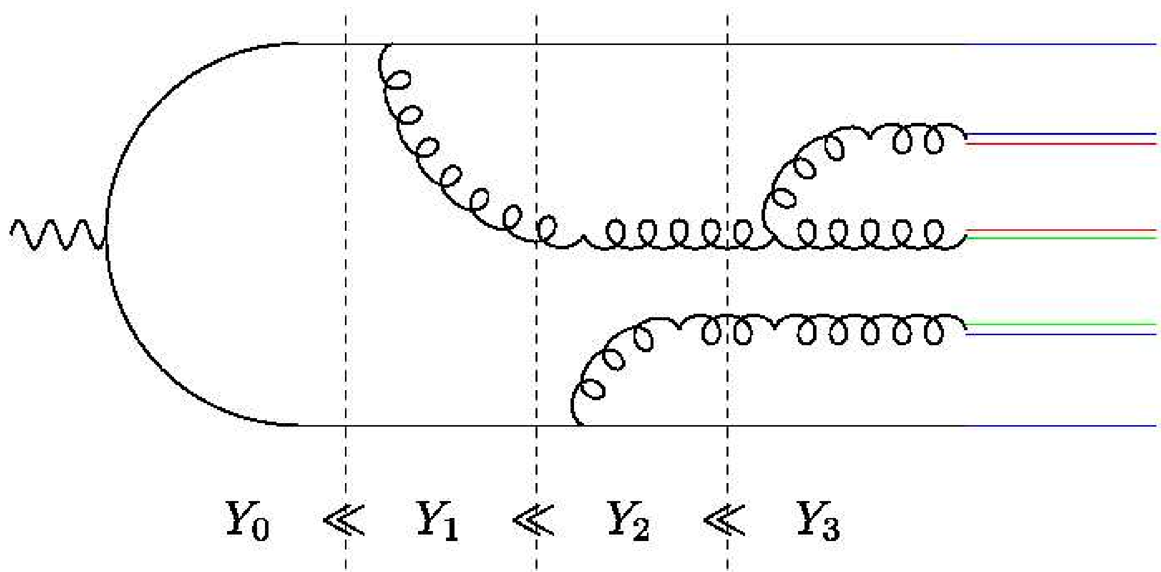
Dipole-proton cross section

- The dipole-proton cross section

$$\sigma_{dip}^{\gamma^* p}(\mathbf{r}, Y) = 2 \int d^2\mathbf{b} \, \langle T(\mathbf{r}, \mathbf{b}) \rangle_Y \qquad (2)$$

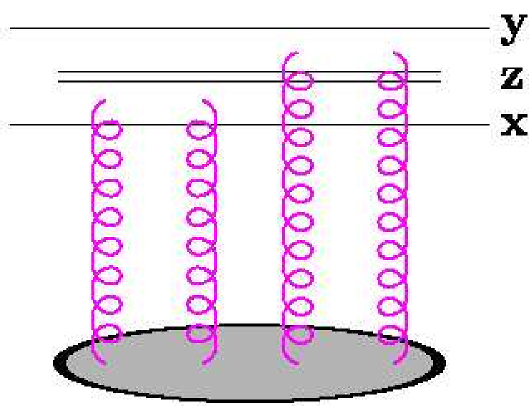
where $\langle T(\mathbf{r}, \mathbf{b}) \rangle_Y$ is the scattering amplitude for the dipole-proton scattering at a given impact parameter $\mathbf{b}$

- Large N_c limit: the gluons emitted can be replaced by quark-anti-quark pairs, which interact with the target via two gluon exchanges



Evolution of the scattering amplitude

Multiple scattering



- $\bar{\alpha} = \alpha_s N_c/\pi$
- Splitting probability

$$\mathcal{M}(\mathbf{x},\mathbf{y},\mathbf{z}) = \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2}$$

- Evolution equation for $\langle T(\mathbf{x},\mathbf{y})\rangle_Y$ [Balitsky, 1996]

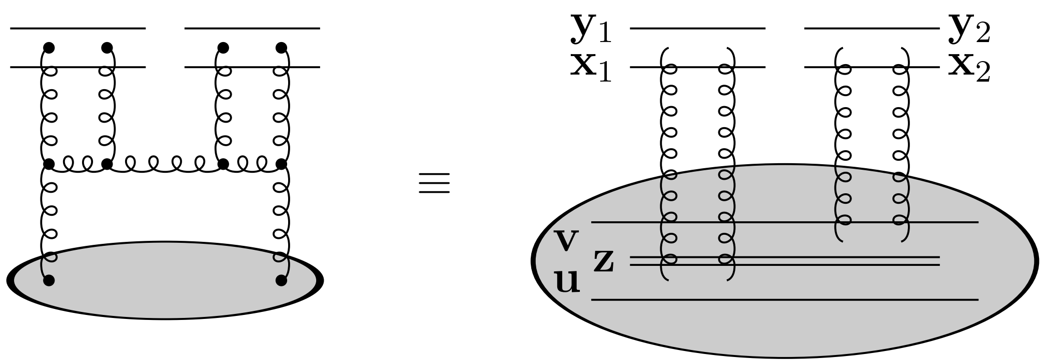
$$\partial_Y \langle T(\mathbf{x},\mathbf{y})\rangle_Y = \bar{\alpha}\int d^2z\,\mathcal{M}(\mathbf{x},\mathbf{y},\mathbf{z})\langle T(\mathbf{x},\mathbf{z}) + T(\mathbf{z},\mathbf{y}) - T(\mathbf{x},\mathbf{y}) - T(\mathbf{x},\mathbf{z})T(\mathbf{z},\mathbf{y})\rangle_Y$$

- First equation of the infinite Balitsky-JIMWLK hierarchy
- Mean field approximation $\langle T(\mathbf{x},\mathbf{z})T(\mathbf{z},\mathbf{y})\rangle \approx \langle T(\mathbf{x},\mathbf{z})\rangle\,\langle T(\mathbf{z},\mathbf{y})\rangle \rightarrow$ Balitsky-Kovchegov (BK) equation [Kovchegov, 1999, 2000]

$$\partial_Y \langle T_{\mathbf{xy}}\rangle_Y = \bar{\alpha}\int d^2z\,\mathcal{M}(\mathbf{x},\mathbf{y},\mathbf{z})\left[\langle T_{\mathbf{xz}}\rangle_Y + \langle T_{\mathbf{zy}}\rangle_Y - \langle T_{\mathbf{xy}}\rangle_Y - \langle T_{\mathbf{xz}}\rangle\,\langle T_{\mathbf{zy}}\rangle_Y\right] \quad (3)$$

Beyond the Balitsky hierarchy

- Balitsky-JIMWLK hierarchy is not complete: it misses **gluon number fluctuations** [Iancu, Triantafyllopoulos, 2005]



- Pomeron Loop Equations: $\left\langle T^{(k)}\right\rangle$ depends on $\left\langle T^{(k)}\right\rangle$, $\left\langle T^{(k+1)}\right\rangle$ and $\left\langle T^{(k-1)}\right\rangle$

$$\frac{\partial}{\partial Y}\langle T\rangle_Y \quad \propto \quad \alpha_s\left[\langle T\rangle_Y - \langle TT\rangle_Y\right] \tag{4}$$

$$\frac{\partial}{\partial Y}\langle TT\rangle_Y \quad \propto \quad \alpha_s\left[\langle TT\rangle_Y - \langle TTT\rangle_Y + \alpha_s^2\,\langle T\rangle_Y\right] \tag{5}$$

- The "mean field" BK equation is not consistent with fluctuations, related to discreteness in small-x evolution

BK equation in momentum space

- If one neglects the dependence on the impact parameter $\rightarrow$ equation for $\langle T(r,Y)\rangle \equiv T_Y(r)$, where $r = |\mathbf{r}| = |\mathbf{x}-\mathbf{y}|$
- After performing the Fourier transform

$$\tilde{T}_Y(k) = \frac{1}{2\pi}\int \frac{d^2r}{r^2}\, e^{i\mathbf{k}\cdot\mathbf{r}}\, T(r,Y) = \int_0^\infty \frac{dr}{r} J_0(kr) T_Y(r), \tag{6}$$

the amplitude $\widetilde{T}_Y(k)$ obeys the BK equation in momentum space

$$\partial_Y \tilde{T}_Y(k) = \frac{\bar{\alpha}}{\pi}\int \frac{dp^2}{p^2}\left[\frac{p^2\tilde{T}_Y(p) - k^2\tilde{T}_Y(k)}{|k^2-p^2|} + \frac{k^2\tilde{T}_Y(k)}{\sqrt{4p^4+k^4}}\right] - \bar{\alpha}\tilde{T}_Y^2(k) \tag{7}$$

or

$$\partial_Y \tilde{T} = \bar{\alpha}\chi(-\partial_L)\tilde{T} - \bar{\alpha}\tilde{T}^2 \tag{8}$$

where

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma) \tag{9}$$

and $L = \log(k^2/k_0^2)$, with k_0 some fixed soft scale.

BK and FKPP equations

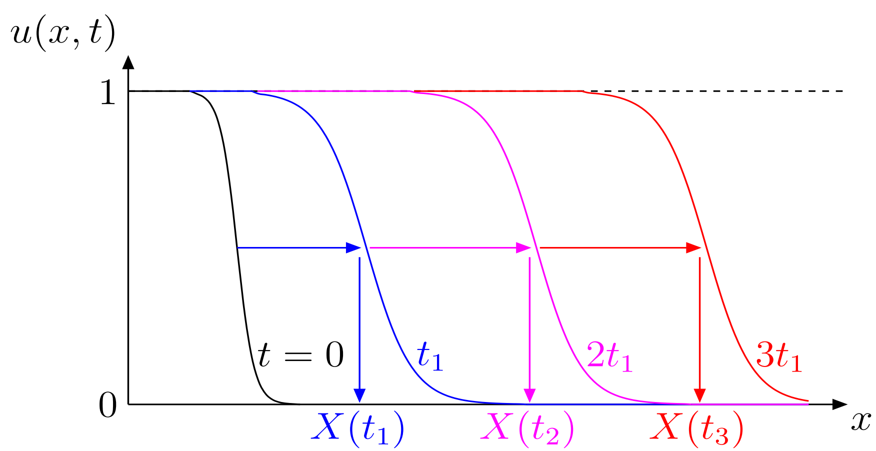
- $\chi(\gamma)$ is the characteristic function of the Balitsky-Fadin-Kuraev-Lipatov (BFKL) kernel
- After an approximation in the kernel and the change of variables [Munier and Peshcanski, 2003]

$$t \sim \bar{\alpha} Y, \quad x \sim \log(k^2/k_0^2), \quad u \sim \tilde{T} \tag{10}$$

BK equation reduces to Fisher and Kolmogorov-Petrovsky-Piscounov (FKPP) equation $\rightarrow$ reaction-diffusion dynamics,

$$\partial_t u(x,t) = \partial_x^2 u(x,t) + u - u^2, \tag{11}$$

- It admits the so-called traveling wave solutions: at large times, the shape of a traveling wave is preserved during its propagation: function of the scaling variable $x - v_c t$



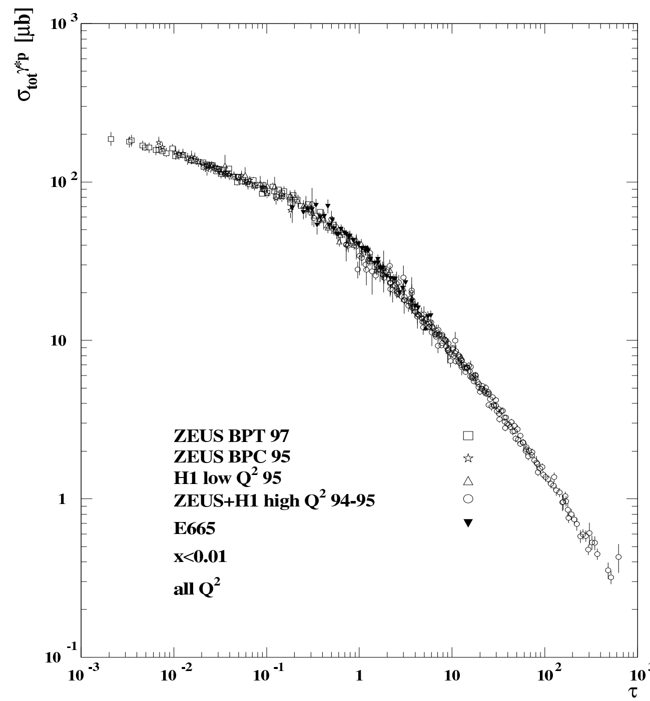
Traveling waves and saturation

- At asymptotic rapidities, $\tilde{T}(k,Y)$ depends only on the scaling variable $k^2/Q_s^2(Y)$: **Geometric Scaling**

$$\sigma^{\gamma^* p}(Y,Q) = \sigma^{\gamma^* p}(\tau)\,, \quad \tau = \frac{Q^2}{Q_s^2(Y)}$$

- The position of the front, for which $\tilde{T} = \mathcal{O}(1)$ is given by

$$\log(Q_s^2(Y)/k_0^2) = \lambda Y$$



- The expression for the tail of the scattering amplitude

$$\tilde{T}_Y(k) \overset{k \gg Q_s}{\approx} \left(\frac{k^2}{Q_s^2(Y)}\right)^{-\gamma_c} \log\left(\frac{k^2}{Q_s^2(Y)}\right) \exp\left[-\frac{\log^2\left(k^2/Q_s^2(Y)\right)}{2\bar{\alpha}\chi''(\gamma_c)Y}\right] \qquad (12)$$

- Geometric scaling window: $\log\left(k^2/Q_s^2(Y)\right) \lesssim \sqrt{2\chi''(\gamma_c)\bar{\alpha}Y}$

Stochasticity in high energy QCD (I)

- After a coarse-graining approximation [Iancu, Triantafyllopoulos 2005], one can Fourier transform the Pomeron Loop Equations : the resulting hierarchy becomes equivalent to the Langevin equation

$$\partial_Y \tilde{T}(L) = \bar{\alpha}\left[\chi(-\partial_L)\tilde{T}(L) - \tilde{T}^2(L) + \sqrt{\kappa\alpha_s^2\tilde{T}(L)}\eta(L,Y)\right], \tag{13}$$

where η is a Gaussian noise satisfying the following commutation relations:

$$\begin{aligned} \langle\eta(L,Y)\rangle &= 0, \\ \langle\eta(L_1,Y_1)\eta(L_2,Y_2)\rangle &= \frac{4}{\bar{\alpha}}\delta(L_1-L_2)\delta(Y_1-Y_2) \end{aligned} \tag{14}$$

- Stochastic equation describing an event-by-event picture
- Formally equivalent to BK equation with an additional noise term
- Each realization of the noise corresponds to a particular evolution of the target
- Diffusive approximation: same universality class as the **stochastic FKPP equation**

Stochasticity in high energy QCD (II)

- The stochastic evolution generates an ensemble of fronts which differ by the saturation momentum $\rho_s \equiv \ln\left(Q_s^2(Y)/Q_0^2\right)$
- Each individual event is a traveling wave with a speed smaller than the one predicted from BK equation

$$\lambda^* \simeq \lambda - \frac{\pi^2 \gamma_c \chi''(\gamma_c)}{2\ln^2(1/\alpha_s^2)} \quad (15)$$

- ρ_s is a random variable

$$\langle \rho_s \rangle = \ln\left(\bar{Q}_s^2(Y)/Q_0^2\right) = \lambda^* Y \quad (16)$$

and dispersion

$$\sigma^2 \equiv \langle \rho_s^2 \rangle - \langle \rho_s \rangle^2 = \bar{\alpha} D Y \quad (17)$$

- The probability distribution of ρ_s is, to a good approximation [Marquet, Soyez, Xiao 2006], a Gaussian

$$P_Y(\rho_s) \simeq \frac{1}{\sqrt{\pi\sigma^2}} \exp\left[-\frac{(\rho_s - \langle\rho_s\rangle)^2}{\sigma^2}\right] \quad (18)$$

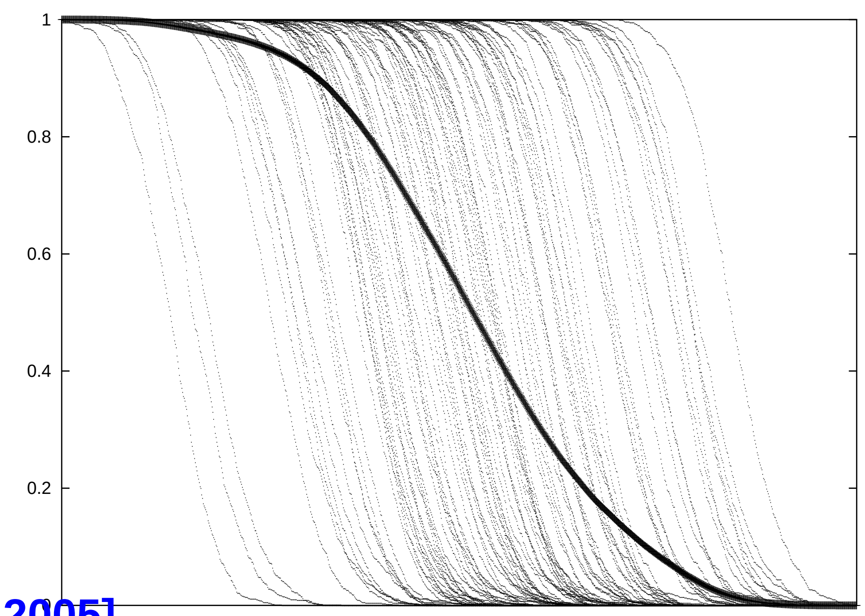
Diffusive scaling

- Single event amplitude: $T(\rho, \rho_s)$, where $\rho \equiv \ln(1/r^2 Q_0^2)$
- Average (physical) amplitude:

$$\mathcal{T}(\rho, \langle\rho_s\rangle) \equiv \langle T(\rho, \rho_s)\rangle = \int_{-\infty}^{+\infty} d\rho_s \, P_Y(\rho_s) T(\rho, \rho_s) \quad (19)$$

- At very high energies, $\sigma \gg 1$, geometric scaling is washed out and replaced by the **diffusive scaling**

$$\mathcal{T}(\rho, \langle\rho_s\rangle, Y) = \mathcal{T}\left(\frac{\rho - \langle\rho_s\rangle}{\sqrt{DY}}\right) \quad (20)$$



[Iancu, Mueller,Munier 2005]

Amplitudes and description of data

- The F_2 proton structure function can be written in terms of the $\sigma^{\gamma^* p}$ cross section

$$F_2(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}}\sigma^{\gamma^* p}(x,Q^2)$$
$$= \frac{Q^2}{4\pi^2\alpha_{em}}\int d^2r\int_0^1 dz\,\left[\left|\Psi_T(r,z;Q^2)\right|^2+\left|\Psi_L(r,z;Q^2)\right|^2\right]\sigma_{dip}^{\gamma^* p}(r,Y)$$

- If one treats the proton as an homogeneous disk of radius R_p, one can write the dipole-proton cross section in terms of the dipole-proton scattering amplitude $T(r,Y)$

$$\sigma_{dip}^{\gamma^* p}(r,Y) = 2\pi R_p^2 T(r,Y)$$

- We are left with teh parametrization of $T(r,Y)$: different approaches have already been proven successful, giving good description of the data

Dipole models: coordinate space

- GBW [Golec-Biernat, Wusthoff 1999] model

$$T^{\rm GBW}(r,x) = 1 - \exp\left[-\frac{r^2\,Q_s^2(x)}{4}\right]\;, \tag{21}$$

- IIM [Iancu, Itakura, Munier 2004] model

$$T^{\rm IIM}(r,x) = \begin{cases} 1 - \exp\left[-a\ln^2(b\,r\,Q_s(x))\right] & ,\quad r\,Q_s(x) > 2 \\ T_0\left(\frac{r\,Q_s(x)}{2}\right)^{2\left(\gamma_c + \frac{\ln(2/r\,Q_s(x))}{\kappa\,\lambda\,Y}\right)} & ,\quad r\,Q_s(x) < 2\;, \end{cases} \tag{22}$$

where the saturation scale is given by

$$Q_s^2(x) = \left(\frac{x_0}{x}\right)^{\lambda} \tag{23}$$

- Good agreement with HERA data with contributions from light and heavy quarks

Effects of fluctuations

- Recent investigation [Kozlov, Shoshi, XIang 2007] of effect of fluctuations in DIS data: GBW and IIM models as event-by-event amplitudes
- FLuctuations: average over all the events

model/parameters	χ^2	χ^2/d.o.f	x_0 ($\times 10^{-4}$)	λ	R(fm)	D
$T^{\rm GBW}$	266.22	1.74	4.11	0.285	0.594	0
$\langle T^{\rm GBW} \rangle$	173.39	1.14	0.0546	0.225	0.712	0.397

model/parameters	χ^2	χ^2/d.o.f	x_0 ($\times 10^{-4}$)	λ	R(fm)	D
$T^{\rm IIM}$	150.45	0.983	0.5379	0.252	0.709	0
$\langle T^{\rm IIM} \rangle$	122.62	0.807	0.0095	0.198	0.812	0.325

- GBW: large improvement after including fluctuations, fitting parameters reasonable
- IIM: not so large improvement, already contains geometric scaling vilations via BK-diffusion
- Diffusion coefficient: values similar to those found in numerical simulations [Soyez 2007, Iancu, De Santana Amaral, Soyez, Triantafyllopoulos 2007]

Dipole model in momentum space (I)

[De Santana Amaral, Gay Ducati, Betemps, Soyez 2007]

- $\gamma^* p$ cross section in terms of the scattering amplitude in momentum space, $\tilde{T}(k, Y)$, through the Fourier transform

$$\tilde{T}_Y(k) = \int_0^\infty \frac{dr}{r} J_0(kr)\, T_Y(r) \qquad (24)$$

- After a bit of algebra one obtains

$$F_2(x, Q^2) = \frac{Q^2 R_p^2 N_c}{4\pi^2} \int_0^\infty \frac{dk}{k} \int_0^1 dz\, |\tilde{\Psi}(k, z; Q^2)|^2 \tilde{T}_Y(k) \qquad (25)$$

  where the photon wavefunction is now expressed in momentum space

- The scattering amplitude $\tilde{T}_Y(k)$ obeys the BK equation in momentum space
- The asymptotic behaviours of the solutions to BK equation are naturally expressed in momentum space

Dipole model in momentum space (II)

- Analytical interpolation between the saturated and the traveling wave regimes predicted from BK equation

$$\tilde{T}_Y^{\rm AGBS}(k) = \left[\log\left(\frac{k}{Q_s}+\frac{Q_s}{k}\right)+1\right]\left(1-e^{-T_{\sf dil}}\right) \qquad (26)$$

where

$$T_{\sf dil} = \exp\left[-\gamma_c \log\left(\frac{k^2}{Q_s^2(Y)}\right) - \frac{L_{\sf red}^2 - \log^2(2)}{2\bar{\alpha}\chi''(\gamma_c)Y}\right]$$

and

$$L_{\sf red} = \log\left[1+\frac{k^2}{Q_s^2(Y)}\right] \qquad Q_s^2(Y) = k_0^2\, e^{\lambda Y}$$

- γ_c fixed at its LO value $\gamma_c = 0.6275$
- Good descrition of the data with light and heavy quarks included
- Decrease of the saturation exponent λ with respect to its predicted (NLO BFKL) value $\lambda \sim 0.3$
- Positive Fourier transform

Investigating fluctuations

- Evaluation of the average amplitude ($\rho \equiv \ln(k^2/k_0^2)$)

$$\left\langle \tilde{T}_Y^{\mathrm{AGBS}}(\rho,\rho_s) \right\rangle = \int_{-\infty}^{+\infty} d\rho_s\, P_Y(\rho_s) \tilde{T}_Y^{\mathrm{AGBS}}(\rho,\rho_s) \qquad (27)$$

- Some tests describing the last HERA data

Model	k_0^2 (10^{-3} GeV^2)	λ	χ_c''	R_p (GeV^{-1})	χ^2/nop	D
$\tilde{T}_Y^{\mathrm{AGBS}}$	7.155 ± 0.624	0.193 ± 0.003	2.196 ± 0.161	3.215 ± 0.065	0.988	0
$\left\langle \tilde{T}_Y^{\mathrm{AGBS}} \right\rangle$	6.431 ± 0.709	0.166 ± 0.004	1.766 ± 0.186	3.417 ± 0.088	1.097	fixed 1.3

- Effects of fluctuations at LHC energies: geometric or diffusive scaling
- Running coupling + fluctuations [Dumitru, Iancu, Portugal, Soyez, Triantafyllopoulos 2007] – are fluctuations really important?