QCD saturation predictions in momentum space: heavy quarks at HERA

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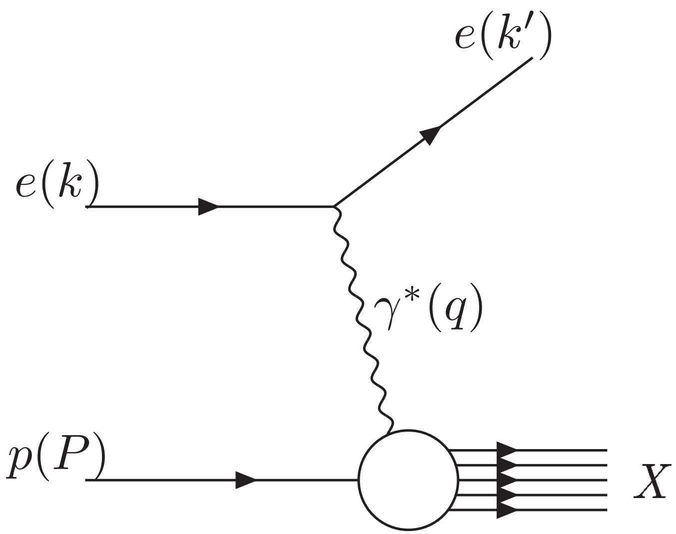
Deep Inelastic Scattering (DIS)

Kinematics and variables



- The total energy squared of the photon-nucleon system

$$s = (P + q)^2$$

- Photon virtuality

$$q^2 = (k - k') = -Q^2 < 0$$

- The Bjorken variable

$$x \equiv x_{Bj} = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{Q^2 + s}$$

- The rapidity variable

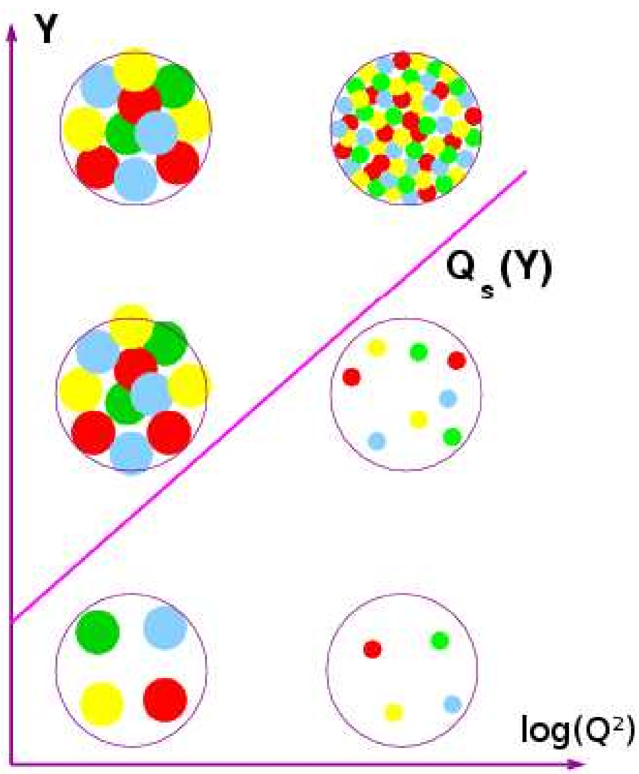
$$Y \equiv \ln(1/x)$$

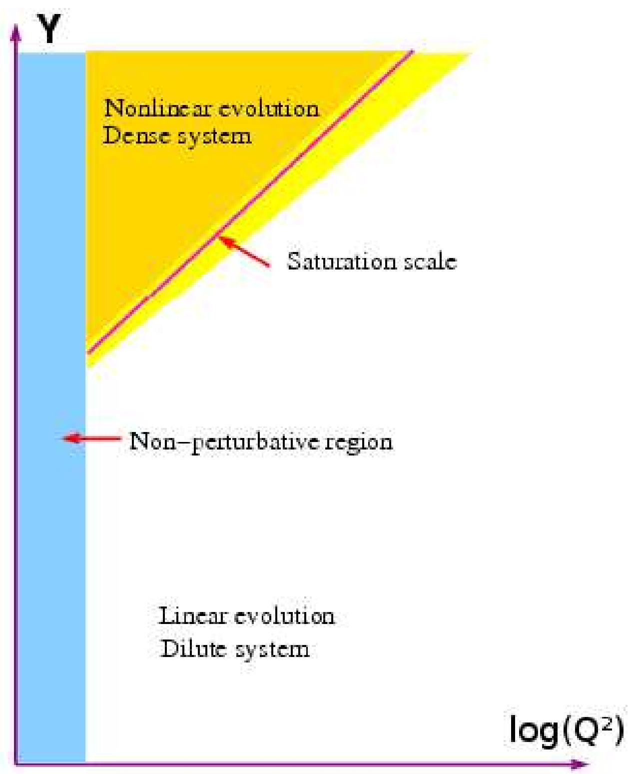
- The high energy limit:

$$s \to \infty, \quad x \approx \frac{Q^2}{s} \to 0$$

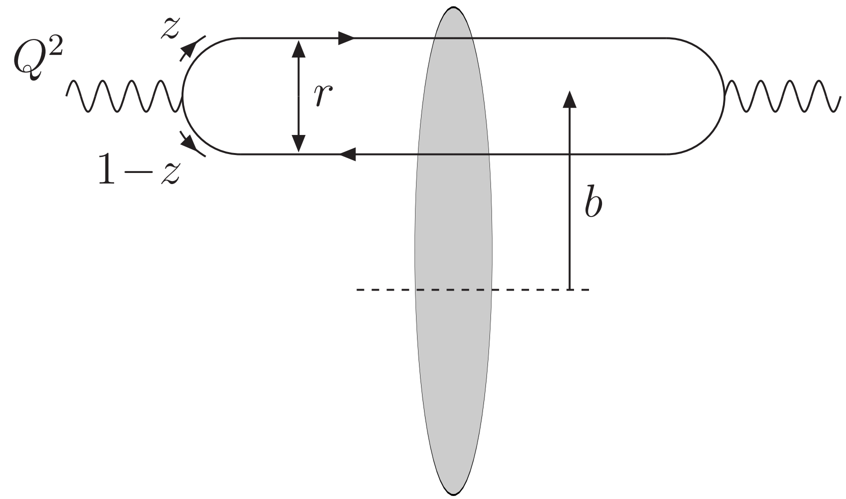
QCD at high energies

- One of the most intriguing problems in Quantum Chromodynamics is the growth with energy (and Q fixed) of the gluon density, and consequently of the cross sections, for hadronic interactions
- At very high energies gluon recombination and multiple scattering might be important to restore unitarity: **nonlinear evolution equations**





$\sigma^{\gamma^* p}$ cross section: dipole frame



- In this frame, the virtual foton (which travels fast) fluctuates into a $q\bar{q}$ pair of size r which then interacts with the proton
- The cross section is factorized like

$$\sigma_{T,L}^{\gamma^* p}(Y,Q) = \int d^2r \int_0^1 dz \, \left|\Psi_{T,L}(r,z;Q^2)\right|^2 \sigma_{dip}(r,Y), \tag{1}$$

where $\sigma_{dip}^{\gamma^* p}(Y,r)$ is the dipole-proton cross section, z is the fraction of photon's momentum carried by the quark, is the size of the dipole and b is the impact parameter and $\Psi_{T,L}(r,z;Q^2)$ stands for the transverse and longitudinal photon wavefunctions

F_2 structure function

- Assuming an independence on the impact paremeter, the dipole-proton cross section is proportional to the dipole-proton forward scattering amplitude $T(r,Y)$ through the relation

$$\sigma_{dip}(r,Y) = 2\pi R_p^2 T(r,Y)$$

where $R_p$is the proton radius.

- The proton structure function F_2 can be obtained from the $\gamma^* p$ cross section through the relation

$$\begin{aligned} F_2(x,Q^2) &= \frac{Q^2}{4\pi^2\alpha_{em}}\left[\sigma_T^{\gamma^* p}(x,Q^2) + \sigma_L^{\gamma^* p}(x,Q^2)\right] \\ &= \frac{Q^2}{4\pi^2\alpha_{em}}\sigma^{\gamma^* p}(x,Q^2) \end{aligned} \quad (2)$$

- It is possible to express the $\gamma^* p$ cross section in terms of the scattering amplitude in momentum space, $\tilde{T}(k,Y)$, through the Fourier transform

$$\tilde{T}(k,Y) = \int_0^\infty \frac{dr}{r} J_0(kr)\, T(r,Y) \quad (3)$$

F_2 in momentum space

- The proton structure function is related to $\tilde{T}(k,Y)$ through

$$F_2(x,Q^2) = \frac{Q^2 R_p^2 N_c}{4\pi^2} \int_0^\infty \frac{dk}{k} \int_0^1 dz\, |\tilde{\Psi}(k,z;Q^2)|^2 \tilde{T}(k,Y) \tag{4}$$

  where the photon wavefunction is now expressed in momentum space [1]

- The scattering amplitude $\tilde{T}(k,Y)$ obeys the The Balitsky-Kovchegov (BK) nonlinear equation in momentum space ($\bar{\alpha} = \alpha_s N_c/\pi$)

$$\partial_Y \tilde{T} = \bar{\alpha}\chi(-\partial_L)\tilde{T} - \bar{\alpha}\tilde{T}^2 \tag{5}$$

  where

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma) \tag{6}$$

  is the characteristic function of the leading-order (LO) Balitsky-Fadin-Kuraev-Lipatov (BFKL) kernel and $L = \log(k^2/k_0^2)$

- The BK equation lies [2] in the equivalence class of the Fisher-Kolmogorov-Petrovsky-Piscounov (F-KPP) equation, which admits the the ***traveling wave solutions***

Scattering Amplitude $\tilde{T}(k)$

- The asymptotic behaviours of the solutions to BK equation are naturally expressed in momentum space
- At asymptotic rapidities, the amplitude $\tilde{T}(k, Y)$, instead of depending separately on k and Y, depends only on the scaling variable $k^2/Q_s^2(Y)$, where we have introduced the *saturation scale* $Q_s^2(Y) = k_0^2 \exp(\lambda Y)$, measuring the position of the wavefront
- The expression for the tail of the scattering amplitude

$$\tilde{T}\,(k, Y) \overset{k \gg Q_s}{\approx} \left(\frac{k^2}{Q_s^2(Y)}\right)^{-\gamma_c} \log\left(\frac{k^2}{Q_s^2(Y)}\right) \exp\left[-\frac{\log^2\left(k^2/Q_s^2(Y)\right)}{2\bar{\alpha}\chi''(\gamma_c)Y}\right] \tag{7}$$

- In the infrared domain, one can show that the amplitude behaves like

$$\tilde{T}\left(\frac{k}{Q_s(Y)}, Y\right) \overset{k \ll Q_s}{=} c - \log\left(\frac{k}{Q_s(Y)}\right) \tag{8}$$

where c is an unfixed constant

The model

- The description of the transition to the saturation region is performed by an analytic interpolation between both asymptotic behaviours [1]
- The expression for the amplitude is unitarised – up to a logarithmic factor – by an eikonal *i.e.* $T_{\text{unit}} = 1 - \exp(-T_{\text{dil}})$
- The following choice gives good results:

$$\tilde{T}(k,Y) = \left[\log\left(\frac{k}{Q_s} + \frac{Q_s}{k}\right) + 1\right]\left(1 - e^{-T_{\text{dil}}}\right) \tag{9}$$

where

$$T_{\text{dil}} = \exp\left[-\gamma_c \log\left(\frac{k^2}{Q_s^2(Y)}\right) - \frac{L_{\text{red}}^2 - \log^2(2)}{2\bar{\alpha}\chi''(\gamma_c)Y}\right] \tag{10}$$

and

$$L_{\text{red}} = \log\left[1 + \frac{k^2}{Q_s^2(Y)}\right] \quad \text{and} \quad Q_s^2(Y) = k_0^2\, e^{\lambda Y} \tag{11}$$

- The equations above determine the model for the scattering amplitude, to be inserted into the expression for the F_2 structure function

Parameters and dataset

- The critical slope γ_c and the saturation exponent λ are obtained from the knowledge of the BFKL kernel alone:

$$\lambda = \min_{\gamma} \bar{\alpha} \frac{\chi(\gamma)}{\gamma} = \bar{\alpha} \frac{\chi(\gamma_c)}{\gamma_c} = \bar{\alpha} \chi'(\gamma_c)$$

- For the LO BFKL kernel, one finds $\gamma_c = 0.6275...$, and $\lambda \approx 0.9$
- Our analysis is restricted to the following kinematic range:

$$\begin{cases} x \leq 0.01, \\ 0.045 \leq Q^2 \leq 150 \text{ GeV}^2 \end{cases}$$

- In the original work [1] $\gamma_c = 0.6275$ and $\bar{\alpha} = 0.2$ kept fixed, while λ, χ''_c , k_0^2 and R_p are free parameters
- The parameters obtained from the fit to the experimental data for F_2:

Masses	k_0^2 (10^{-3} GeV^2)	λ	χ''_c	R_p (GeV^{-1})	χ^2/nop
$m_q = 50$ MeV, $m_c = 50$ MeV	3.782 ± 0.293	0.213 ± 0.004	4.691 ± 0.221	2.770 ± 0.045	0.960
$\mathbf{m_q = 50}$ MeV, $\mathbf{m_c = 1.3}$ GeV	**$\mathbf{7.155 \pm 0.624}$**	**$\mathbf{0.193 \pm 0.003}$**	**$\mathbf{2.196 \pm 0.161}$**	**$\mathbf{3.215 \pm 0.065}$**	**0.988**
$m_q = 140$ MeV, $m_c = 1.3$ GeV	3.917 ± 0.577	0.161 ± 0.005	2.960 ± 0.279	4.142 ± 0.167	1.071

Improved IIM model

- The IIM model has been recently improved [3] by fully including heavy quarks contribution (both charm and bottom)
- Parameters:
  - The saturation scale $Q_s^2(Y) = \left(\frac{x_0}{x}\right)^\lambda$ GeV^2
  - x_0 free, R_p free, $T_0 = T(r = 1/Q_s) = 0.7$ fixed
  - λ free: LO BFKL predicts $\lambda = \bar{\alpha}_s \chi_c' \approx 0.9$ and NLO BFKL analysis gives $\lambda \sim 0.3$ [7]
  - The parameter $\kappa = \chi_c''/\chi_c'$ was set from the LO BFKL kernel, which gives $\kappa \approx 9.9$ [5]
  - Allowing γ_c to vary, one recovers a saturation scale similar to that found with only light quarks
  - A Good fit is obtained
  - In addition, the value for γ_c coming out of the fit is rather close to what one expects from NLO BFKL ($\gamma_c \gtrsim 0.7$)

		γ_c	λ	x_0 (10^{-4})	R_p (GeV^{-1})	χ^2/n.o.p.
light+heavy quarks	γ_c fixed	0.6275	0.1800 $\pm$ 0.0026	0.0028 $\pm$ 0.0003	3.819 $\pm$ 0.017	1.116
	γ_c free	**0.7376 $\pm$ 0.0094**	**0.2197 $\pm$ 0.0042**	**0.1632 $\pm$ 0.0471**	**3.344 $\pm$ 0.041**	**0.900**

New analysis: momentum space

- In order to fully include the heavy quark effects in the original model, we add the bottom contribution with $m_b = 4.5$ GeV
- To search for better results for HERA, we used the modified Bjorken

$$x_{eff} = x\left(1 + \frac{4m_q^2}{Q^2}\right) \quad (12)$$

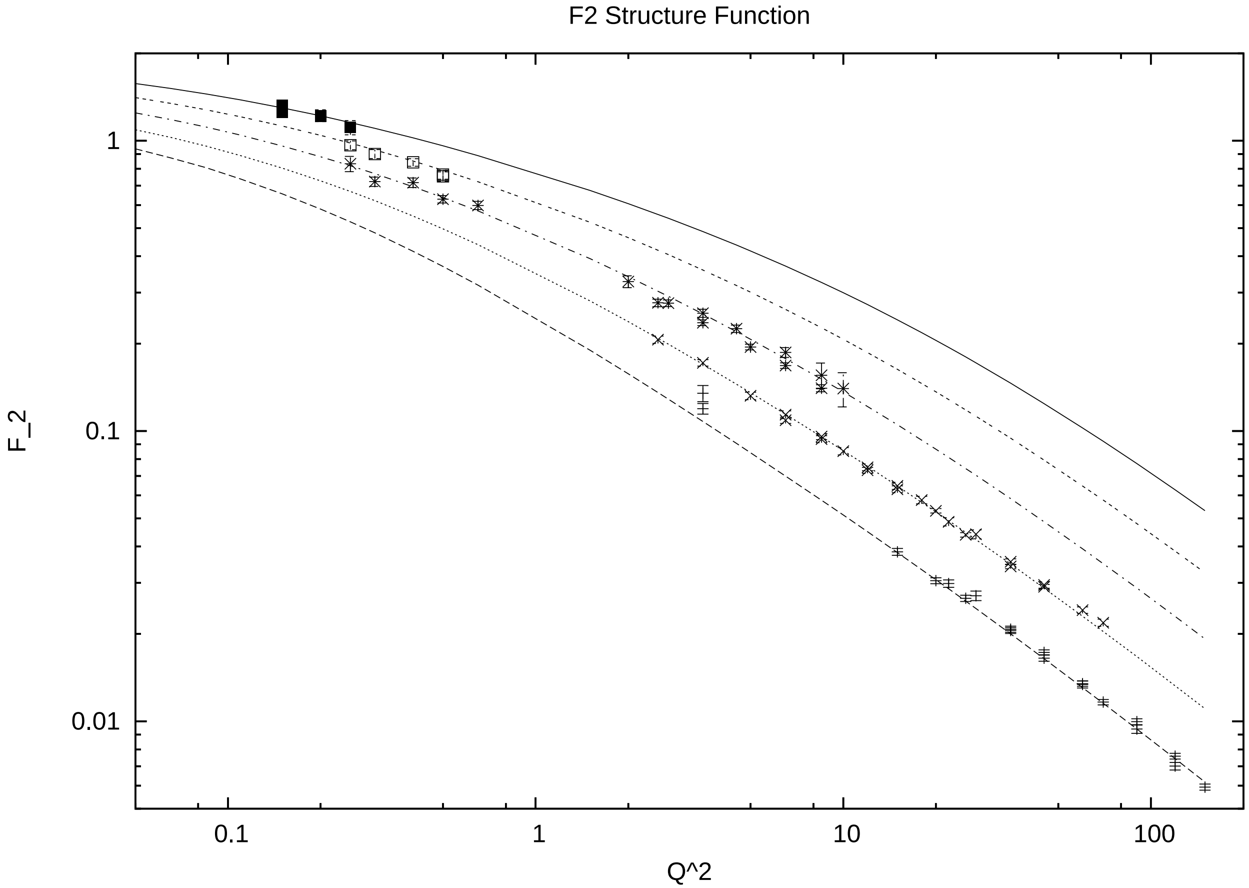
  to correctly explain the threshold for heavy-quark production
- We perform the following analysis:

  [A] To compare with the original model [1] we fix $\gamma_c = 0.6275$ and $\bar{\alpha} = 0.2$ and allowed λ, χ_c'', k_0^2 and R_p to vary

  [B] Unlike in [4], make γ_c freely to vary not result in good parameters. In a first test, we fix $\gamma_c = 0.7$ and allow λ, χ_c'', k_0^2 and R_p to vary

Results A

Values of rapitity (from bottom to top): $Y = 0,\ 2,\ 4,\ 6,\ 8$



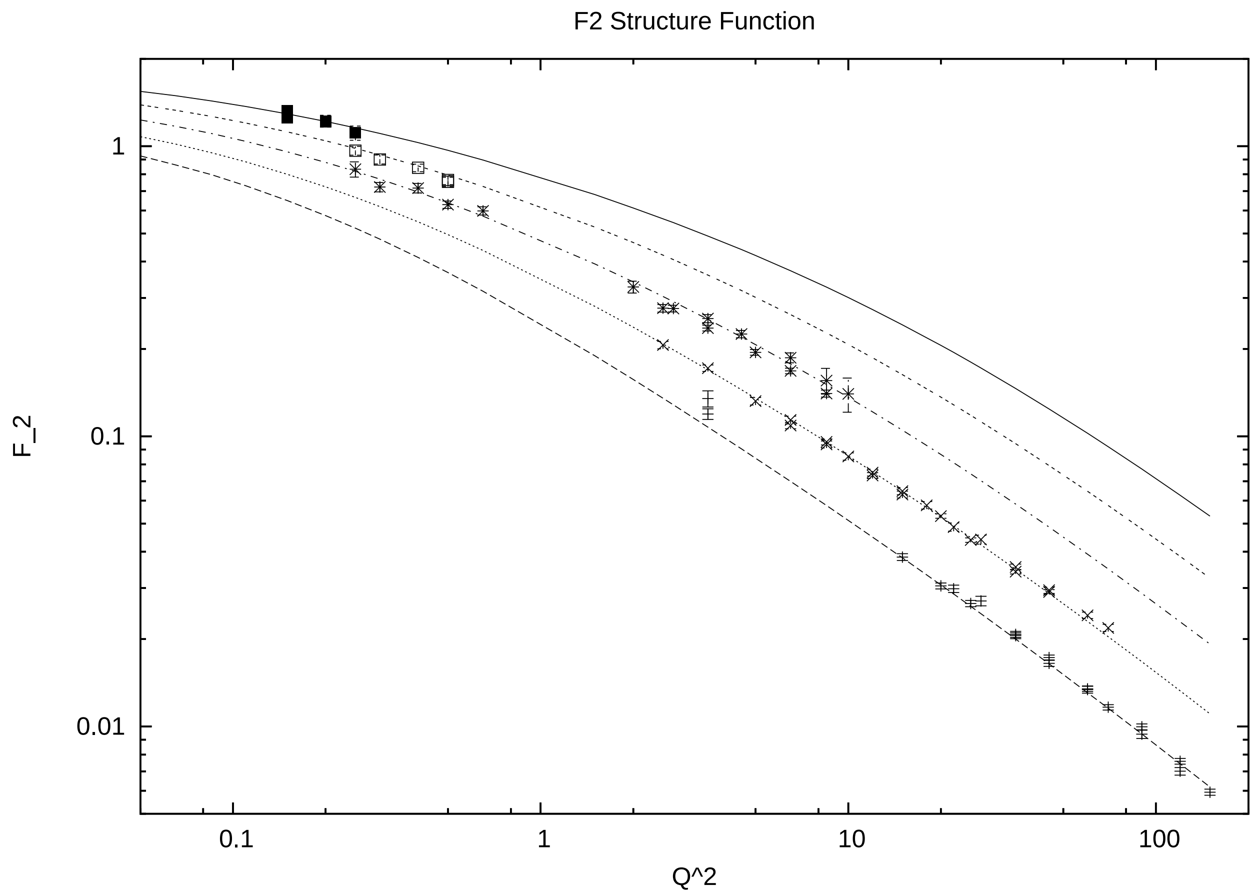
H1 [EPJC 21, 2002] and ZEUS [EPJC 12, 2000; EPJC 21, 2001]

Parameters: $\gamma_c = 0.6275$ fixed

k_0^2 (10^{-3} GeV^2)	λ	χ_c''	R_p (GeV^{-1})	$\chi^2/n.o.p.$
17.625 ± 0.892	0.242 ± 0.016	0.948 ± 0.073	2.793 ± 0.038	1.276

Results B

Values of rapitity (from bottom to top): $Y = 0,\ 2,\ 4,\ 6,\ 8$



H1 [EPJC 21, 2002] and ZEUS [EPJC 12, 2000; EPJC 21, 2001]

Parameters: $\gamma_c = 0.7$ fixed

k_0^2 (10^{-3} GeV^2)	λ	χ_c''	R_p (GeV^{-1})	$\chi^2/n.o.p.$
18.229 ± 0.8162	0.249 ± 0.013	0.983 ± 0.072	2.720 ± 0.030	1.299

Discussion

- We have obtained good values for the parameters:
  - In particular, one can see that the value of the saturation exponent is not so small as it was obtained in [1] and it is similar to the one obtained in [4]
- However the $\chi^2/n.o.p.$ is still poor and is enhanced when including γ_c expected for NLO BFKL
- Making γ_c free to vary does not result in a good fit like occurs in [4]
- Also, for a near future: to investigate R_{pA} at RHIC and fluctuations effects

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