Integrable models in atomtronics: Atomtronic toolbox

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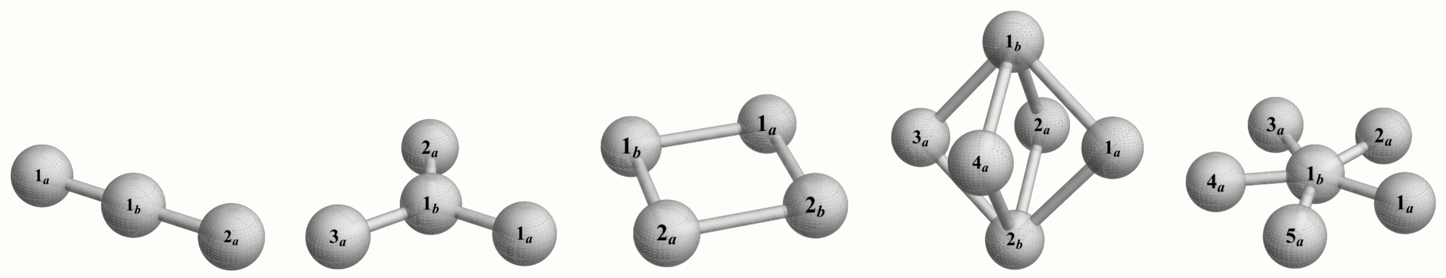
Integrable Multi-Well Hamiltonian [1]

A bipartite model obtained through the Quantum Inverse Scattering Method

$$H_{n,m} = U(N_A - N_B)^2 + \mu(N_A - N_B) + \Sigma_{i=1}^{n}\Sigma_{j=1}^{m} t_{i,j}(a_i b_j^\dagger + a_i^\dagger b_j) \quad (1)$$

Some multi-well geometric models (N_A, N_B):

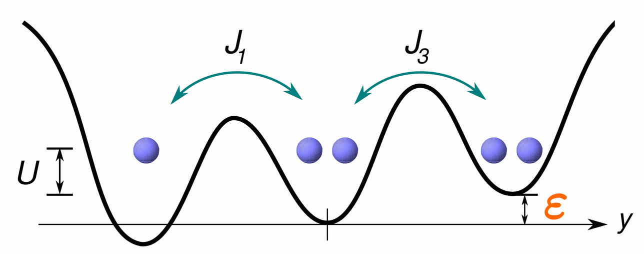


Switching Device: 2+1 modes [2, 3]

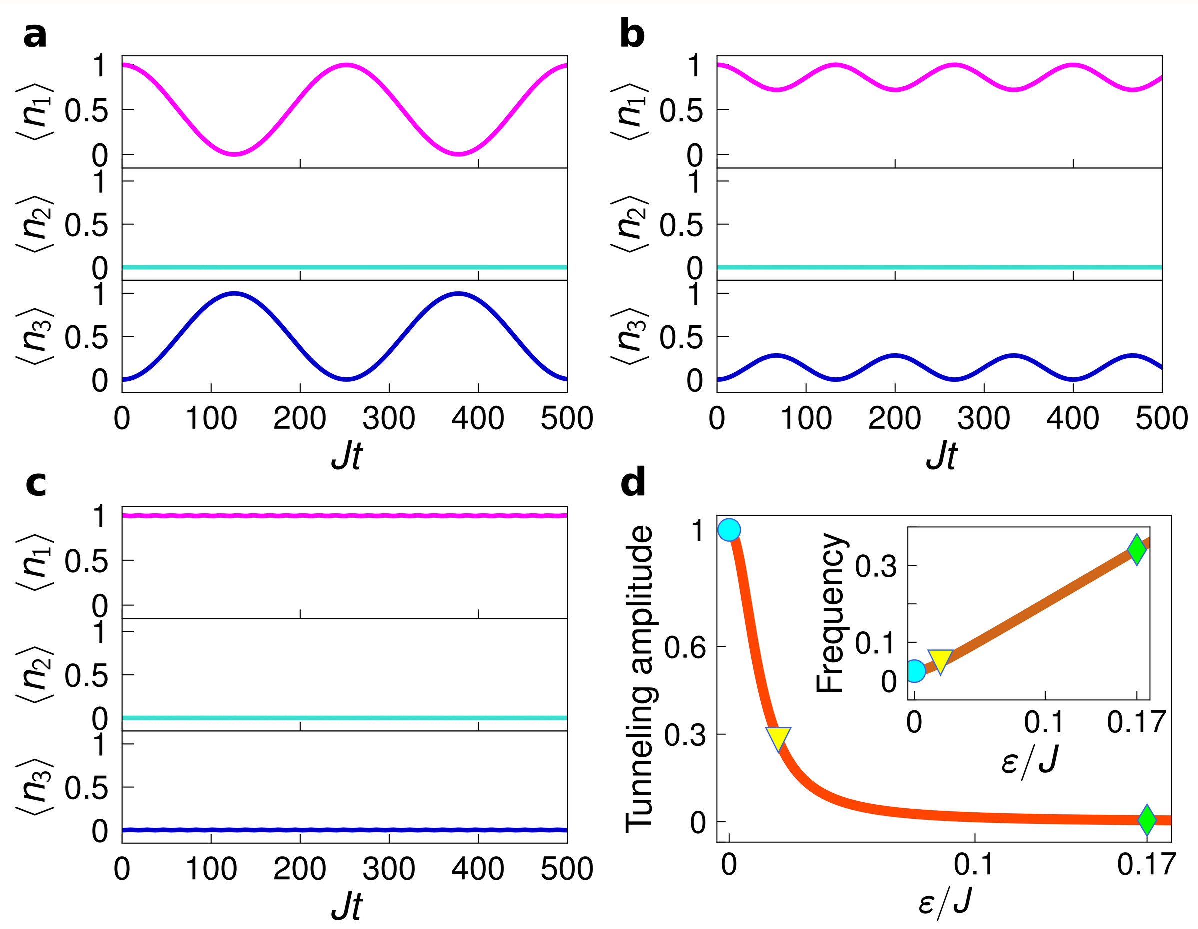
$$H_0 = U(N_1 - N_2 + N_3)^2 + J_1(a_1^\dagger a_2 + a_1 a_2^\dagger) + J_3(a_2^\dagger a_3 + a_2 a_3^\dagger)$$

$[H, N] = [H, Q] = [N, Q] = 0, \quad Q = J_1^2 N_3 + J_3^2 N_1 - J_1 J_3 (a_1^\dagger a_3 + a_3^\dagger a_1)$

The additional conserved quantity Q provides an H_{eff} which, in the resonant regime, yield analytical frequency and amplitude equations.



By applying an external field, $\varepsilon(N_3 - N_1)$, the second-order tunneling amplitude between wells 1 and 3 can be controlled while N_2 remains constant.



Abstract: The precise control of quantum systems will play a major role in the realization of atomtronic devices. Here we study models of dipolar bosons confined to three and four wells. The analysis considers both integrable and non-integrable regimes within the models. Through variation of the external field, we demonstrate how the triple well system can be controlled between various "switched-on" and "switched-off" configurations and how the four well system can be controlled to encode a phase into a NOON state

NOON states: 2+2 modes [4, 5]

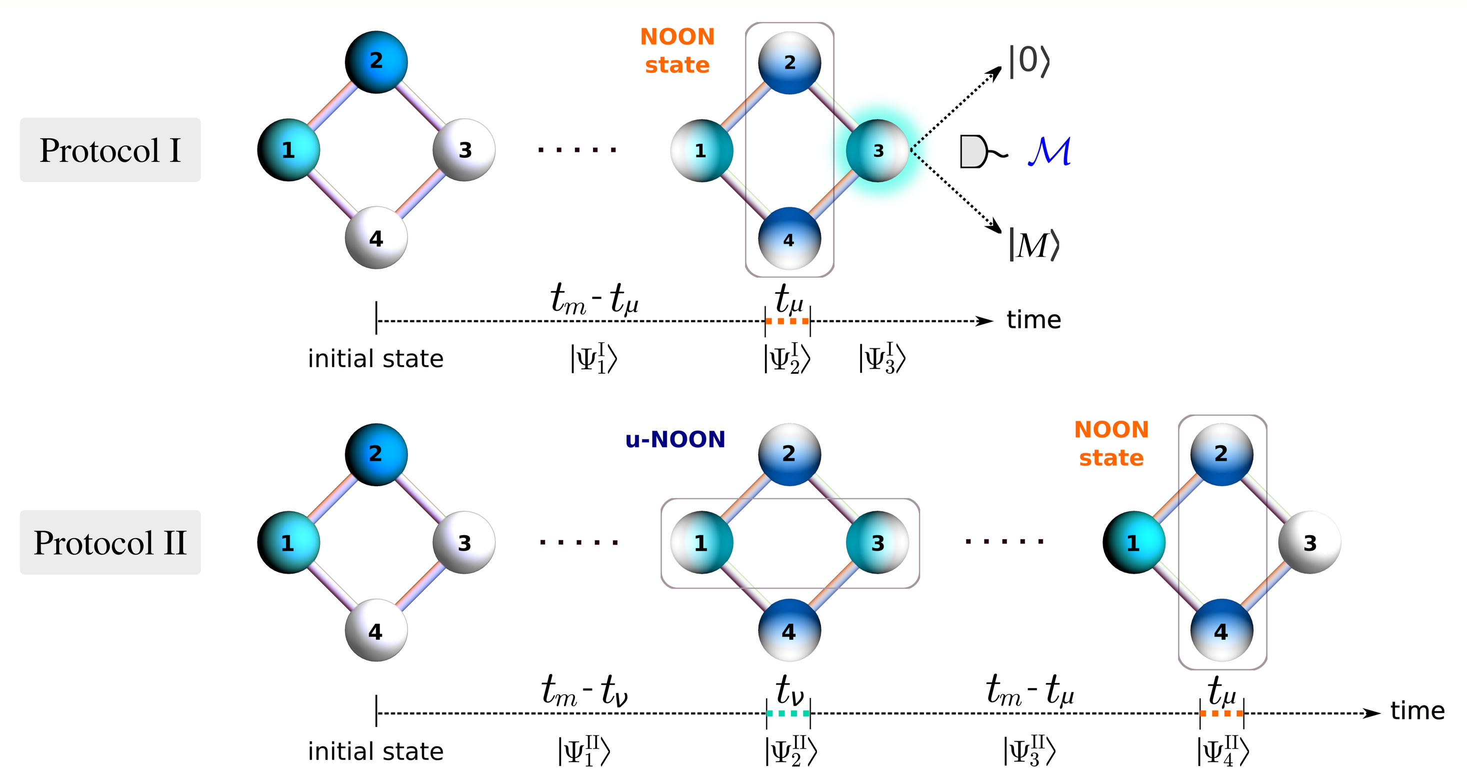
Integrable Extended Bose-Hubbard

$$H = \frac{U_0}{2}\sum_{i=1}^{4} N_i(N_i - 1) + \sum_{i=1}^{4}\sum_{j=1, j\neq i}^{4} \frac{U_{ij}}{2} N_i N_j - \frac{J}{2}\sum_{i=1} (a_i^\dagger a_{i+1} + a_{i+1}^\dagger a_i). \quad (2)$$

where $a_j, a_j^\dagger : j = 1, 2, 3, 4$. Eq.(1) could be derived from EBHM as long as it complies with the integrability condition $U_0 = U_{13} = U_{24}$, which provides four conserved quantities, $[H, N] = [H, Q_k] = [N, Q_k] = 0$, $k = 1, 2$, as many as number modes.

NOON Protocol

Below, we describe two protocols that enable the generation of NOON states. For Protocol I the outcomes are probabilistic while Protocol II are deterministic.



Protocol I

Considering $\mathcal{U}(t, \mu, \nu) = \exp\left(-\frac{it}{\hbar}[H + \mu(N_2 - N_4) + \nu(N_1 - N_3)]\right)$,
here we employ breaking of integrability through an applied field $\mu(N_4 - N_2)$ to subsystem $B = \{2, 4\}$ and a measurement process $\mathcal{M}$:

(i) $|\Psi_1^I\rangle = \mathcal{U}(t_m - t_\mu, 0, 0)\,|\Psi_0\rangle$;
(ii) $|\Psi_2^I\rangle = \mathcal{U}(t_\mu, \mu, 0)\,|\Psi_1^I\rangle$;
(iii) $|\Psi_3^I\rangle = \mathcal{M}\,|\Psi_2^I\rangle$,

where $t_m = \hbar\pi/(2\Omega)$ and $\mathcal{M} = 0, M$ represents a projective measurement of the number of bosons at site 3 which heralds a high-fidelity NOON state in subs. B.

In an idealized limit, with $\beta = (-1)^{(N+1)/2}$ and the initial state $|\Psi_0\rangle = |M, P, 0, 0\rangle$,

$$|\Psi_3^I\rangle = \begin{cases} \frac{1}{\sqrt{2}}\left(\beta\,|M, P, 0, 0\rangle + e^{iP\theta}\,|M, 0, 0, P\rangle\right), & r = 0, \\ \frac{1}{\sqrt{2}}\left(|0, P, M, 0\rangle - \beta e^{iP\theta}\,|0, 0, M, P\rangle\right), & r = M, \end{cases} \quad (3)$$

These states are recognized as products of a NOON state for subsystem B with Fock basis states for subsystem $A = \{1, 3\}$.

Protocol II

Employing the same initial state $|\Psi_0\rangle$, the following sequence of steps are implemented to arrive at a NOON state in subsystem B and deterministic state in A:

(i) $|\Psi_1^{II}\rangle = \mathcal{U}(t_m - t_\nu, 0, 0)\,|\Psi_0\rangle$;
(ii) $|\Psi_2^{II}\rangle = \mathcal{U}(t_\nu, 0, \nu)\,|\Psi_1^{II}\rangle$;
(iii) $|\Psi_3^{II}\rangle = \mathcal{U}(t_m - t_\mu, 0, 0)\,|\Psi_2^{II}\rangle$;
(iv) $|\Psi_4^{II}\rangle = \mathcal{U}(t_\mu, \mu, 0)\,|\Psi_3^{II}\rangle$.

where ν represent the breaking of integrability through an applied field $\nu(N_3 - N_1)$ to subsystem $A = \{1, 3\}$. In an idealized limit, with $\Upsilon = \beta\exp(i(P\theta - \pi/2))$,

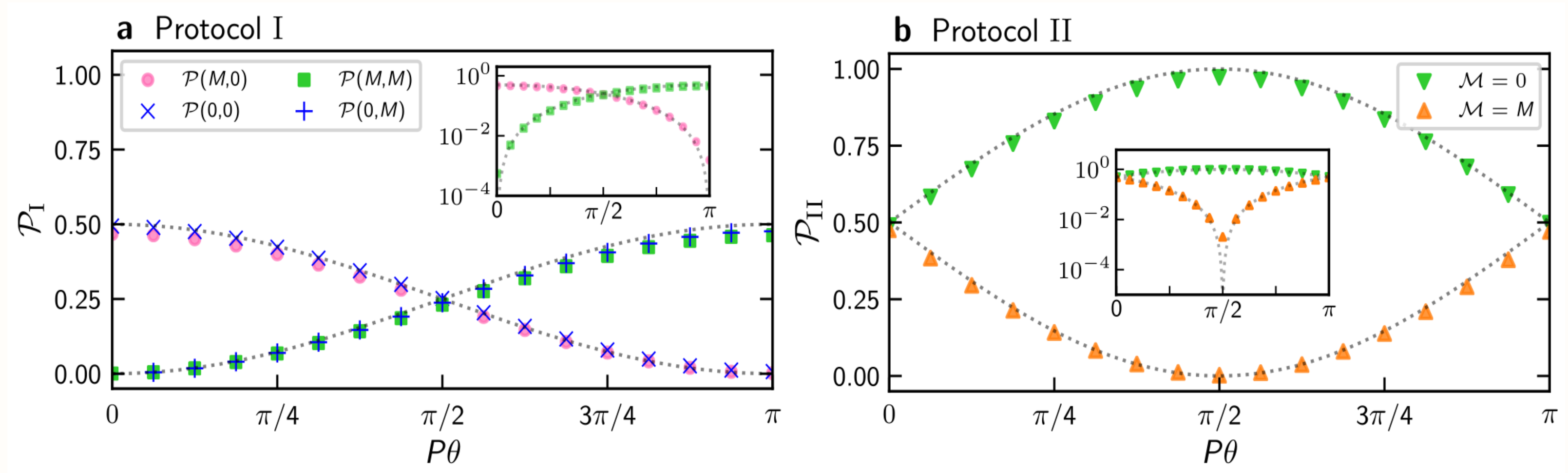
$$|\Psi_4^{II}\rangle = \frac{1}{\sqrt{2}}\Big(|M, P, 0, 0\rangle + \Upsilon\,|M, 0, 0, P\rangle\Big) \quad (4)$$

Fidelity and Probability

The fidelities are computed for $P\theta$ ranging from 0 to π, achieved by varying t_μ.

$$F_I = |\langle\Psi_3^I|\Phi_3^I\rangle| > 0.9 \qquad F_{II} = |\langle\Psi_4^{II}|\Phi_4^{II}\rangle| > 0.9$$

where $|\Psi\rangle$ denotes the analytical states and $|\Phi\rangle$ the numerically state obtained by EBHM (2) time evolution. For physically realistic settings, with M=4 and P=11, the NOON state fidelities for $P\theta \in [0, \pi]$ are greater than 0.9 with probabilities varying as:



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[5] Daniel S. Grun, Karin Wittmann W., et al. arXiv preprint arXiv:2102.02944 (2021).

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