

Reply to “Incommensurate vortices and phase transitions in two-dimensional XY models with interaction having auxiliary minima” by S. E. Korshunov

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We present a rigorous proof and extensive numerical simulations showing the existence of a transition between the paramagnetic and nematic phases, in a class of generalized XY models. This confirms the topology of the phase diagram calculated by Poderoso et al. [PRL 106(2011)067202]. The results disprove the heuristic argument presented by Korshunov in arXiv:1207.2349v1, against the existence of the generalized-nematic phase in a model with $q = 3$.

In a recent Letter [1], we have studied the phase diagram of a generalized XY model with Hamiltonian $H = -\sum_{\langle ij \rangle} [\Delta \cos(\theta_i - \theta_j) + (1 - \Delta) \cos(q\theta_i - q\theta_j)]$, where $q > 1$ is an integer and $0 \leq \Delta \leq 1$. Using Monte Carlo (MC) simulations, we showed that for $q = 3$ and $0 < \Delta \lesssim 0.4$, the model exhibits — depending on the temperature — three possible phases: paramagnetic (P), generalized-nematic (N), and ferromagnetic (UF). The phase transition between P and N was found to be in the Kosterlitz-Thouless (KT) universality class, while the transition between N and UF was found to belong to the 3-state Potts universality class. In his Comment on our work [2], Dr. Korshunov argued that the N phase cannot exist for $q > 2$, and that there should only be one “genuine phase transition” between the P and UF phases. We will now show that the argument of Ref. [2] is incorrect.

Let us first consider $\Delta = 0$. In this case the Hamiltonian becomes purely q -nematic. Changing variables in the partition function, $q\theta_i \rightarrow \theta_i$, shows that the model is isomorphic to the usual XY model, but with the low temperature phase N, instead of UF. The phase transition from P to N will, therefore, occur at $T_0 \simeq 0.893$, the same temperature as for the standard XY model and will belong to the KT universality class. This, clearly demonstrates that the N phase exists for $\Delta = 0$. Using Ginibre’s inequality [3] it is possible to show that the P to N transition will also extend to finite Δ [4]. Furthermore, Ginibre’s inequality allows one to derive a rigorous lower bound [4] on the transition temperature between P and N phases, $T_{KT}(\Delta) \geq (1 - \Delta)T_0$. Since at very low temperature the system must be in UF phase, this proves the existence of P, N and UF phases for small, but finite values of Δ , contradicting the heuristic argument of Ref. [2].

To precisely delimit the location of all three phases for the model with $q = 3$, we consider a specific example, $\Delta = 1/4$. For this Δ , and using finite size scaling (FSS), in Ref. [1] we have calculated the critical temperature for the N-UF transition to be $T_{Potts} \simeq 0.365$, which was found to belong to the 3-state Potts universality class. The order parameter m_1 (magnetization) shows clearly this transition, see Fig. 1. On the other hand, at T_{KT} , the nematic order parameter m_3 shows the transition between N and P phases. At the transition tempera-

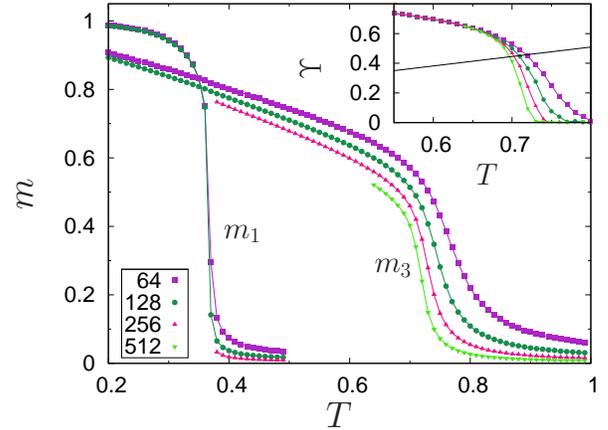


FIG. 1: Order parameters m_1 and m_3 (see Ref. [1]) for several system sizes L showing phase transitions at $T_{Potts} \simeq 0.365$ and $T_{KT} \simeq 0.68$. Inset: Helicity modulus Υ versus T . The crossing with the line $2T/\pi$ at $T_{KT}(L)$, when extrapolated to $L \rightarrow \infty$, gives $T_{KT} \simeq 0.68$.

ture, m_3 decreases with L as $m_3(T_{KT}) \sim L^{-\beta/\nu}$, with $\beta/\nu \simeq 0.117$. The exponent is very close to the theoretical value expected for the KT transition, $1/8 = 0.125$. To further verify the “genuineness” of this transition, we calculated the helicity modulus Υ [5], shown in the inset of Fig. 1 as a function of temperature, for several system sizes. The helicity modulus crosses the straight line $2T/\pi$ [6] at $T_{KT}(L)$ and, extrapolating to $L \rightarrow \infty$, we obtain $T_{KT} \simeq 0.68$, slightly above the lower bound provided by Ginibre’s inequality.

In Fig. 2 we present the susceptibility χ_3 as a function of T for different system sizes. The phase transition is very clear from the divergence of the susceptibility at T_{KT} , as $L \rightarrow \infty$. For a KT phase transition, the FSS predicts that $\chi_3(T_{KT}) \sim L^{1.75}$, while our simulations find $L^{-1.766}$. Finally, if we plot $\chi_3 L^{\eta-2}$, with the KT $\eta = 1/4$, vs. the Binder cumulant, all the susceptibilities for different system sizes should collapse onto a universal curve [8]. This is precisely what is found in our MC simulations, see inset of Fig. 2.

Ref. [2] also questions the transition between the phases F_1 and UF, in the model with $q = 8$, and the ab-

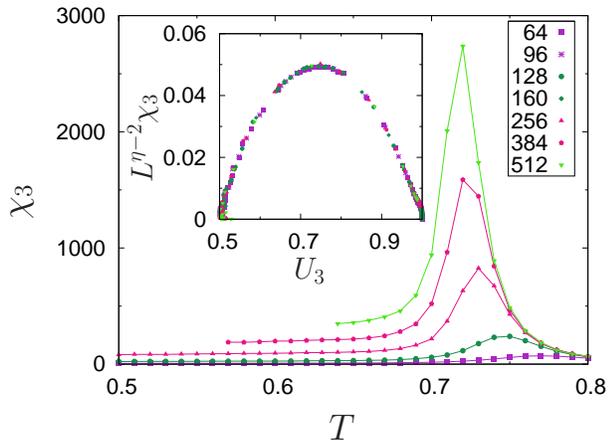


FIG. 2: The susceptibility χ_3 associated with m_3 near the KT transition for various system sizes. Inset: rescaled susceptibility *versus* the Binder cumulant, $U_3 = \langle m_3^2 \rangle^2 / \langle m_3^4 \rangle$, showing a perfect collapse with the KT exponent $\eta = 1/4$. A similar collapse is also obtained for the magnetization [7].

sense of any “qualitative” difference between these two phases. This, however, is clearly not an issue, as is exemplified by the usual liquid-gas phase transition — the difference between liquid and gas being only “quantitative”. The Fig. 6 of Ref. [1] shows clearly the transition between F_1 and UF, which again belongs to the KT universality class.

In conclusion, we have presented a rigorous proof, as well as, numerical evidence for the existence of a transition between the N and P phases belonging to the KT universality class, at odds with the heuristic argument of Ref. [2].

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